Topic 7: Circular Motion

Source: Conceptual Physics textbook, lab book and CPO textbook and lab book

Types of Materials: Textbooks, lab books, worksheets, demonstrations/activities, videos and websites, and good stories

Building on: Circular motion uses vectors, kinematics (velocity and acceleration) and dynamics (Newton’s 2nd law). Energy and momentum can also be considered when considering circular motion.

Leading to: Circular motion can be used in a simplistic way to study satellite motion as well as the e- orbiting around the nuclei in an atom. High-energy physics uses circular motion for accelerators that orbit charged particles in circles such as the synchrotron at Fermi National Accelerator Laboratory in Batavia, Illinois.

Links to Physics: As stated above, the motion of any object moving in a circle is understood by the four force laws of nature; gravity, electricity and magnetism, weak, and strong force. Examples include satellite motion, e- orbiting around the nuclei, and charged particles orbiting in circular accelerators.

Links to Chemistry: Circular motion at an introductory level in chemistry is explained as electrons moving in circles about the nucleus. Quantum mechanics then replaces the circular motion idea, but circular motion does put a visual image in a young person’s mind.

Links to Biology: Circular motion has no direct effect on biology.

Materials:
(a) Hewitt
       Lab 30 – Going in Circles

(b) Hsu*

(c) My Lab
       Conceptual Circular Motion

(d) Worksheet
       Circular Motion

(e) Demonstrations
       Tangential Velocity in Circular Motion
(f) Websites and Videos
   1. GPS Lab Sim Activity (Shockwave)
   2. ESPN SportsFigures “Walking on Water” (Water Skiing)
   3. The G-Force Experience
      Funderstanding & Question 3-D Coaster Lab Sim (Shockwave/Java)

(g) Good Stories
   Circular Motion and the “Funky Chicken”
Topic 7: Lab – Circular Motion, Conceptual 2

Purpose:
Spin a mass and:
(a) Discover what physical factors affect the inward centripetal force keeping the mass “in orbit.”
(b) Determine if those factors directly or indirectly (inversely) affect that inward force.

Procedure:
(a) How does velocity affect centripetal force?

1. Securely attach one end of a 1.5 m strong, thin, braided string to a #5 one-hole rubber stopper.
2. Cut a 15 cm long by 1/4” inner diameter hollow glass tube and fire polish each end to make them smooth.
3. Duct tape the glass tube to a meter stick leaving 1 cm above the 0 cm end of the meter stick.
4. Thread the string through the top of the glass tube and create a permanent loop about 2 cm in diameter.
5. Attach a spring scale calibrated in Newton’s (0-5 N or 0-20 N depending on the trial) to the loop.
6. Hold this apparatus as shown in the sketch with your most coordinated hand at the top of the meter stick and glass tube.
7. Hold tight and rotate in small horizontal circles at a constant rate.
8. Now adjust that rate using a radius of 60 cm to 10 revolutions in 5 seconds (2 rev/s). Record your centripetal force and velocity in the table.
9. Repeat at 60 cm, but do 15 revolutions in 5 seconds (3 rev/s). Record.
10. Now repeat at 60 cm, but do 20 revolutions in 5 seconds (4 rev/s). Record.
11. Sketch a force-velocity graph and from the shape of the graph determine the math relationship between force and velocity.
Data:

<table>
<thead>
<tr>
<th>Frequency (representing velocity) (Rev/s)</th>
<th>Centripetal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td></td>
</tr>
</tbody>
</table>

![Force-Velocity Graph](image)

Analysis:
Centripetal force is affected by velocity in the following way: ____________________________
(b) How does mass affect centripetal force?

1. Horizontally spin one #5 single hole stopper at a radius of 60 cm at 10 revolutions in 5 seconds (2 rev/s). Record the centripetal force below.
2. Remove the string from the glass tube and thread a second #5 stopper onto the string. Again, at 60 cm radius and 2 rev/s, spin the double mass. Make sure to keep the radius at 60 cm (think). Record.
3. Repeat step 11, but with three stoppers.

Data:

<table>
<thead>
<tr>
<th>Mass (Stopper)</th>
<th>Centripetal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 1</td>
<td></td>
</tr>
<tr>
<td>(b) 2</td>
<td></td>
</tr>
<tr>
<td>(c) 3</td>
<td></td>
</tr>
</tbody>
</table>

Force-Mass Graph

\[ F \]

\[ M \]
Analysis:
How does the mass affect the centripetal force needed to maintain circular motion?

(c) How does the radius affect centripetal force?

1. Use one #5 stopper. Rotate the one-hole stopper at a radius of 60 cm at 10 revolutions in 5 s (2 rev/s). Record below.
2. Change the radius to 30 cm, but keep the same velocity. Thus, at 30 cm radius you will have to rotate at 4 rev/s (if the circumference halves then so must the time to revolve). Record the values below.
3. Change the radius to 20 cm, but keep the same velocity. Thus, at 20 cm radius, you will have to rotate at 6 rev/s (if the circumference is 1/3 as much, then the time must be 1/3 as much). Record the values below.

Data:

<table>
<thead>
<tr>
<th>Radius (M)</th>
<th>Centripetal Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td></td>
</tr>
</tbody>
</table>
Analysis:
How does the radius affect the centripetal force needed to maintain circular orbit?

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

Summary:
Combine all three analyses from (a), (b), and (c) in a mathematical form to show how velocity, mass and radius affect centripetal force.
Topic 7: Lab – Circular Motion Answer Sheet

(a) Data: (SAMPLE) – Force vs. Velocity

<table>
<thead>
<tr>
<th>Velocity</th>
<th>Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>

Analysis:
The centripetal force increases faster than the velocity, suggesting that a power law might be at work such as \( F \propto V^2 \). (This is correct.)

(b) Data: (SAMPLE) – Force vs. Mass

<table>
<thead>
<tr>
<th>Mass</th>
<th>Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

Analysis:
The force increases linearly with mass, which suggests a direct proportionality such as \( F \propto M \). (This is correct.)
Analysis:
The centripetal force increases directly with the mass \((F \propto M)\).

(c) Data: (SAMPLE) – Force vs. Radius

<table>
<thead>
<tr>
<th>Centripetal Force</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>60</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
</tr>
<tr>
<td>12</td>
<td>20</td>
</tr>
</tbody>
</table>

Analysis:
As the radius gets smaller, the force gets larger when keeping the same velocity. This is an inverse relationship or also called an indirect relationship and can be written as \(F \propto 1/R\).

Summary:
The centripetal force is affected by velocity as the square of the velocity, and affected directly by the mass, and inversely affected by the radius.
Topic 7: Worksheet – Circular Motion

A. Linear Speed, Tangential Speed and Rotational (Angular) Speed:

You ride this merry-go-round and first stand at point (a) and then at point (b) and rotate counterclockwise. Point (a) is halfway between the center, O, and the outer edge (b) of the bar that you hold on to.

1. How does the time of one rotation compare for point (a) and point (b)?

_____________________________________________________________________

2. What is the shape of the path for points (a) and (b) during one rotation?

_____________________________________________________________________

3. How does the distance traveled by you at points (a) and (b) compare for one rotation?

_____________________________________________________________________

4. How would your linear speeds compare at points (a) and (b)? (Linear speed is distance traveled divided by the time of travel.)

_____________________________________________________________________

5. Is your direction of motion at (a), parallel or perpendicular to the bar? So as pictured, your direction of motion is: back, to the right, to the left or forward?

_____________________________________________________________________

6. Is this rotational motion when direction is included, a scalar or vector?

_____________________________________________________________________

7. If the motion at (a) or (b) includes rate and direction, should this motion be called (a) radial (out from the center) speed, (b) radial velocity, (c) tangential speed, or (d) tangential velocity?

_____________________________________________________________________
Rotational speed for a rotating object is measured by the angle moved through (sweep out) divided by the time to rotate that angle. Rotational angles can be measured by degrees, revolutions (rotations), and radians. One time around a circle is measured in units of 360 degrees, 1 revolution or 2 \( \pi \) radians. For now, we will use revolutions.

8. If you stand at point (a) and move through 5 revolutions in 10 seconds, your rotational speed is equal to:


9. If you stand at point (b) and move through 5 revolutions in 10 seconds your rotational speed is equal to:


10. As you move from the center of the rotating merry-go-round to the outside, your linear speed _______________ and your rotational speed _______________.

B. Centripetal Force and Centrifugal Force

Centripetal force is defined as an inward force on a body that causes it to move in circles. Centrifugal force is defined as an outward force on a body that “appears real” to the body rotating in a circle, however, it is fictitious (fake).

11. What is the type of force exerted on you by the bar at points (a) and (b) that keep you moving in a circle?


12. When the merry-go-round is turning and you let go at point (a) or (b), you see yourself flying out from the merry-go-round. From someone’s point of view nearby the merry-go-round, did you move outward from (a) a centrifugal force, or by (b) continuing along the same motion you had just before release, which would be tangent to the bar’s motion?


13. If you change “velocity” as you rotate on the merry-go-round, are you accelerating \( a = \frac{\Delta v}{\Delta t} \)?
14. An accelerating frame of reference is called non-inertial because the law of inertia doesn’t work and any constant velocity frame of reference is called inertial because the law of inertia does work. While rotating on the merry-go-round are you (the rider) in an inertial or non-inertial frame of reference?

15. To YOU, the rider, as you rotate on the merry-go-round, the reason you “fly off” after letting go is: centripetal force (b) centrifugal force

C. Centripetal Acceleration

Centripetal acceleration is defined as the change in the tangential velocity by the time to change that velocity. (If an object changes its rotational speed, it also changes its angular acceleration, but that will be left for another physics class.)

\[ \vec{a}_T = \frac{\Delta \vec{v}_T}{\Delta t} \]

Sketched here is a “top view” of our merry-go-round rotating counterclockwise.

16. Draw tangential velocities at (a) and (b) on sketch O.

Review: Adding two vectors can be done by drawing one vector to scale on a sheet of paper and connecting a second vector drawn to scale. Connect the second vector’s tail to the head of the first vector. The sum, or resultant, is the vector produced when it is drawn from the tail of the first vector to the head of the second vector.

Practice: Add this vector \( \vec{2} \) to this vector: \( \vec{1} \)
Solution:

1

Resultant

(The resultant is larger than (1) or (2) and points down and to the right.)

2

17. Add your vectors at (b) and (x) from the sketch in the space below.

18. Compare the size of the resultant to the two original vectors and which way does it point?

New: Subtraction of vectors can be done by “adding the negative“ of the vector to be subtracted.

Example: Vector A

Vector B

So: \( \vec{A} - \vec{B} \) is just \( \vec{A} + (-\vec{B}) \)

OR: A

Resultant

(Larger than the originals and directed down to the right)

Review: Recall that centripetal acceleration is defined as:

\[ \vec{a_c} = \Delta \vec{v} / \Delta t, \text{ and } \vec{v}_T = v_T - v_1 \]

So, when the mass goes from (b) to (x), 

\[ \Delta \vec{v} = \vec{v}_x - \vec{v}_b. \]

19. Find the direction of \( \Delta v \) in the space below, which would be the direction of the centripetal acceleration.
20. This average centripetal acceleration would occur in the middle of the time interval (trust me!) between points (b) and (x). What is that direction?

21. Also draw the $\vec{a}_c$ vector on the sketch O at the center between (b) and (x).

22. What way does the $\vec{a}_c$ point on the drawing?
Topic 7: Circular Motion Worksheet Answer Sheet

A. Linear Speed, Tangential Speed and Rotational (Angular) Speed
   1. Same
   2. Circle
   3. Since $C = 2\pi r$, $C \propto r$, so $r$ is doubled, so is the distance doubled (circumference)
   4. Since $v = \frac{d}{t}$, $v_a = \frac{d}{t}$ and $v_b = 2\ \frac{d}{t}$, $v_b$ is double that of $v_a$.
   5. Perpendicular: Back
   6. Vector
   7. (d) Tangential velocity
   8. $W = \frac{\Delta \theta}{\Delta t} = 5 \ \text{rev} / 10 \ \text{s} = 0.5 \ \text{rev/s}$
   9. $W = \frac{\Delta \theta}{\Delta t} = 5 \ \text{rev} / 10 \ \text{s} = 0.5 \ \text{rev/s}$
   10. Increases, remains constant

B. Centripetal Force and Centrifugal Force
   11. Centripetal force
   12. (b) Same motion before release
   13. Yes, inward
   14. Non-inertial
   15. (b) Centripetal force

C. Centripetal Acceleration
   16.
17.

18. R is larger than v at b or x and points up to the left,

19.

20. Down to the left (actually, inward toward the center of rotation)

21.

22. Inwards, toward the center of the circle
Topic 7: (e) Circular Motion Demonstration

Tangential Velocity in Circular Motion
Sketch of a mass spinning at the end of a string that is rotating in a horizontal circle in a counterclockwise direction:

(a)

(b) Top View of Spinning Mass

(c) Purpose:
Many students (and any person) feel that when a mass is spinning in a circle and let go, it will fly radially outward. So using the above figure (b), the mass would fly downward if let go at the shown position.
To correct this incorrect thinking, this demo will show the mass will fly tangent to the radius of the circle, or perpendicular to the radius of the circle as shown in figure (b).

![Diagram showing radius and velocity vector](image)

Once in a while you will get lucky and hit the bull’s-eye in figure (a).

(d) The teacher should probably do this demo with several practice tries before class. It is possible for some coordinated student to also do the demo. Use a one-hole rubber stopper connected to thin, cotton sewing thread about 1 m long. Use a new single edged razor blade securely attached a test tube clamp. Have a student hold the stand for the razor to provide the maximum force on impact. Spin the stopper around 1 rev/s in the horizontal circle above the razor and slowly lower the mass and string as you lower your arm. Making contact as close the stopper will ensure greater success.
Circular Motion and the “Funky Chicken”

G-Lock, G-suits, and the “funky chicken.” What does any of this have to do with circular motion? One of the most spectacular effects of circular motion can be experienced in the cockpit of a modern fighter plane. Quickly attaining velocities that easily exceed twice the speed of sound places forces on the pilot that border on the limits of human endurance. Add to this, the extreme forces, needed to turn the plane, have often proved fatal.

A pilot executing a high-speed turn can experience forces that exceed nine times the force of gravity. As a pilot turns, his blood rushes to his lower body causing the brain to become oxygen deprived. The first indication of the problem is when the pilot experiences tunnel vision. Next, his vision loses color and fades. This is known as “gray-out.” As his vision fades, the pilot enters G-Lock; he only has hearing before passing out completely. As forces diminish blood returns to the brain and the pilot regains consciousness, but not until his body undergoes a series of severe, violent spasms as part of the recovery process. Pilots refer to this recovery process as “doing the funky chicken.”

To combat these g-forces, pilots anaerobically build lower body muscle in order to squeeze blood to the upper extremities. Additionally, pilots wear g-suits, which inflate air bladders around the lower body, again forcing blood to the upper regions. Using these techniques and technologies, fighter pilots can pull in excess of 12 “g” avoiding “G-Lock” and doing the “funky chicken.”