Neutrinos: The Ghosts of the Universe

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New Mexico State University

5 August 2014
Fermilab Undergraduate Lecture Series

Part 1: Prelude

What is particle physics?

What is particle physics?

- The study of fundamental particles of nature and their interactions.

What is a fundamental particle?

What is a fundamental particle?

- Drilling down from chemistry...

What is a fundamental particle?

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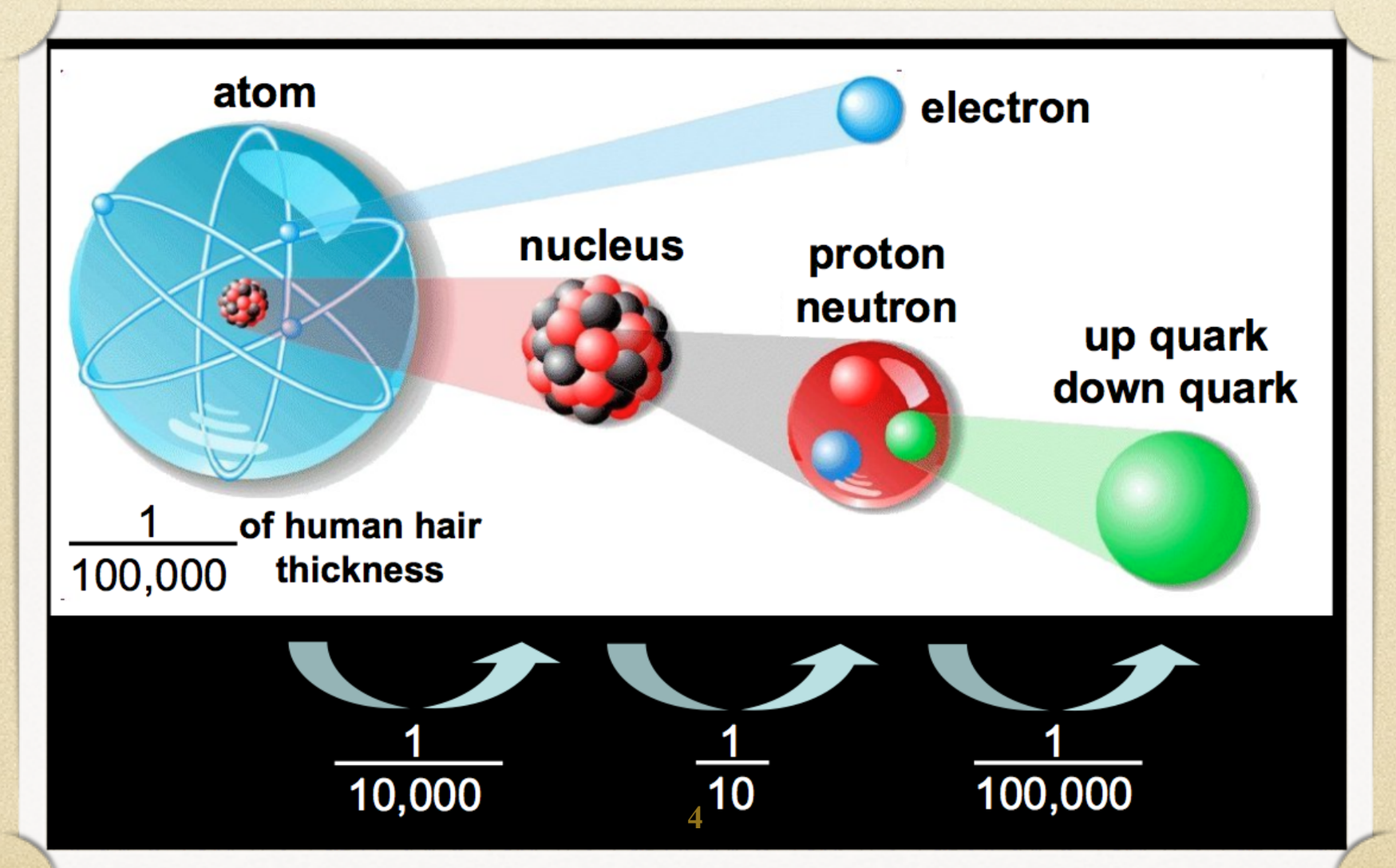
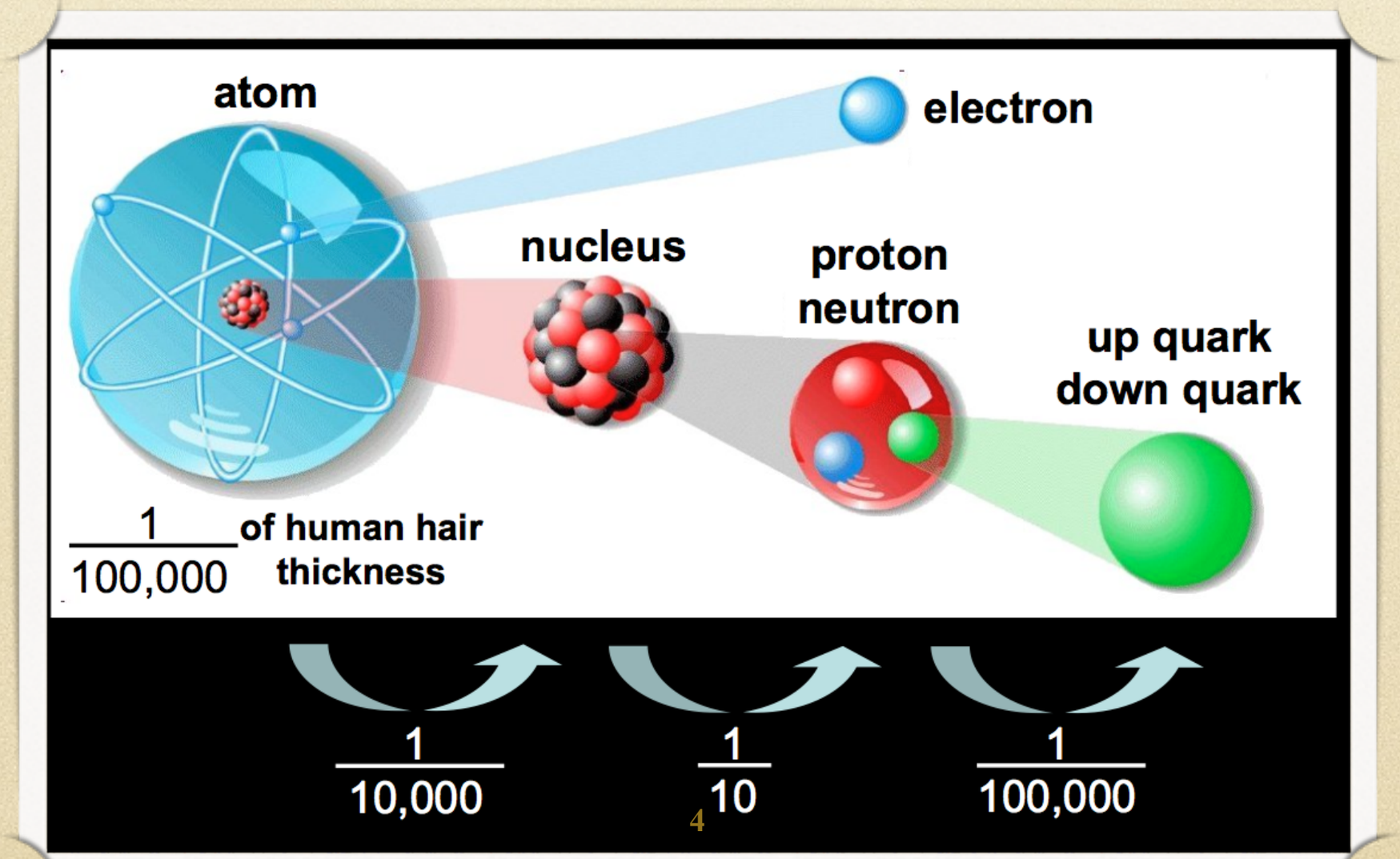


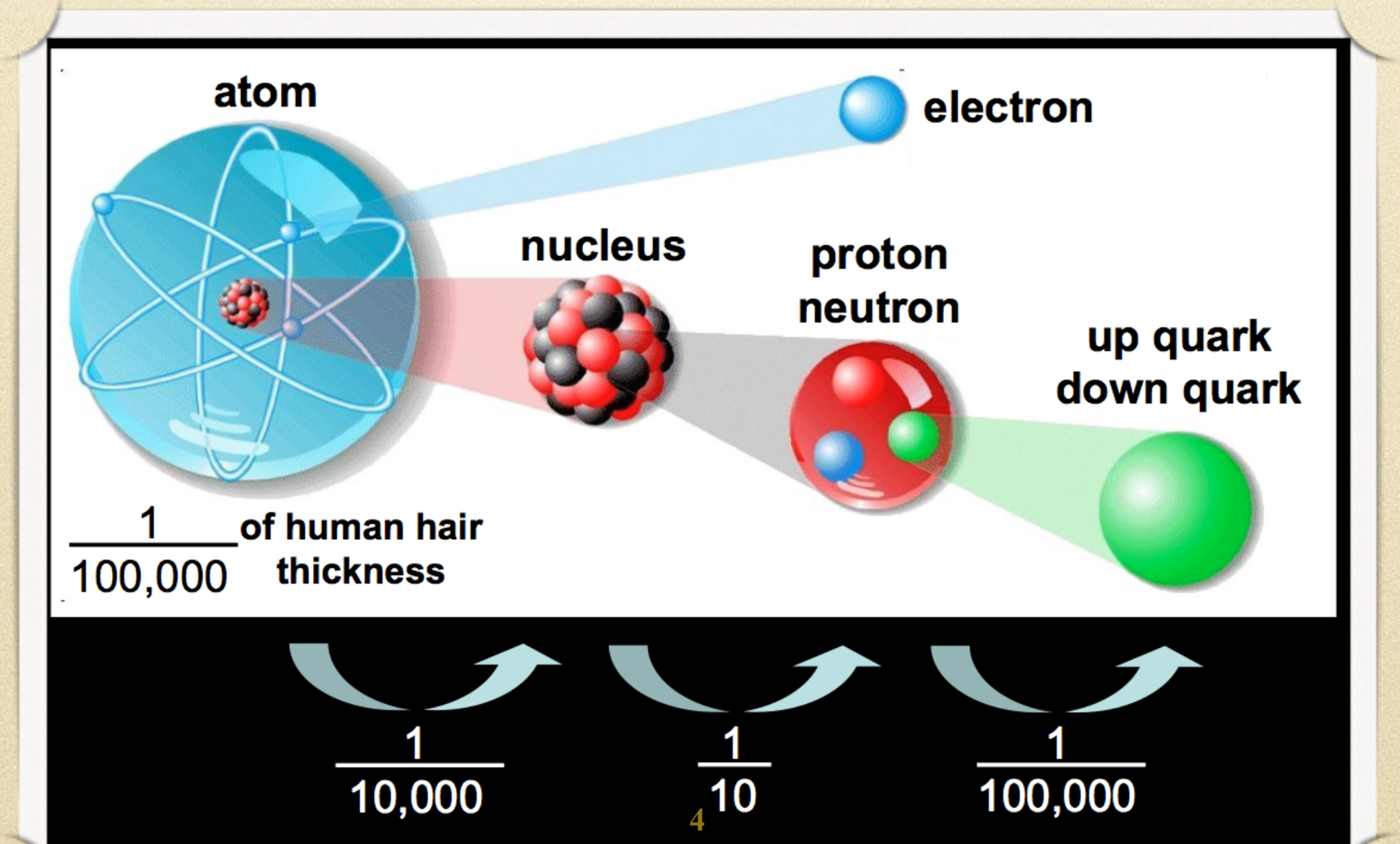
What is a fundamental particle?

- Drilling down from chemistry...
  - we get the stable particles making up nearly everything!



What is a fundamental particle?

- Drilling down from chemistry...
  - we get the stable particles making up nearly everything!
    - electron, up quark, and down quark

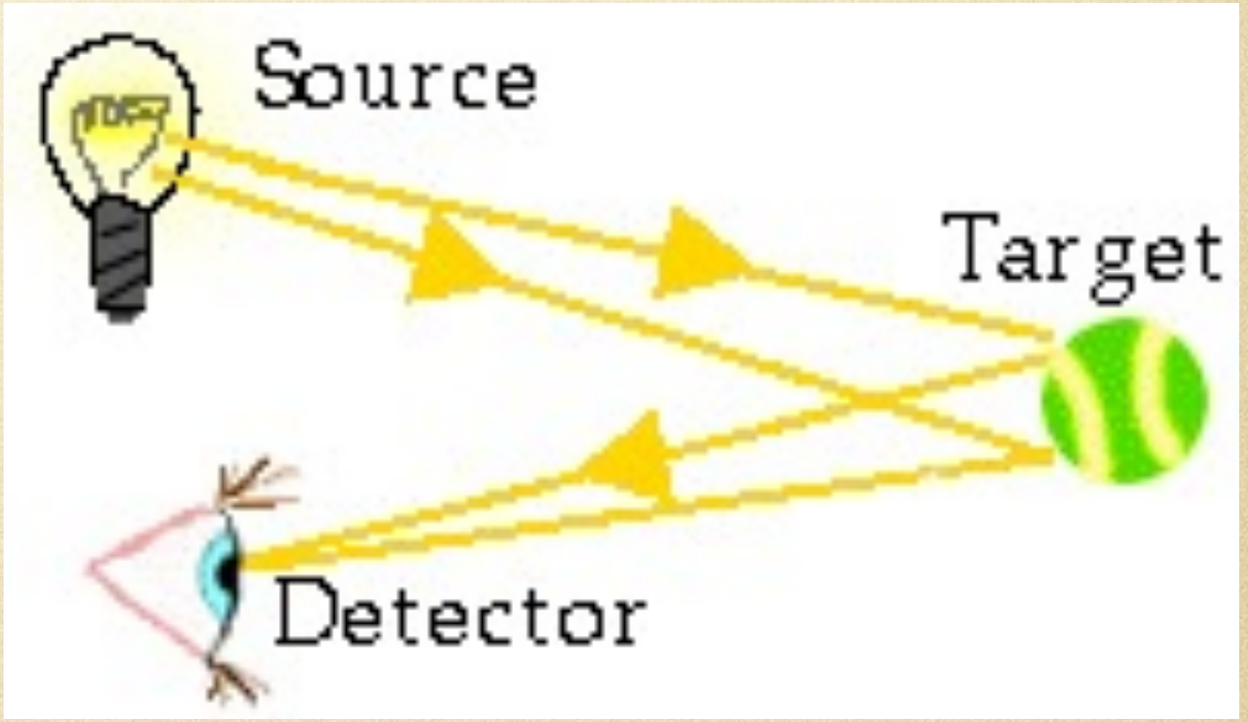


What is a fundamental particle?

- Also, this.

What is a fundamental particle?

- Also, this.
  - The actual photons hitting your retina in your eye from this presentation.



What is a fundamental particle?

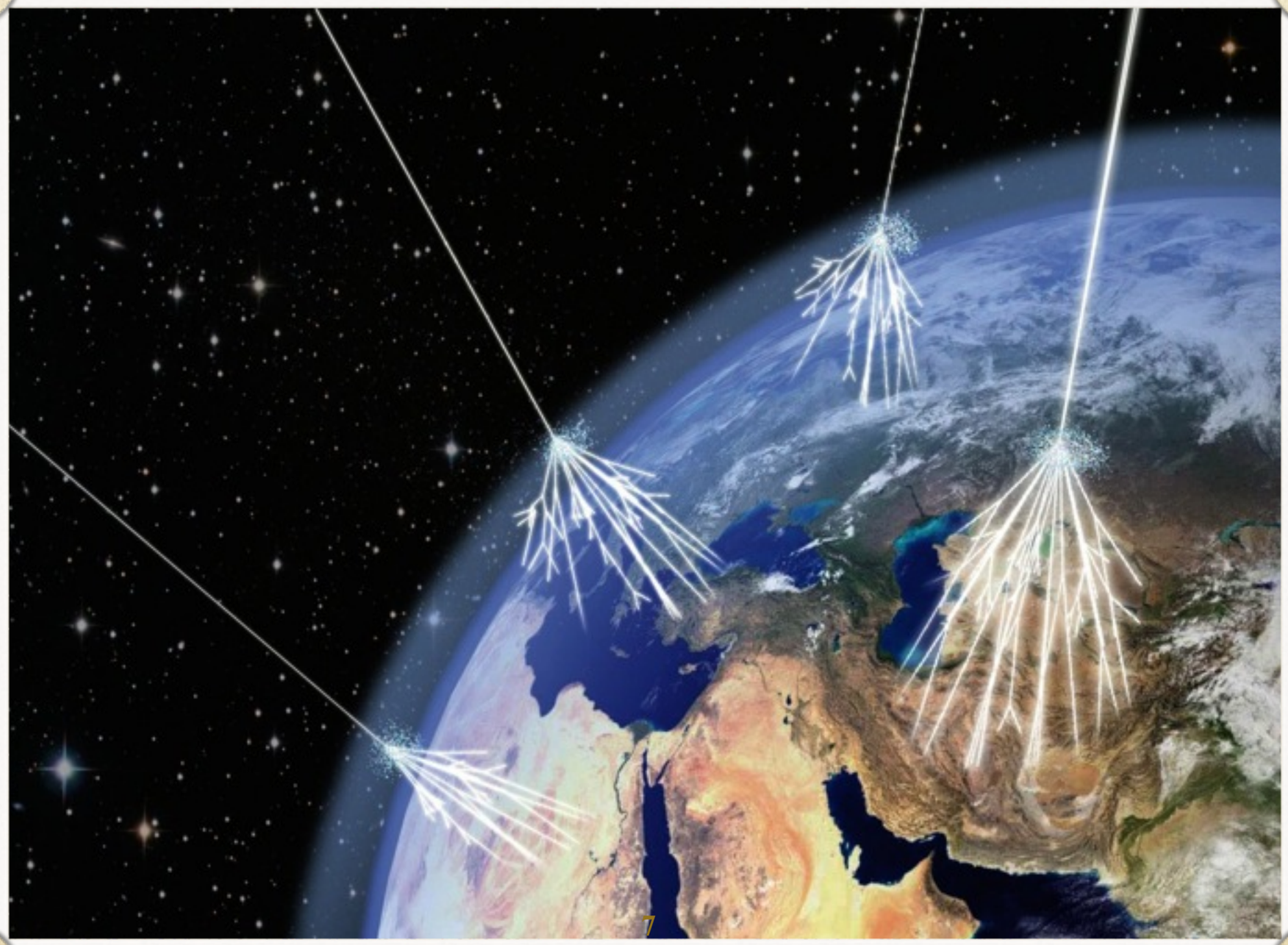
- But there are other fundamental particles too...

What is a fundamental particle?

- But there are other fundamental particles too...
  - What are other sources you've heard of?

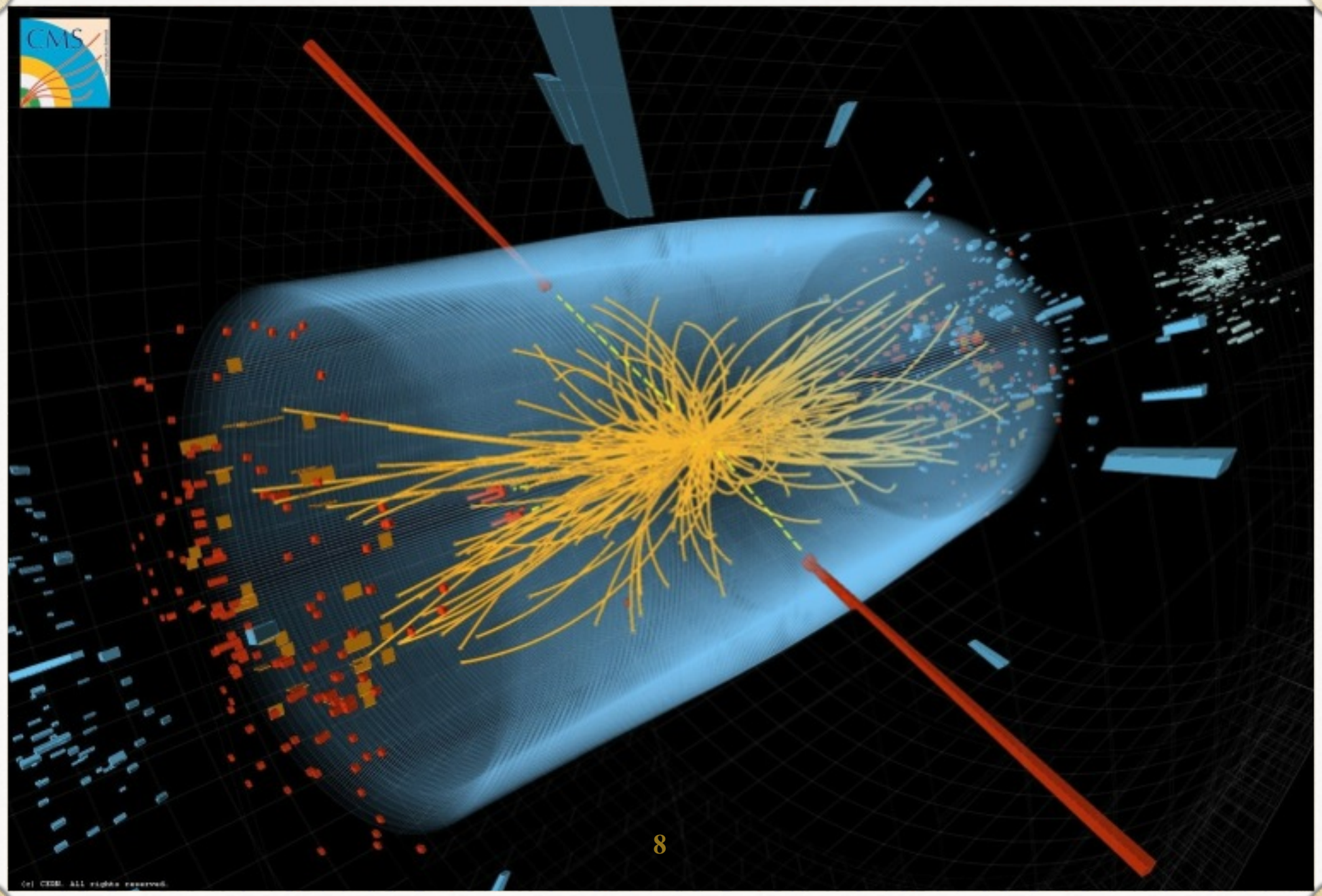
Other fundamental particles appear...

- Cosmic ray showers
  - muons, lifetime ~ 2.2 microseconds



Other fundamental particles appear…

- Fleeting collision debris from smashing protons and/or electrons together (LHC).
  - τ, s, c, t, b, $W^{\pm}$, Z, ν_e, ν_μ, ν_τ, g, H



Other fundamental particles appear...

- Neutrinos sources

Other fundamental particles appear...

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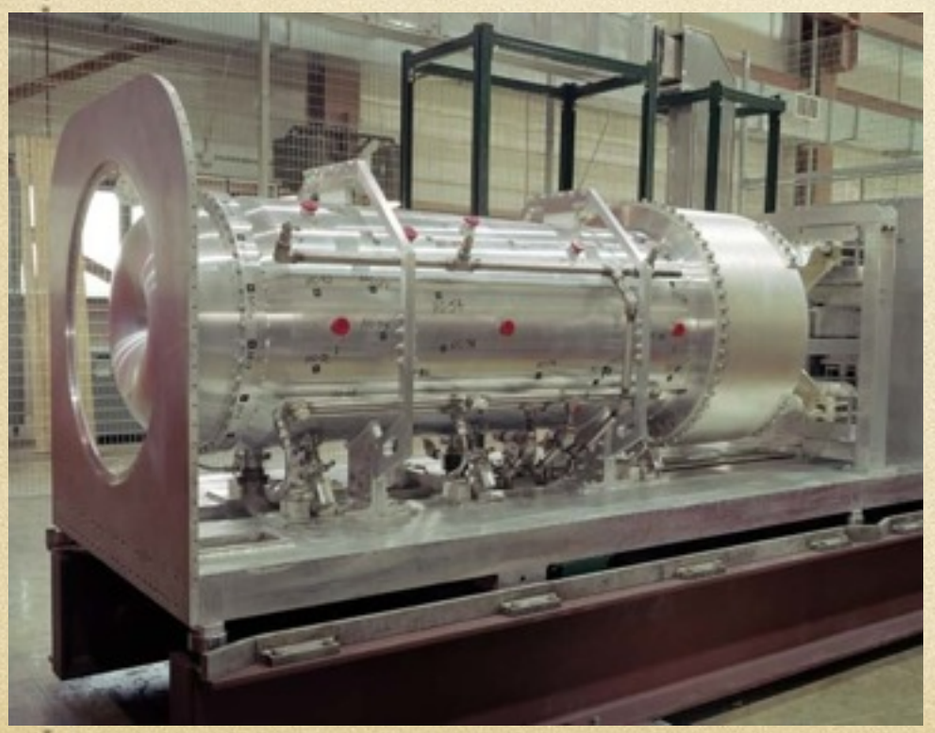
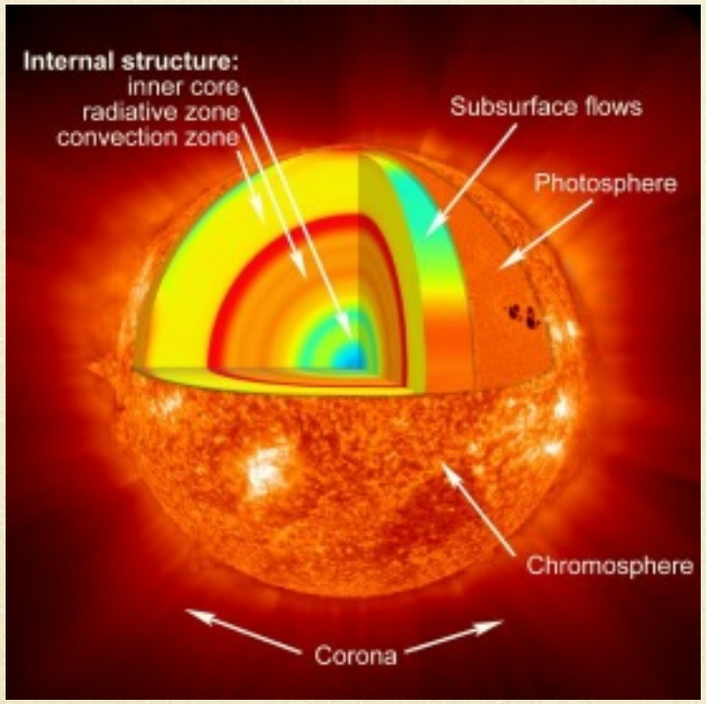
Other fundamental particles appear...

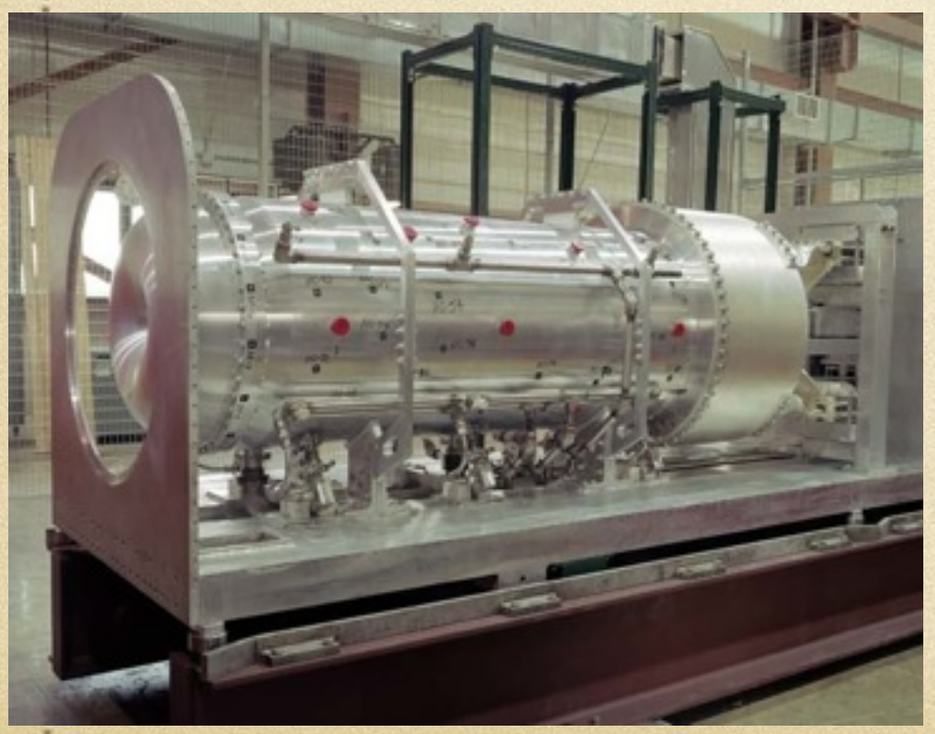
- Neutrinos sources



Other fundamental particles appear...

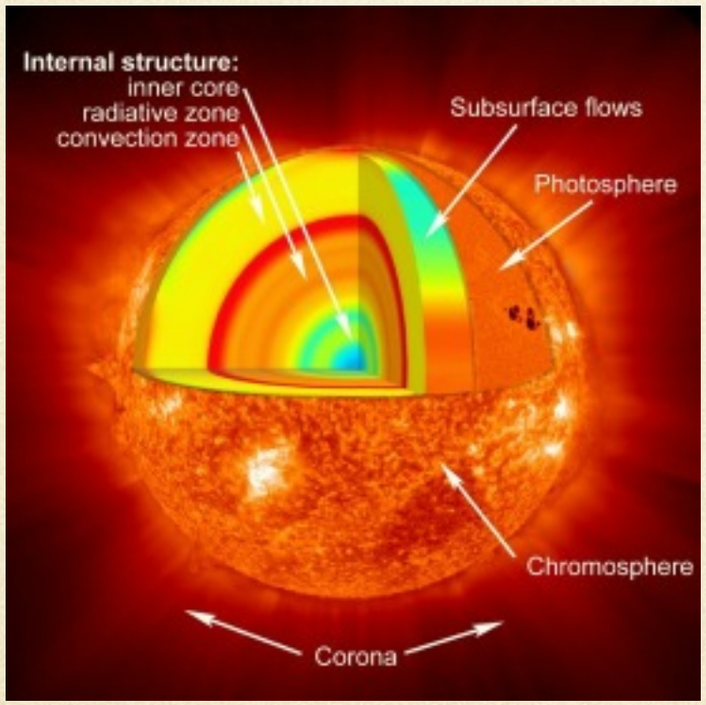
- Neutrinos sources

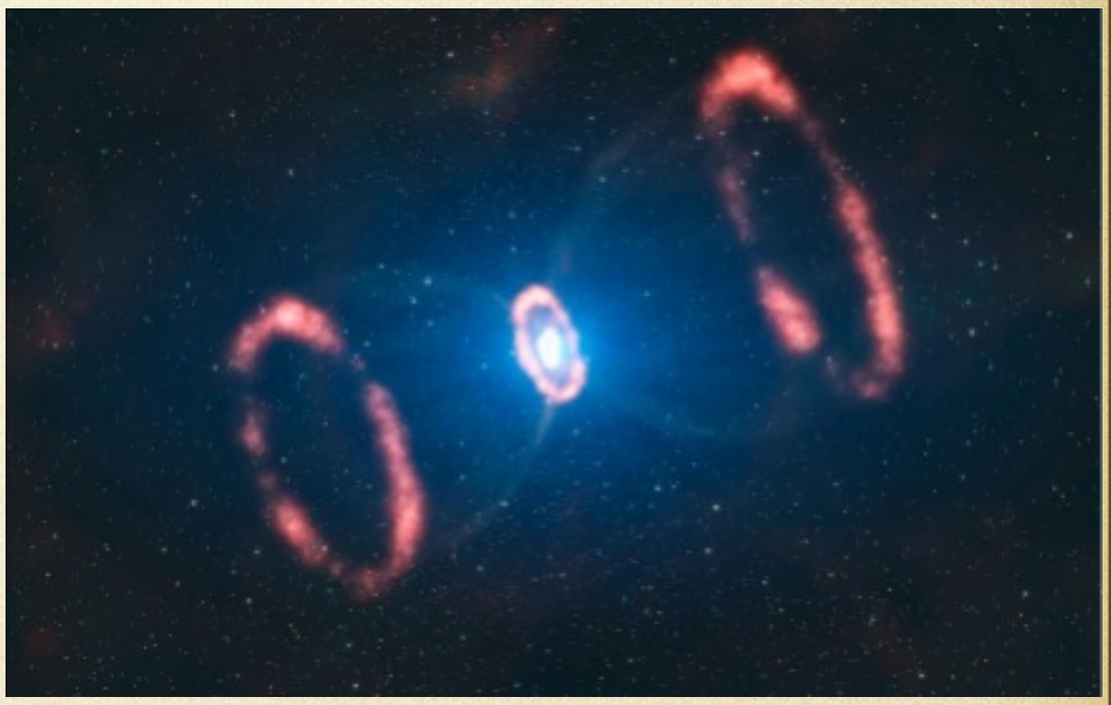


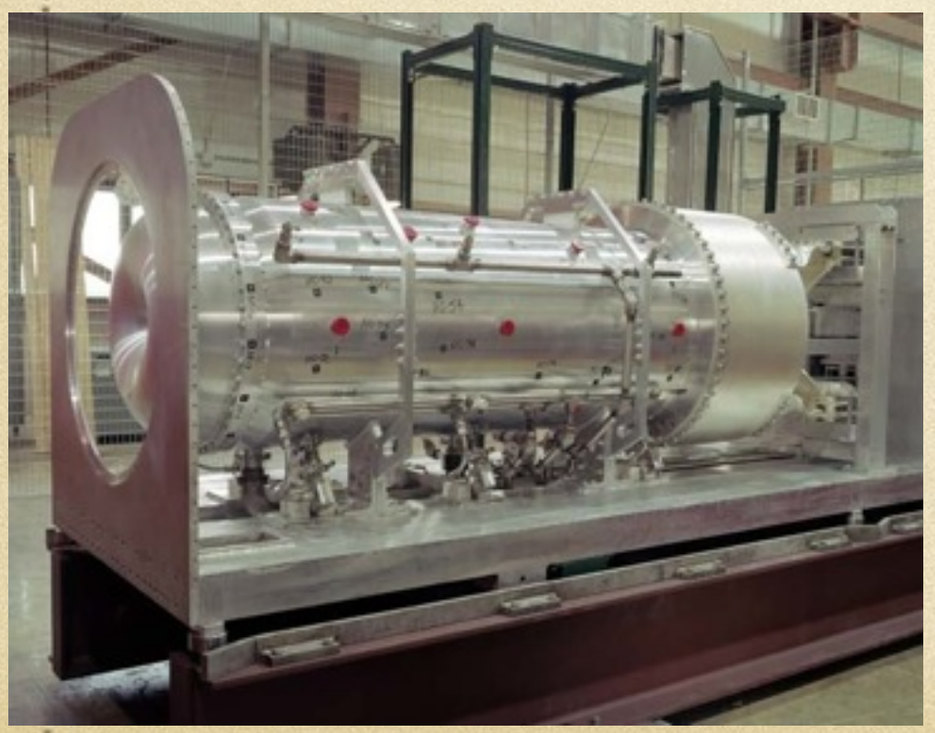


Other fundamental particles appear...

- Neutrinos sources







Other fundamental particles appear...

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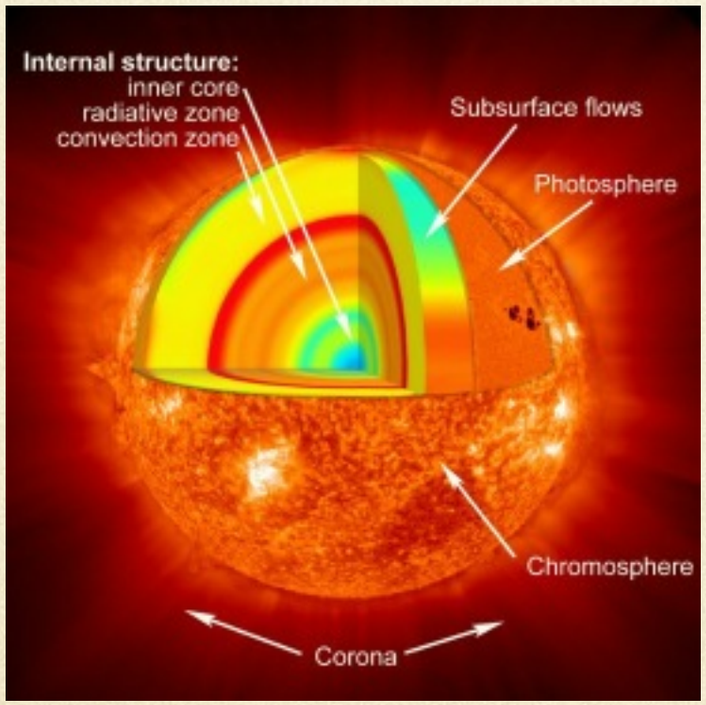
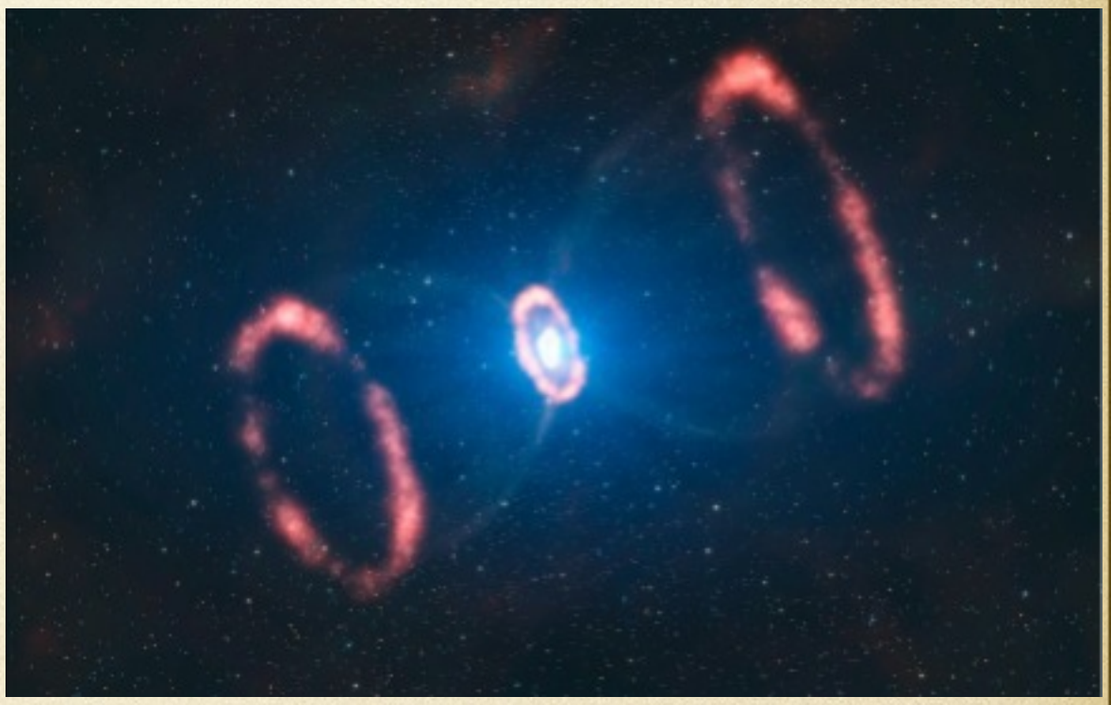
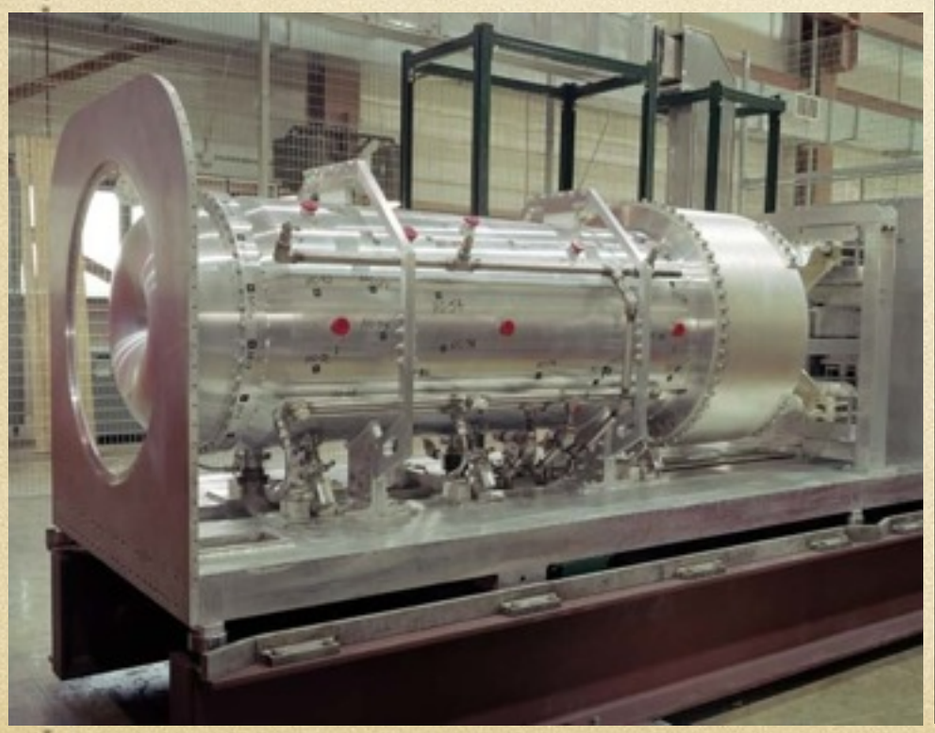
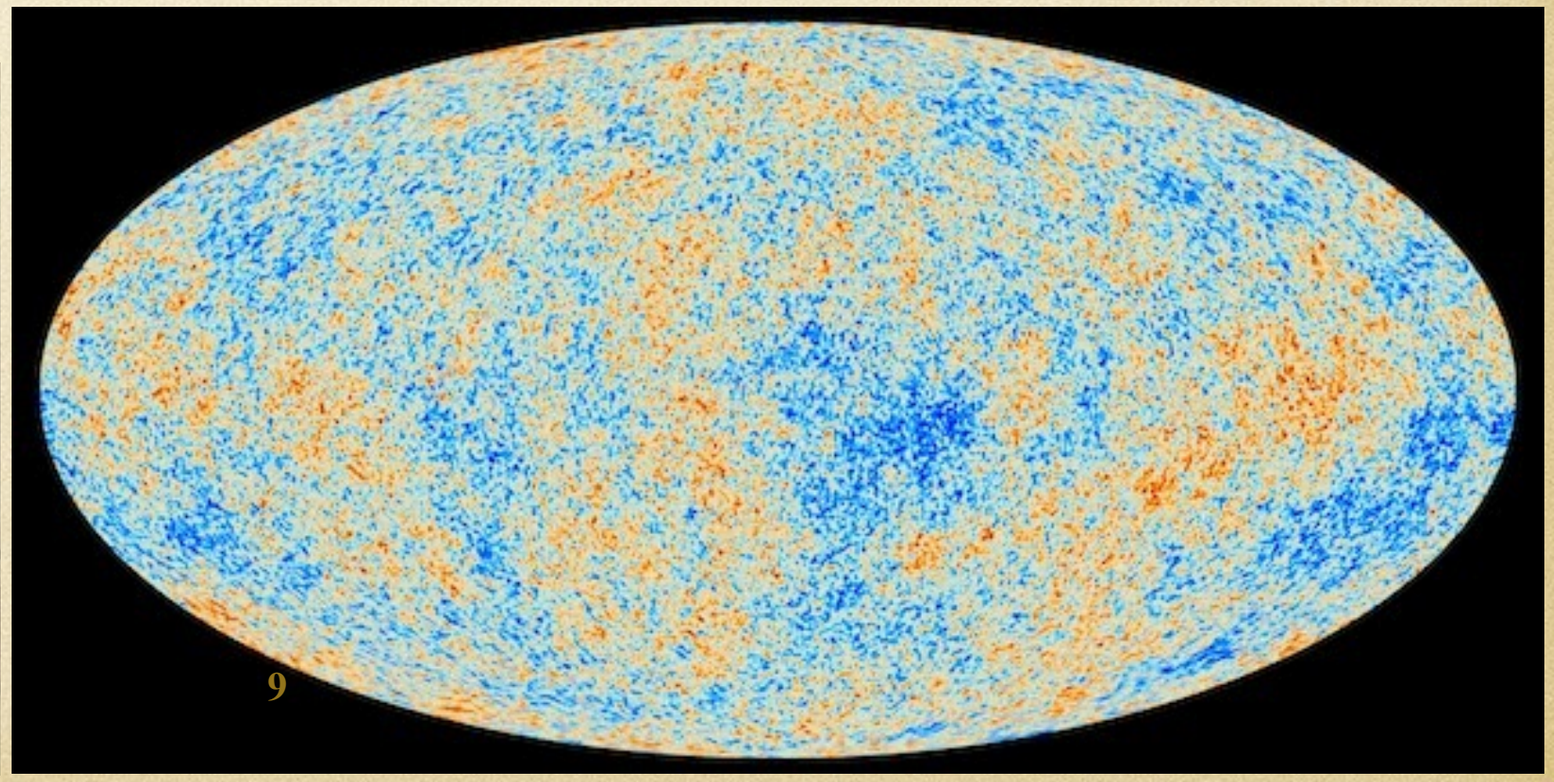








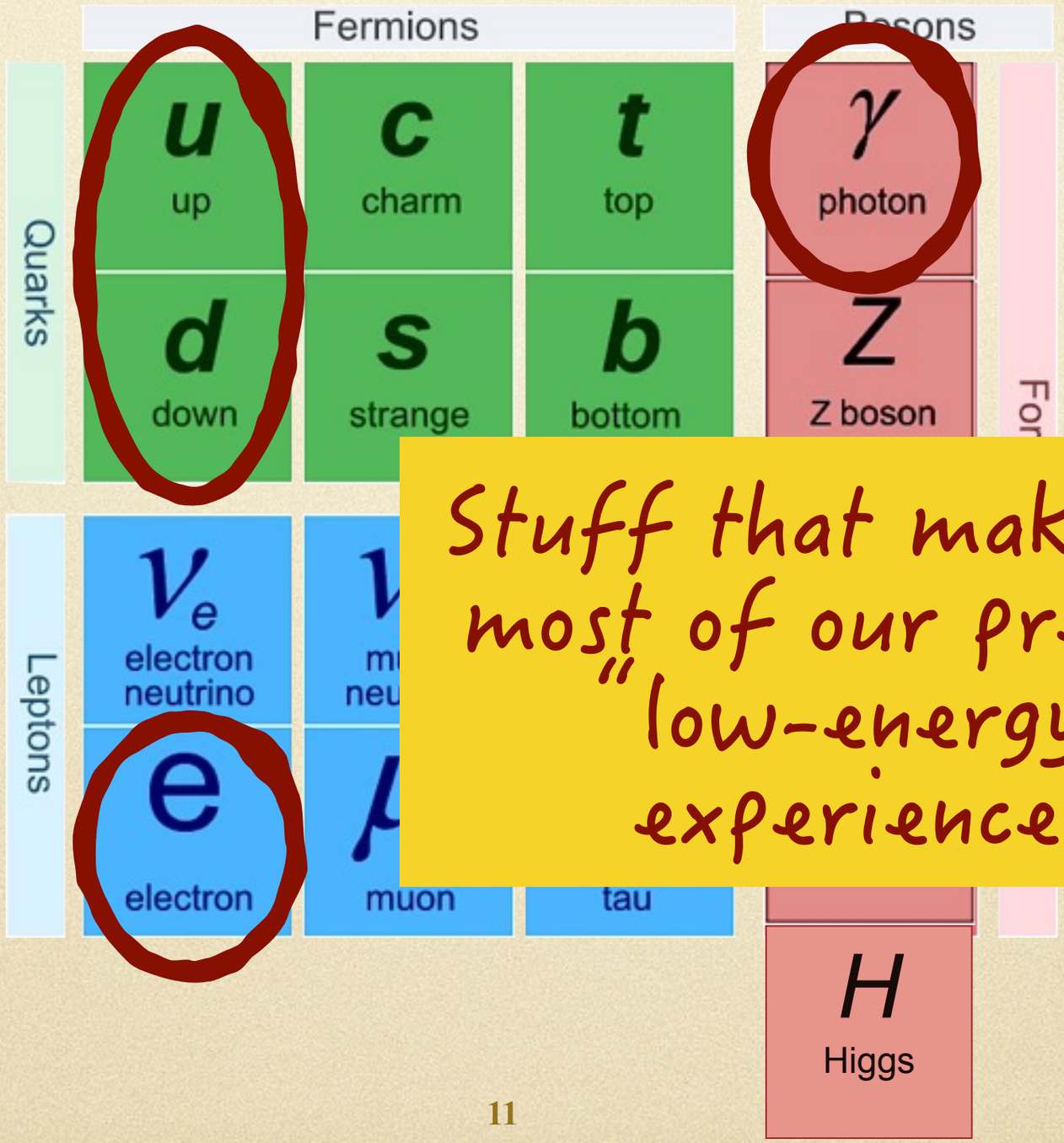
Fundamental Particles

Standard Model

	Fermions			Bosons	
Quarks	u up	c charm	t top	γ photon	Force carriers
	d down	s strange	b bottom	Z Z boson	
Leptons	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	
	e electron	μ muon	τ tau	g gluon	
				H Higgs	

Fundamental Particles

Standard Model



Stuff that makes up most of our present "low-energy" experiences

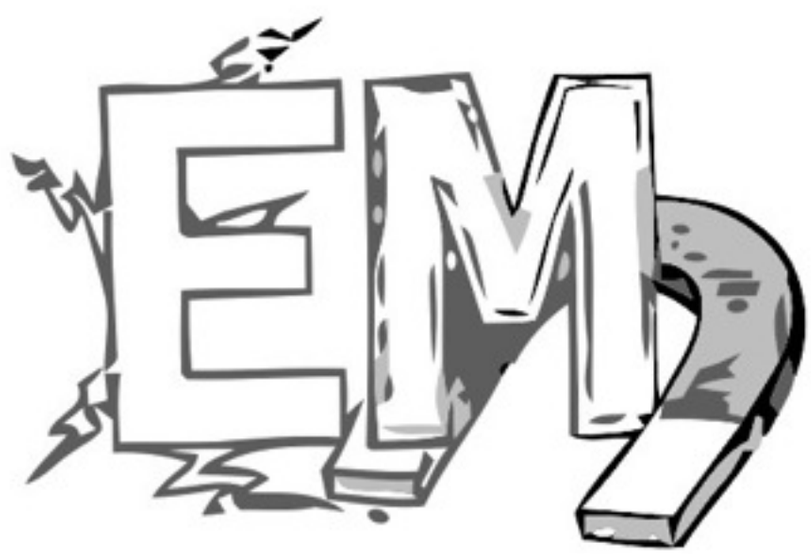
What is particle physics?

- The study of fundamental particles of nature and their interactions.

Fundamental Forces

strong force

weak force



electomagnetism

gravity

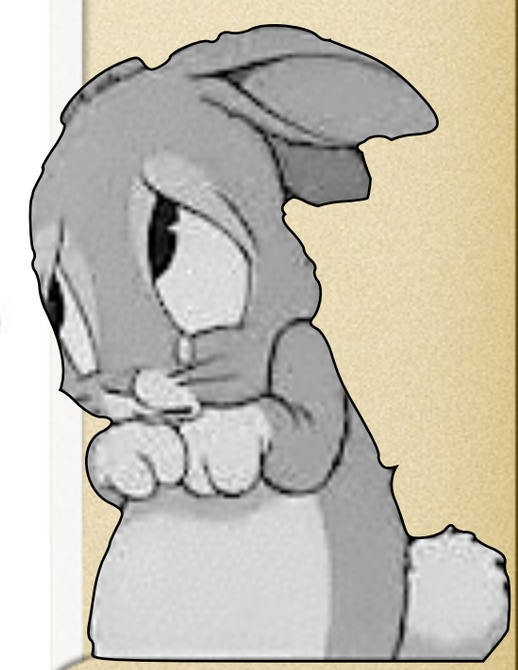
Fundamental Forces

strong force

weak force

electomagnetism

gravity



Fundamental Particles

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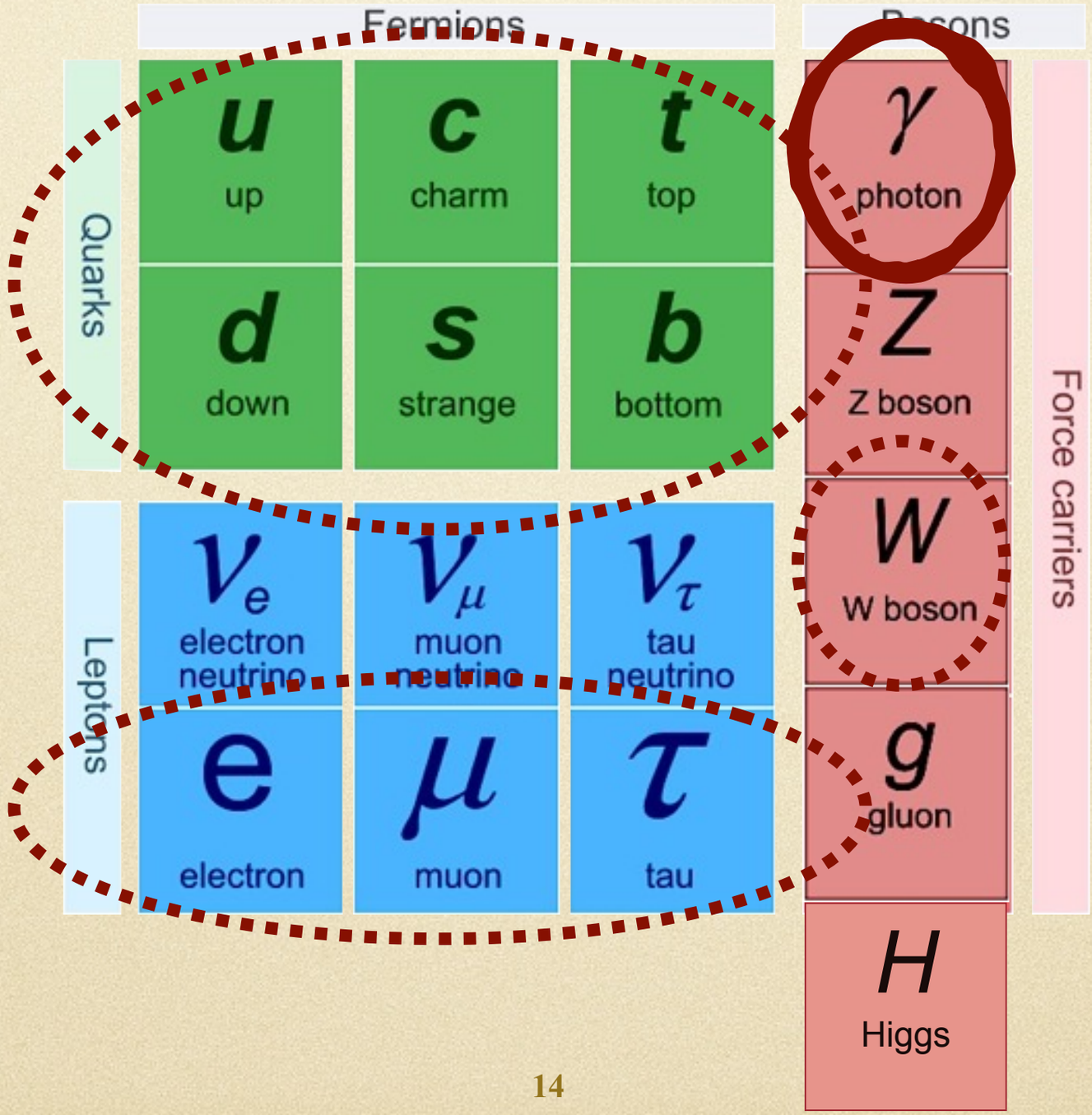

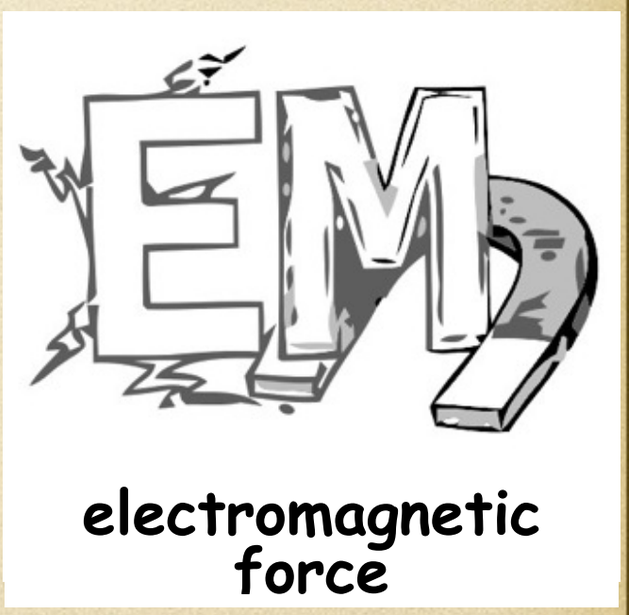

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Fundamental Particles

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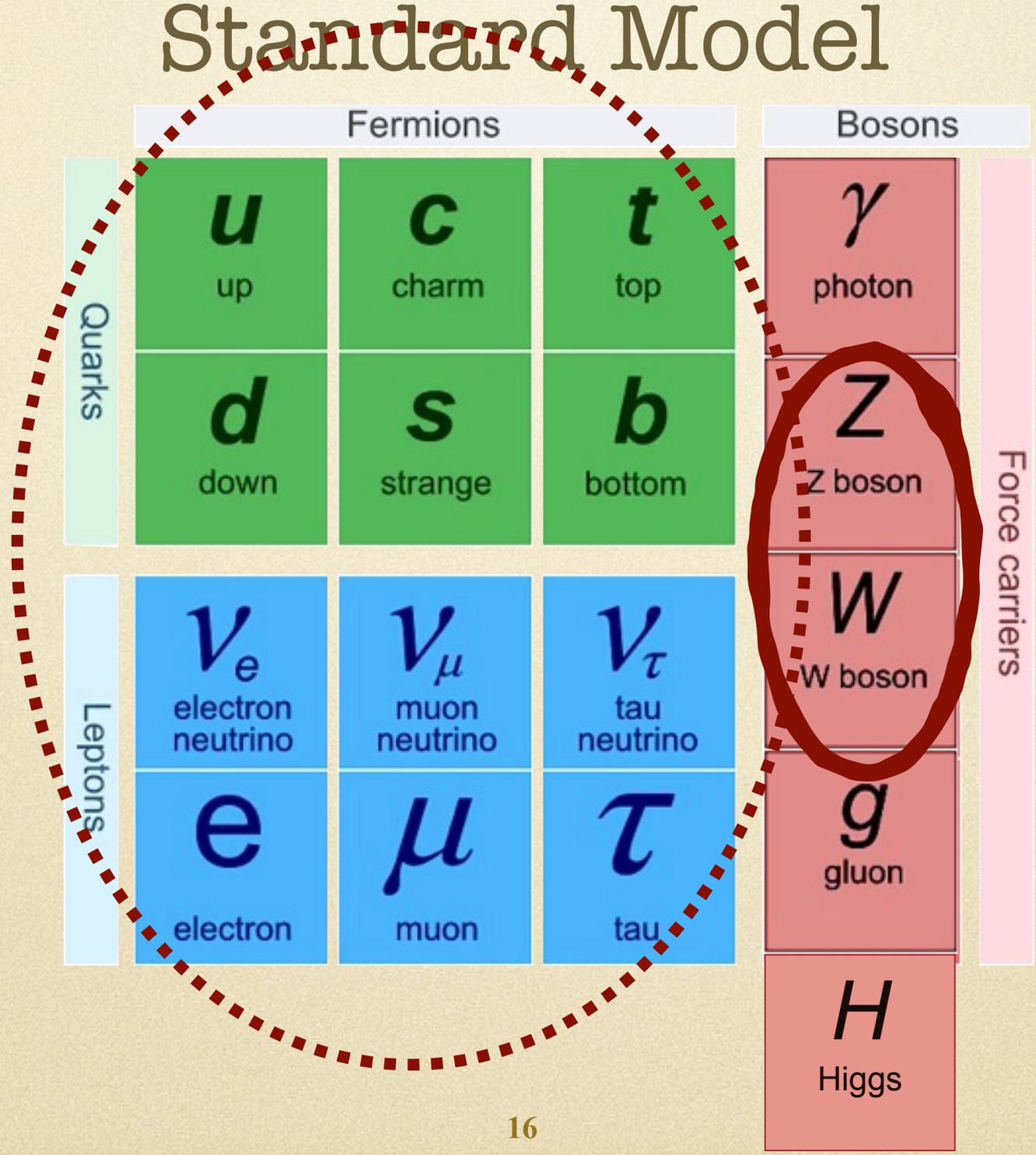

Fundamental Particles

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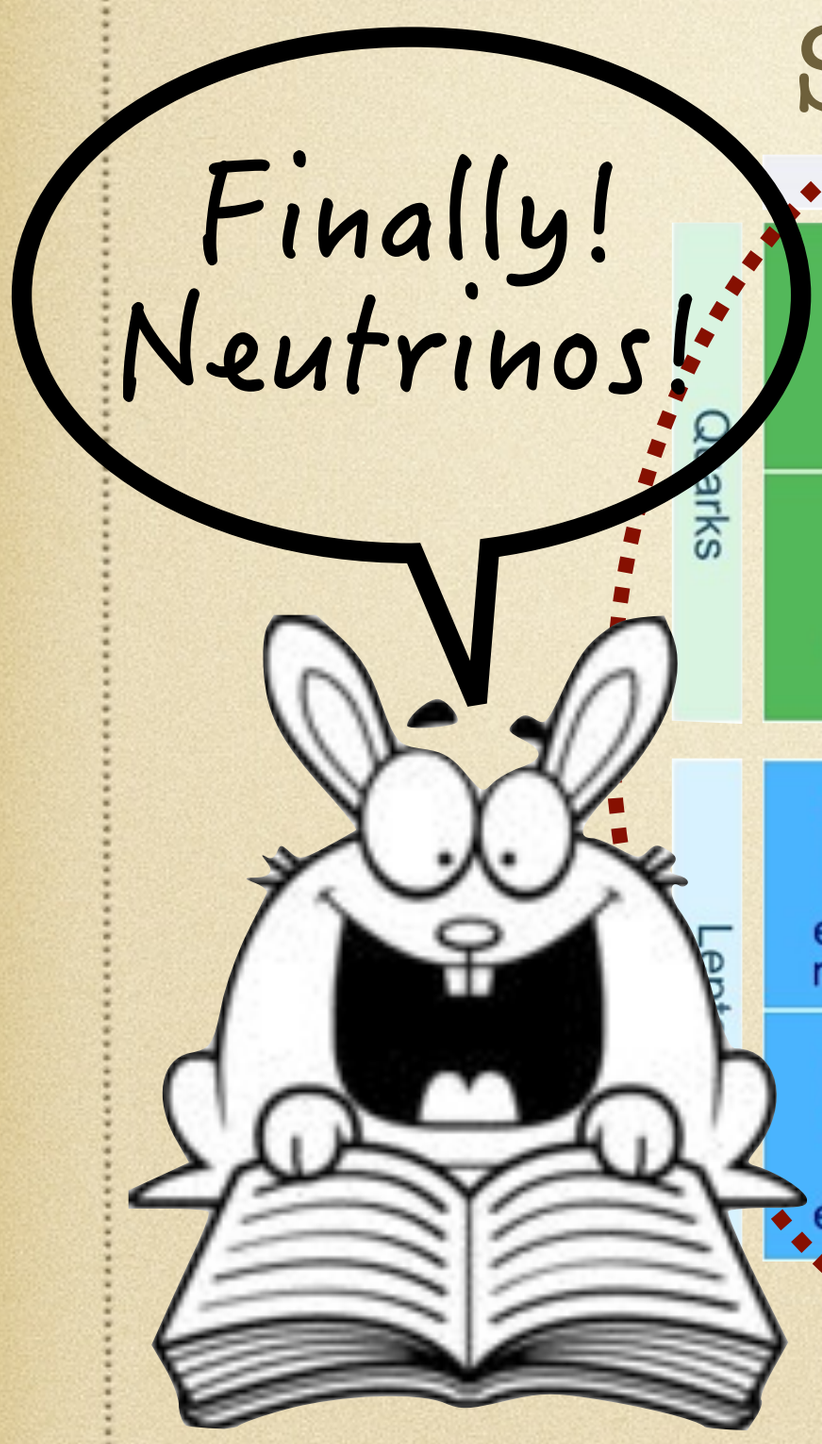

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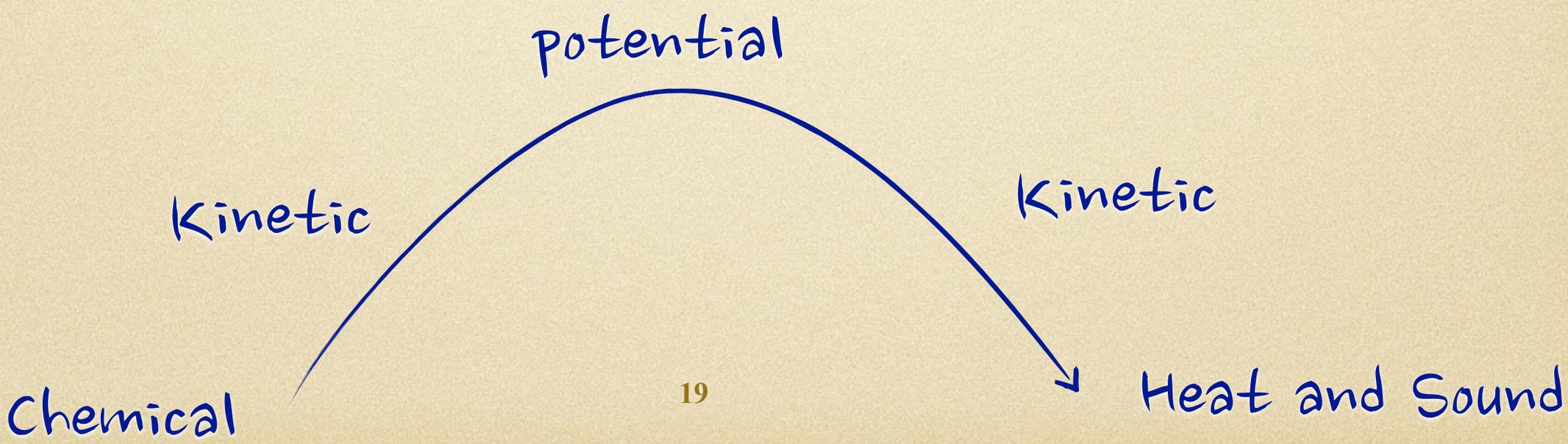
Part 2: Neutrino Discovery

Mystery File 1: Is Energy Conserved?

Fact: Energy is Conserved



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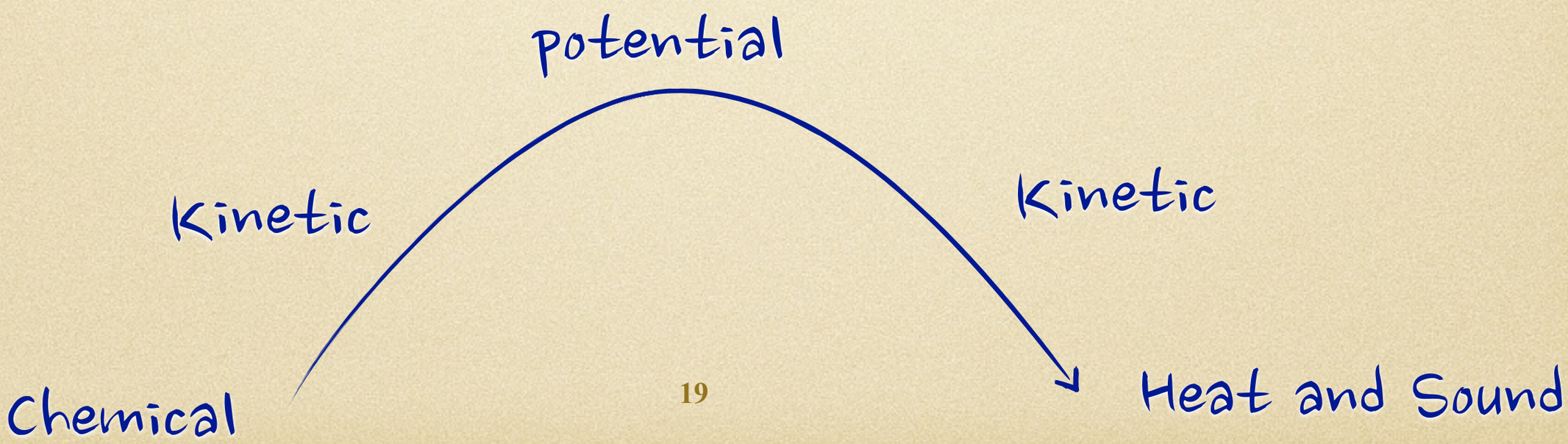


Fact: Energy is Conserved

$$E_{initial} = E_{final}$$

Kinetic

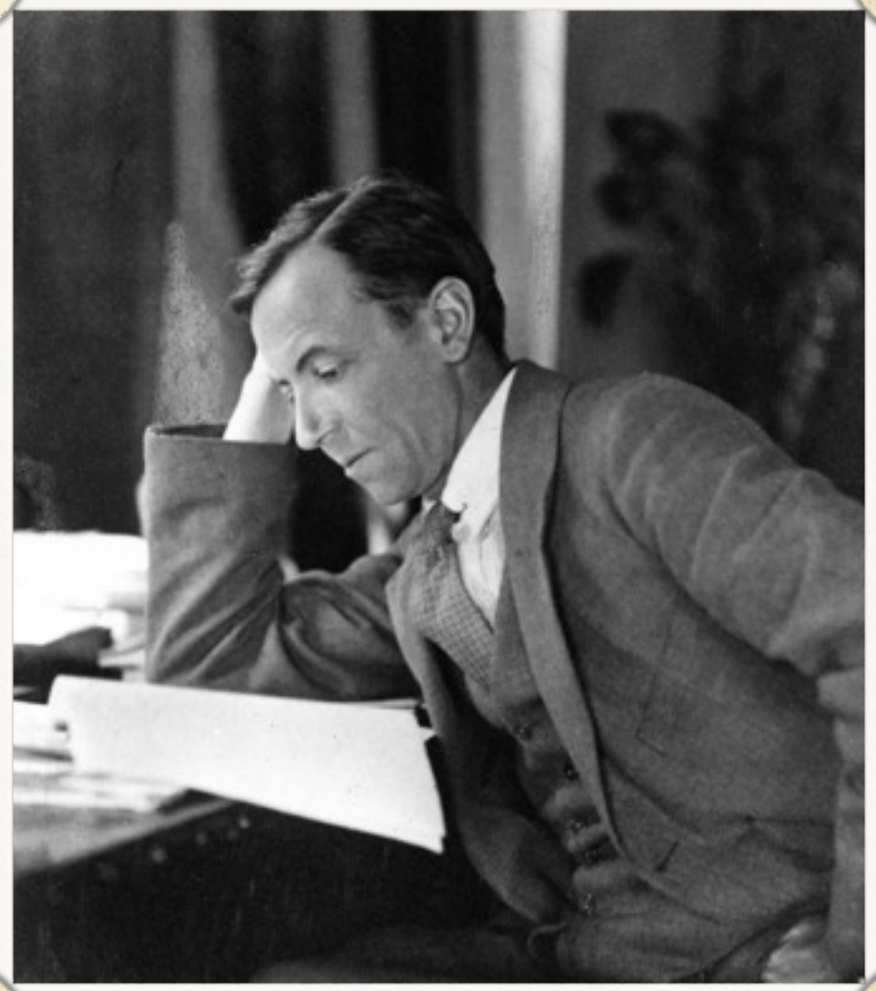
Chemical

Kinetic

Heat and Sound

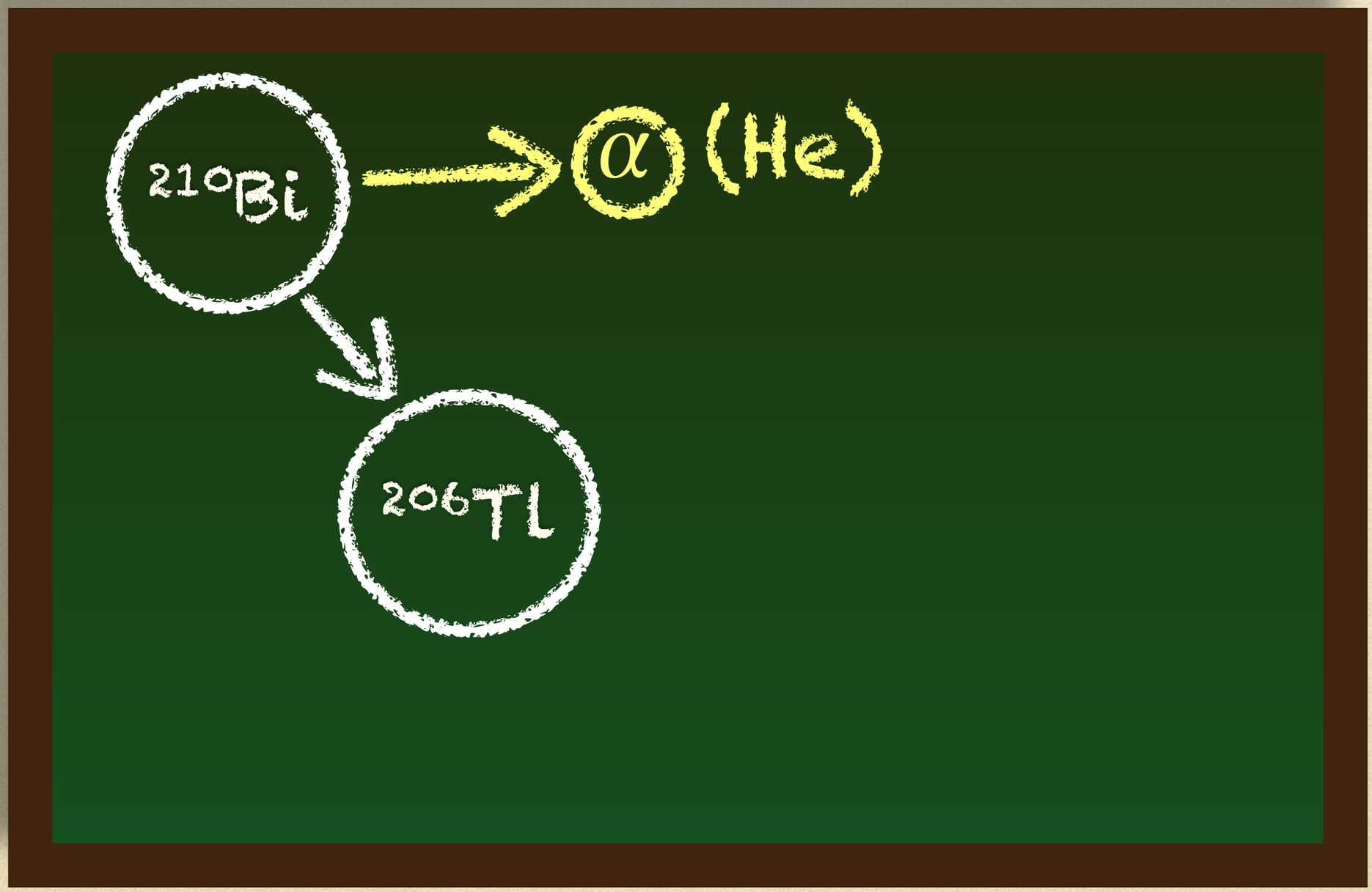
Investigating β-decay

Lise Meitner and Otto Hahn 1911



Sir James Chadwick 1914

Missing Energy in β-decay



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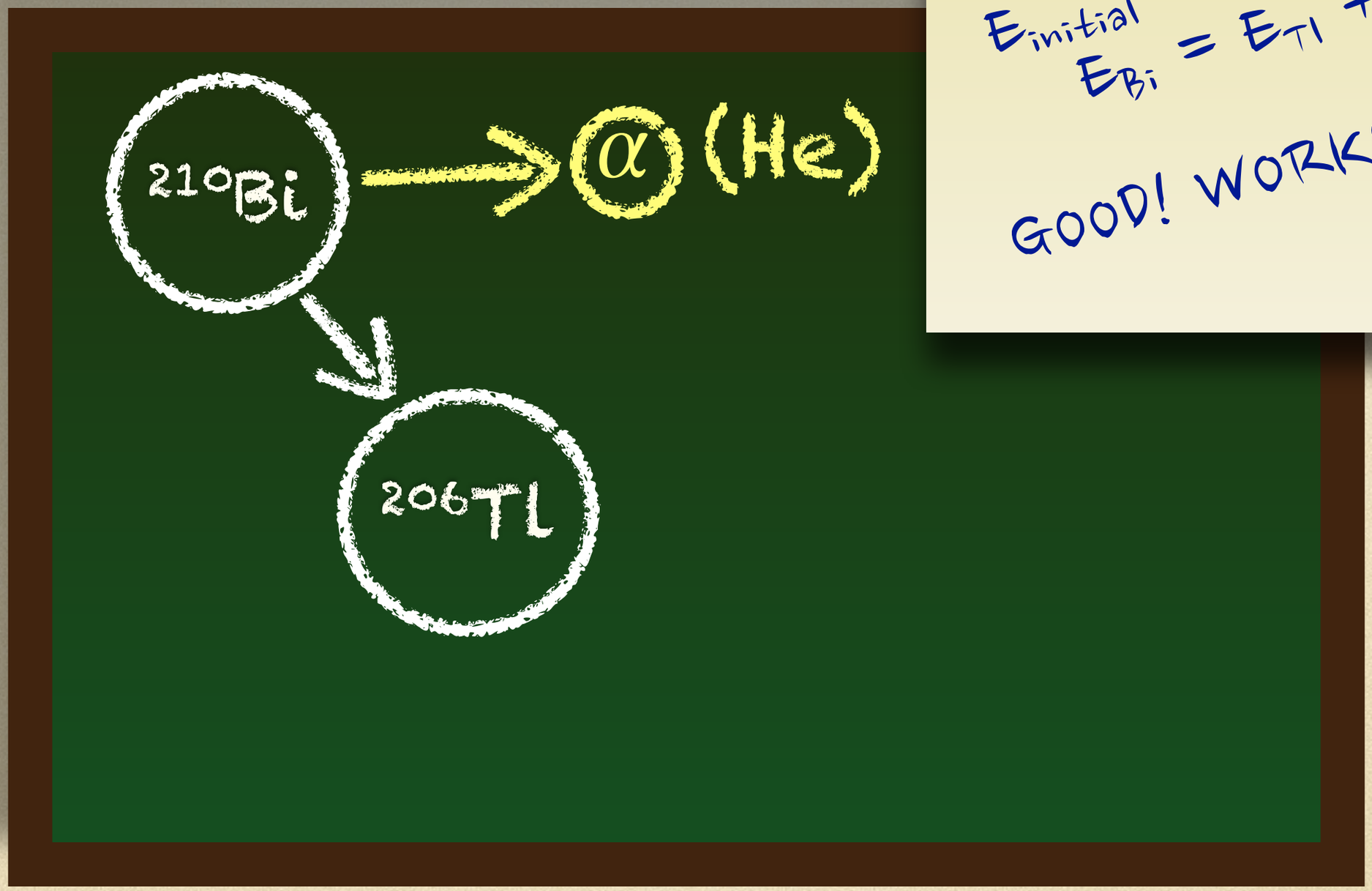


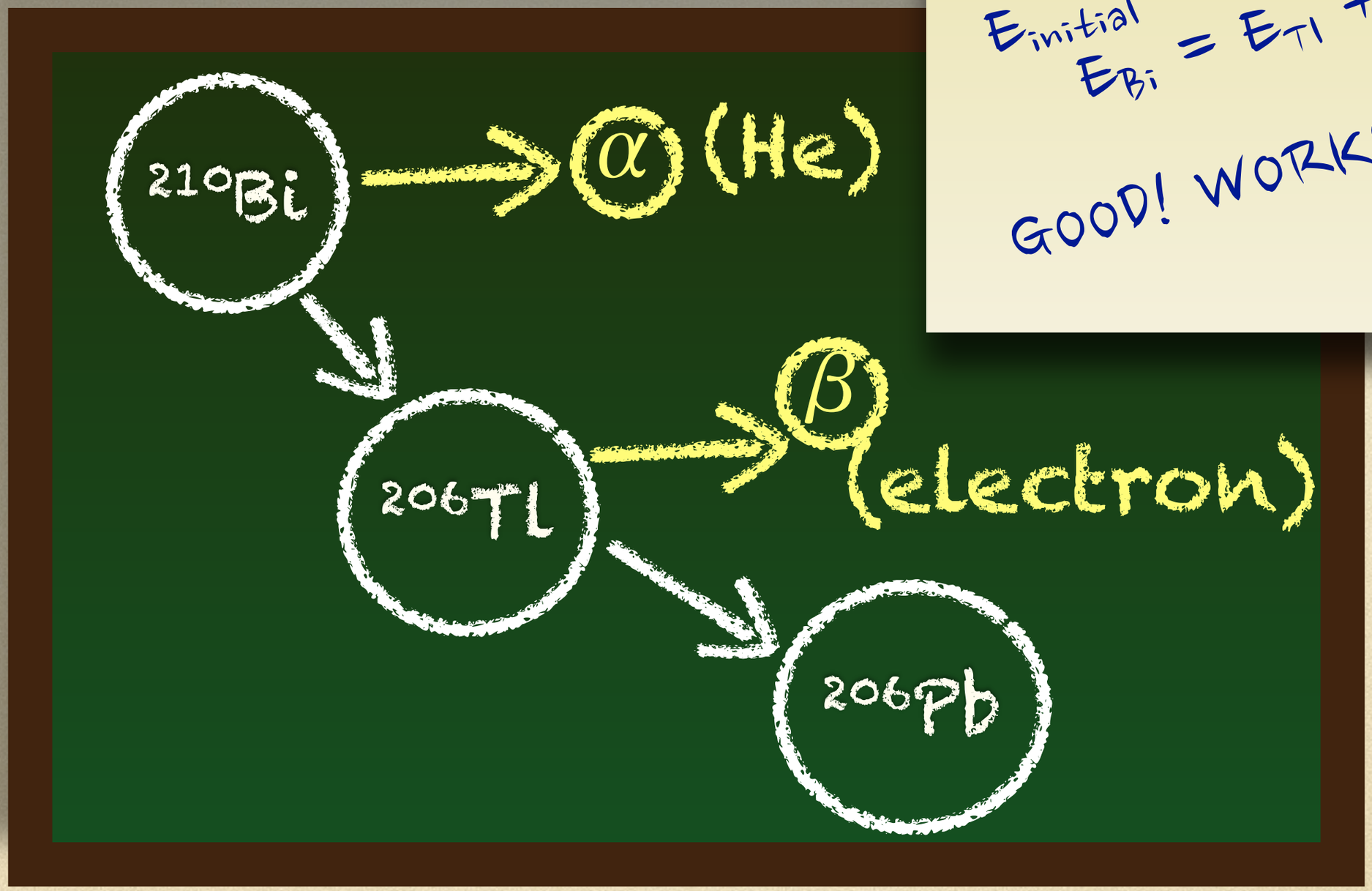
Step 1

$E_{initial} = E_{final}$

$E_{Bi} = E_{Tl} + E_{\alpha}$

GOOD! WORKS!

Missing Energy in β-decay



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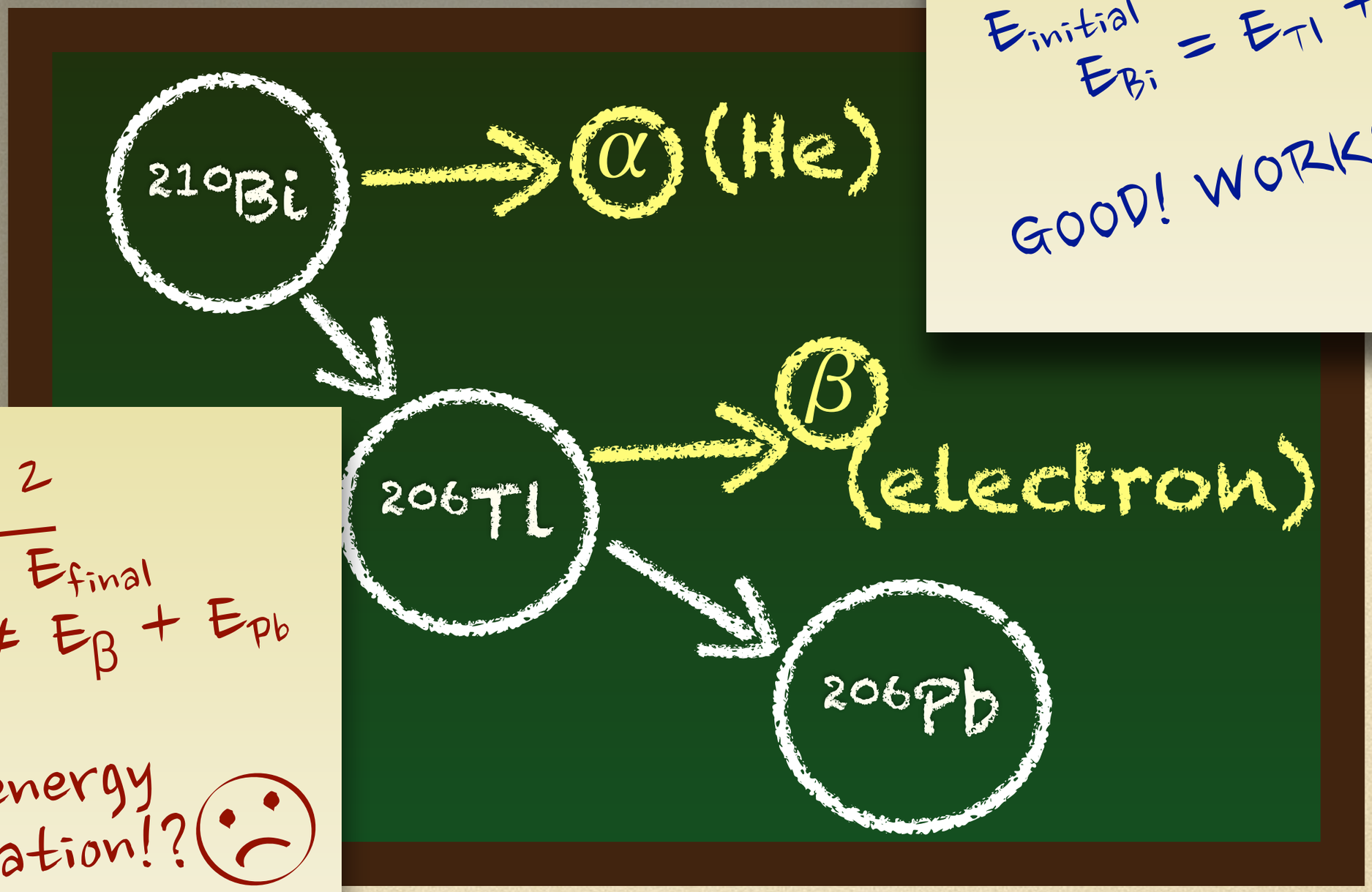


Step 1

$E_{initial} = E_{final}$

$E_{Bi} = E_{Tl} + E_{\alpha}$

GOOD! WORKS!

Step 2

$E_{initial} = E_{final}$

$E_{Tl} \neq E_{\beta} + E_{Pb}$

Fails energy conservation!?

A Party Faux Pas!

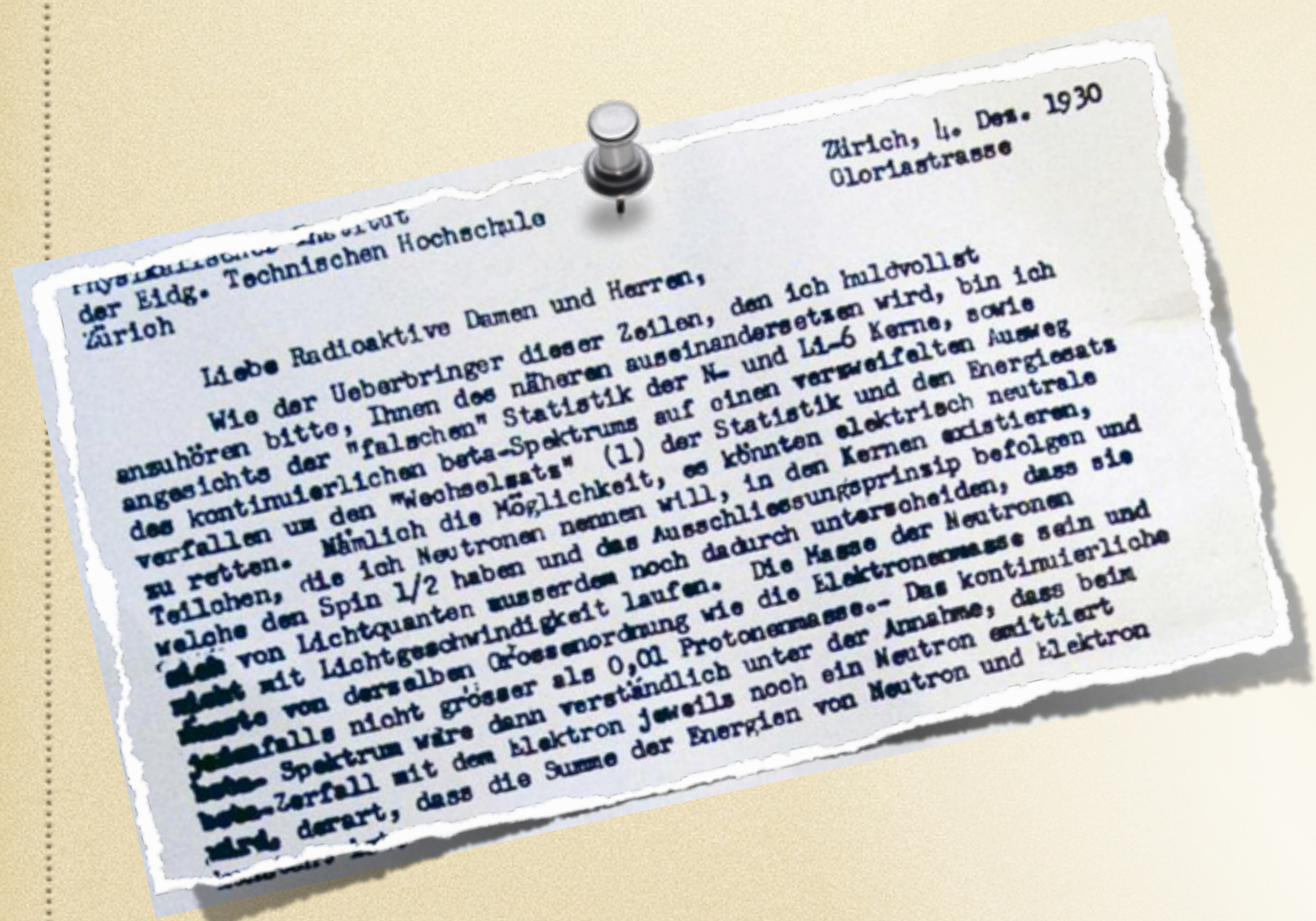
4 December 1930

Wolfgang Pauli

A Desperate Remedy

4 December 1930

- Pauli proposes an INVISIBLE particle, now known as the neutrino to conserve energy!



Zürich, 4. Dez. 1930
Gloriastrasse

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron

Neutrino Wanted!

Dead or Alive!

Neutrino Wanted!

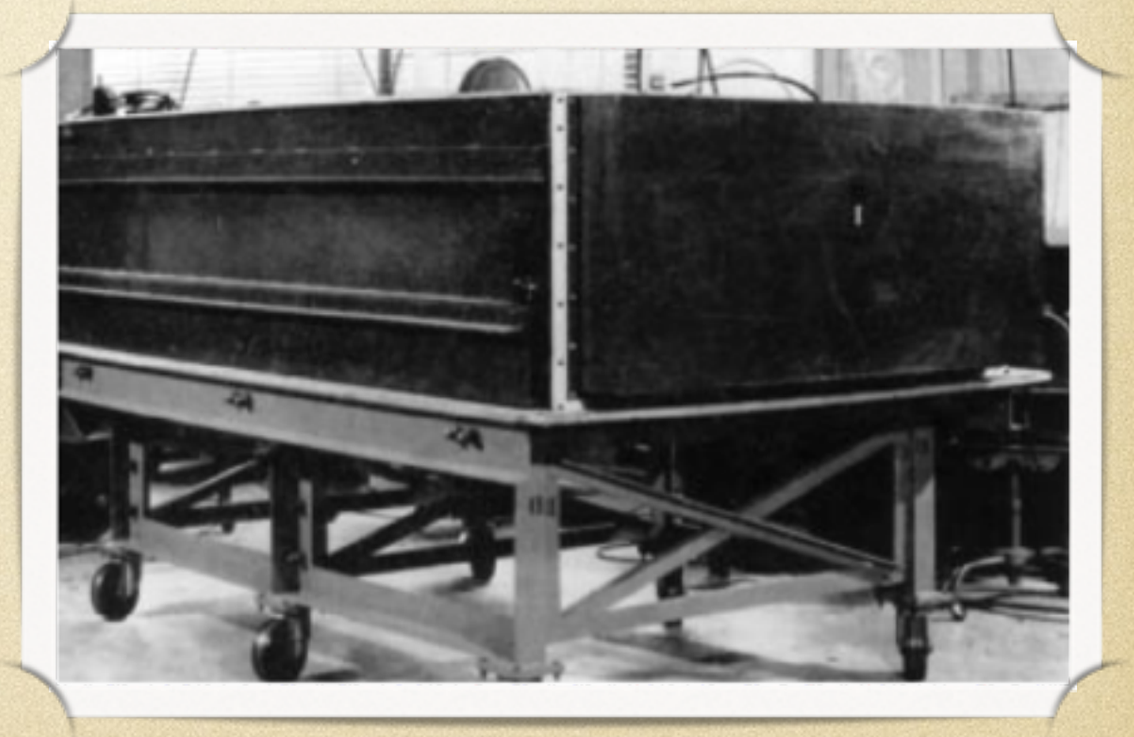
Dead or Alive!

1955

- Savannah River, SC.

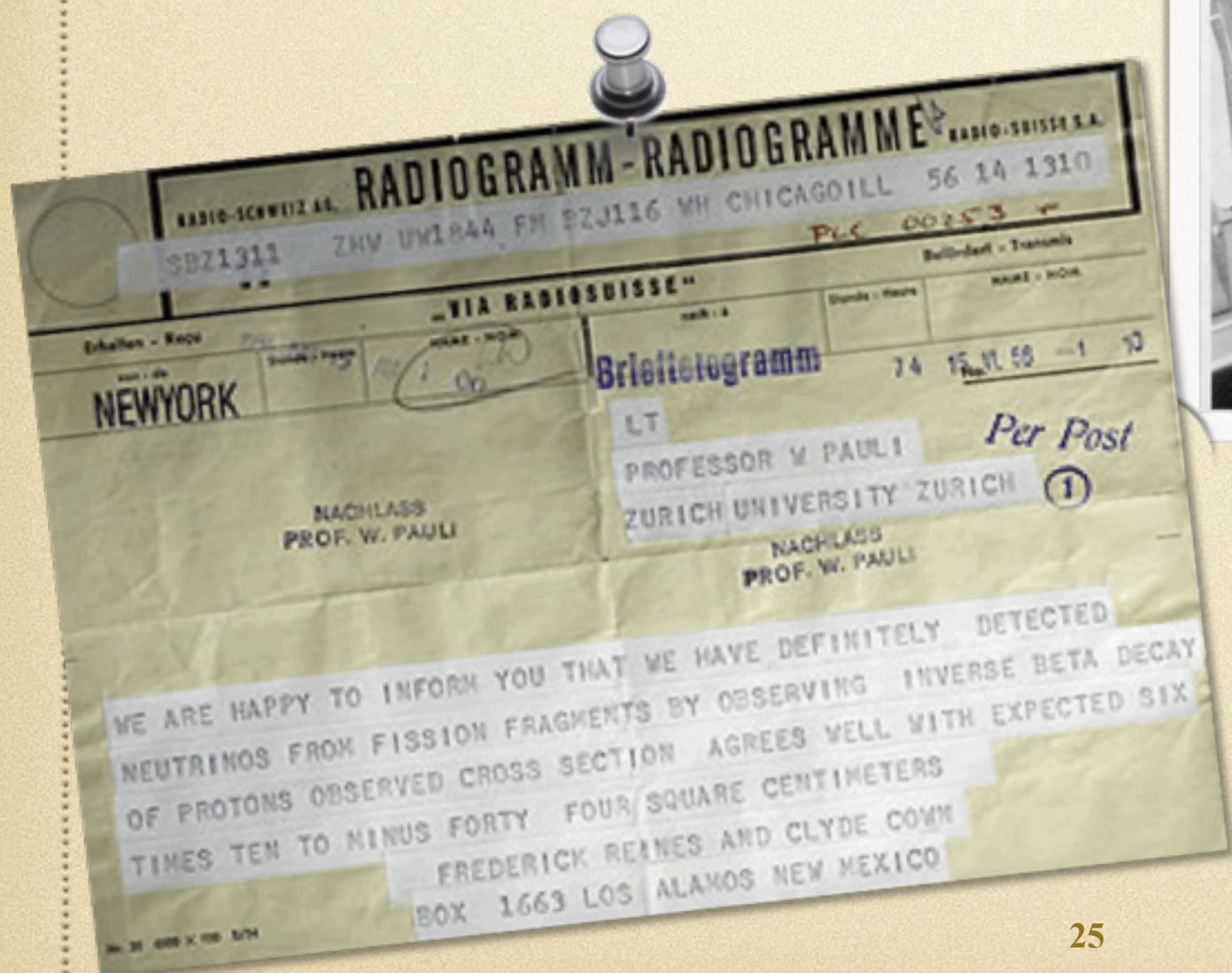
Savannah River
Nuclear Reactor

Neutrino Detector:
tank of water



Neutrino Caught!

14 June 1956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„VIA RADIOSUISSE"

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LT

PROFESSOR W PAULI

ZURICH UNIVERSITY ZURICH

Per Post

1

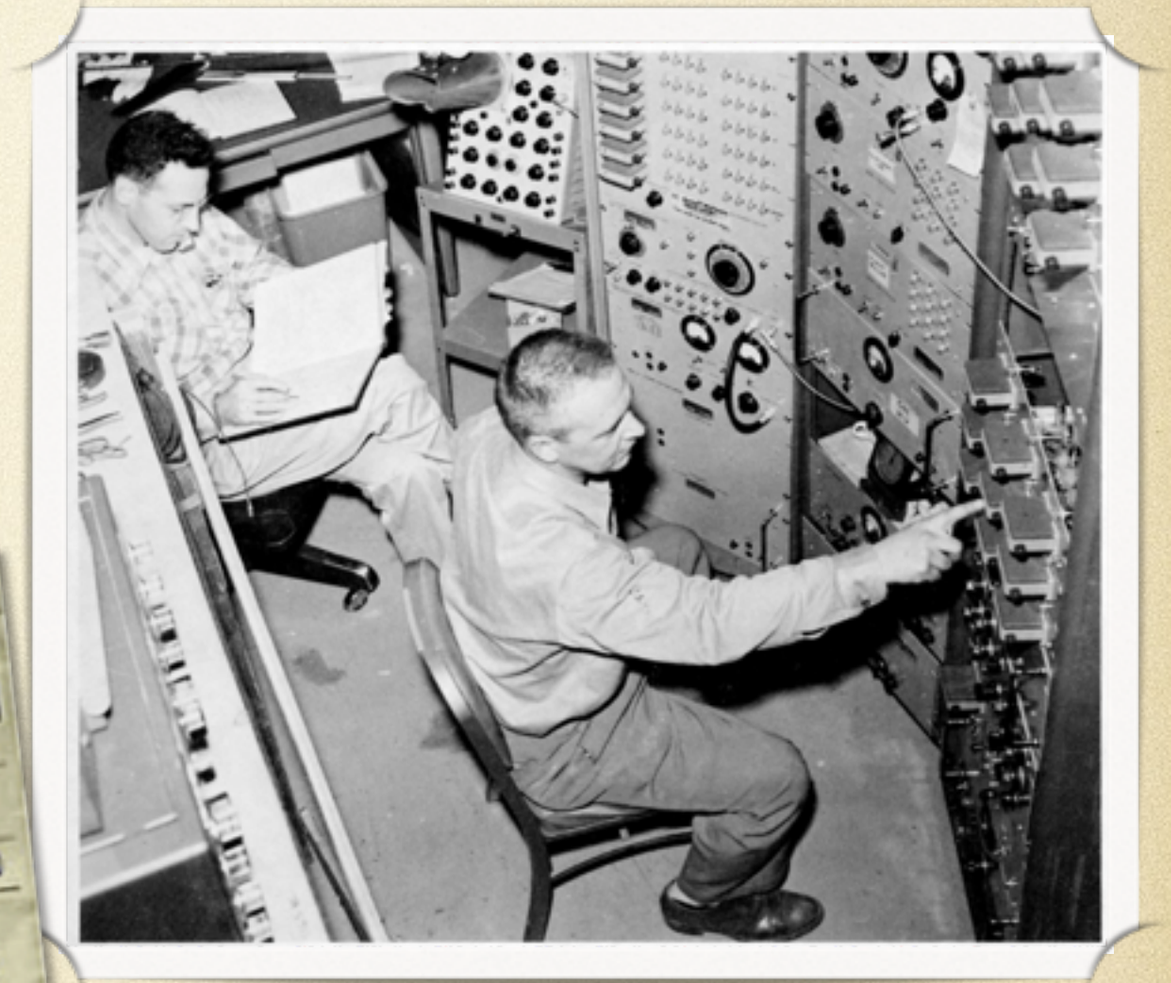
NACHLASS PROF. W. PAULI

NACHLASS PROF. W. PAULI

WE ARE HAPPY TO INFORM YOU THAT WE HAVE DEFINITELY DETECTED NEUTRINOS FROM FISSION FRAGMENTS BY OBSERVING INVERSE BETA DECAY OF PROTONS OBSERVED CROSS SECTION AGREES WELL WITH EXPECTED SIX TIMES TEN TO MINUS FORTY FOUR SQUARE CENTIMETERS

FREDERICK REINES AND CLYDE COWN

BOX 1663 LOS ALAMOS NEW MEXICO

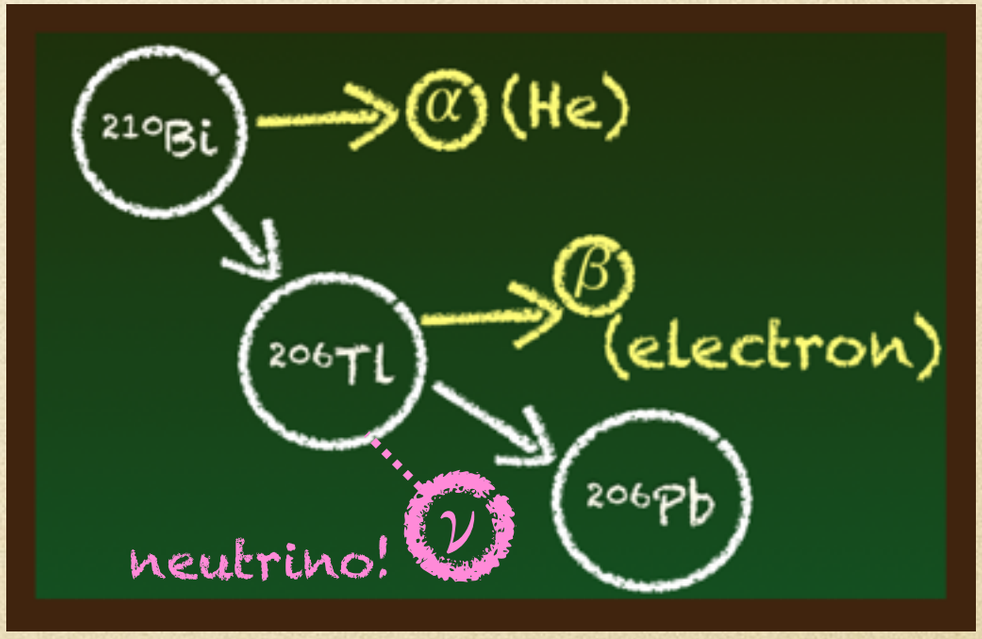


Clyde Cowan &
Frederick Reines

CASE CLOSED

Mystery File 1: Is Energy Conserved?

YES!



Mystery File 2: The Missing Neutrinos

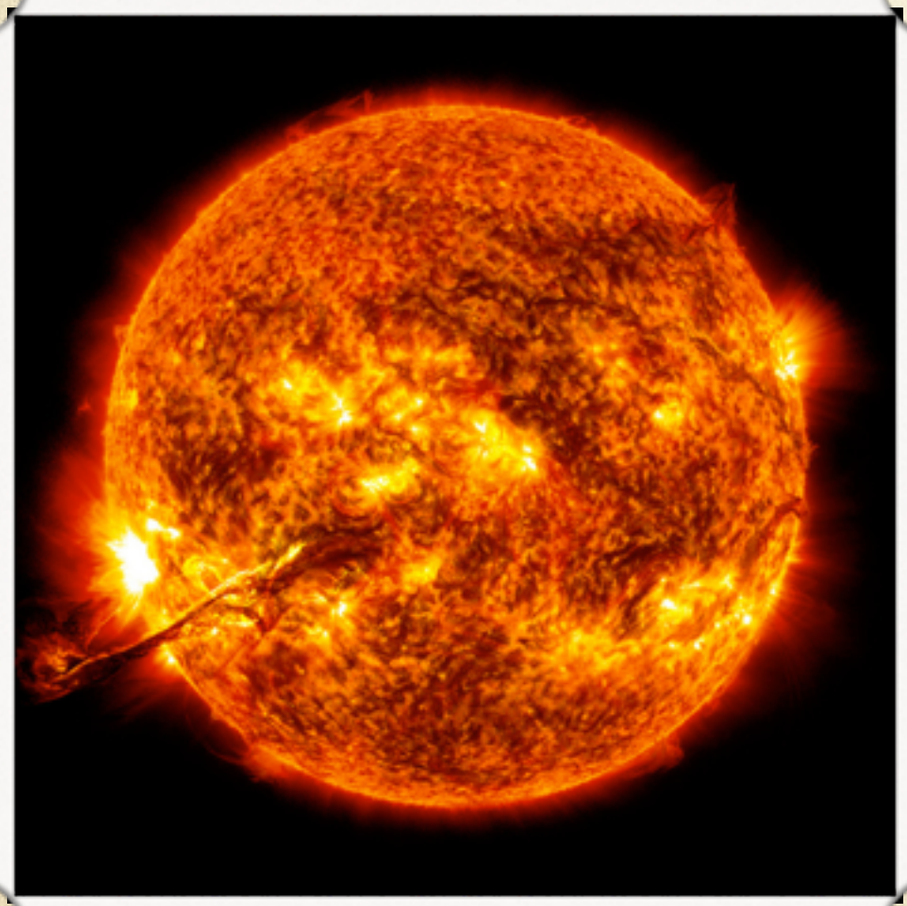
Neutrinos Come in 3 Flavors

Neutrinos Come in 3 Flavors

- electron flavor
- muon flavor
- tau flavor

Hypothesis: The Sun Emits Electron Neutrinos

1960's



The Sun

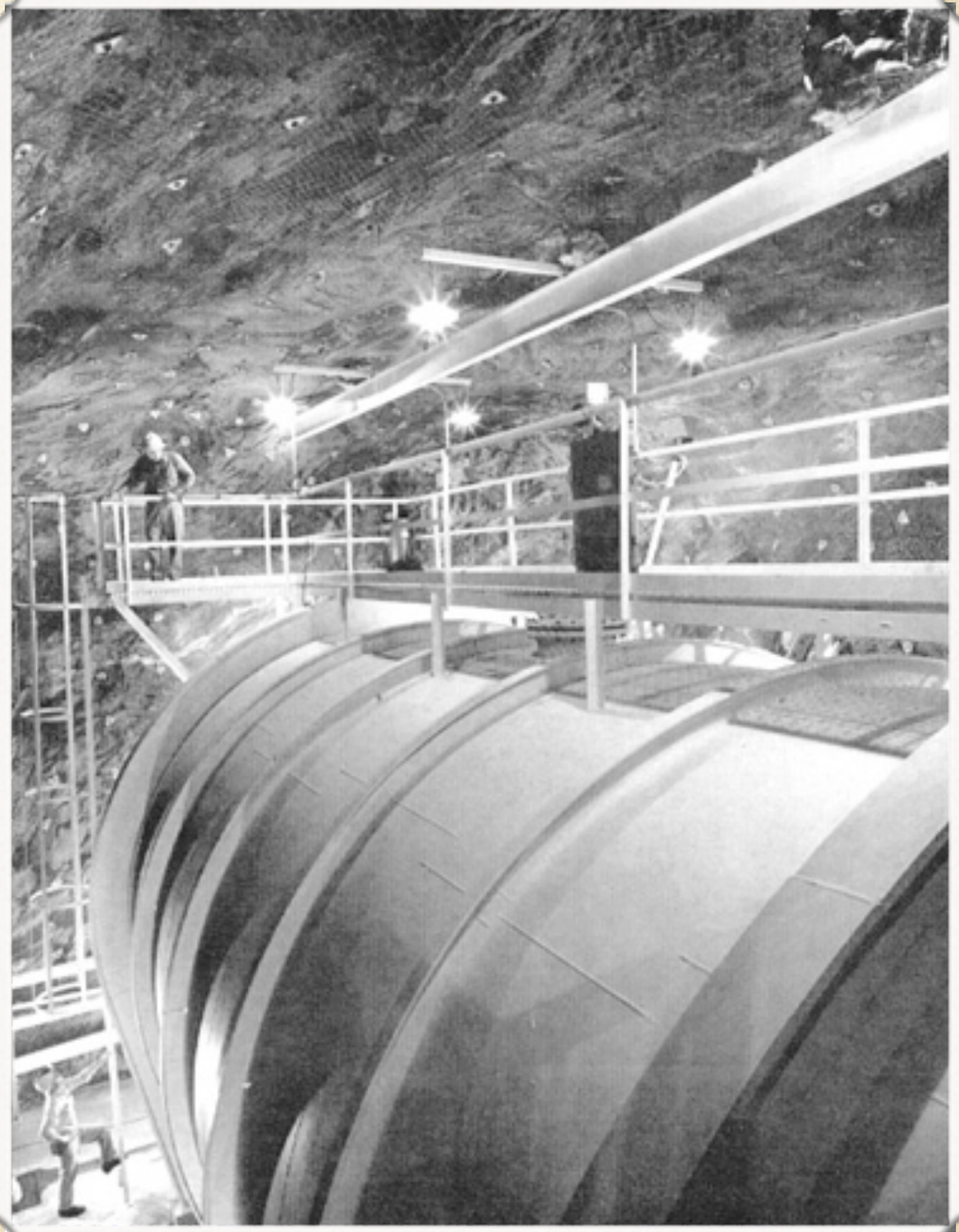
John Bahcall

Clue: The Homestake Mine Experiment

1960's

- Lead, South Dakota

Ray Davis and John Bahcall



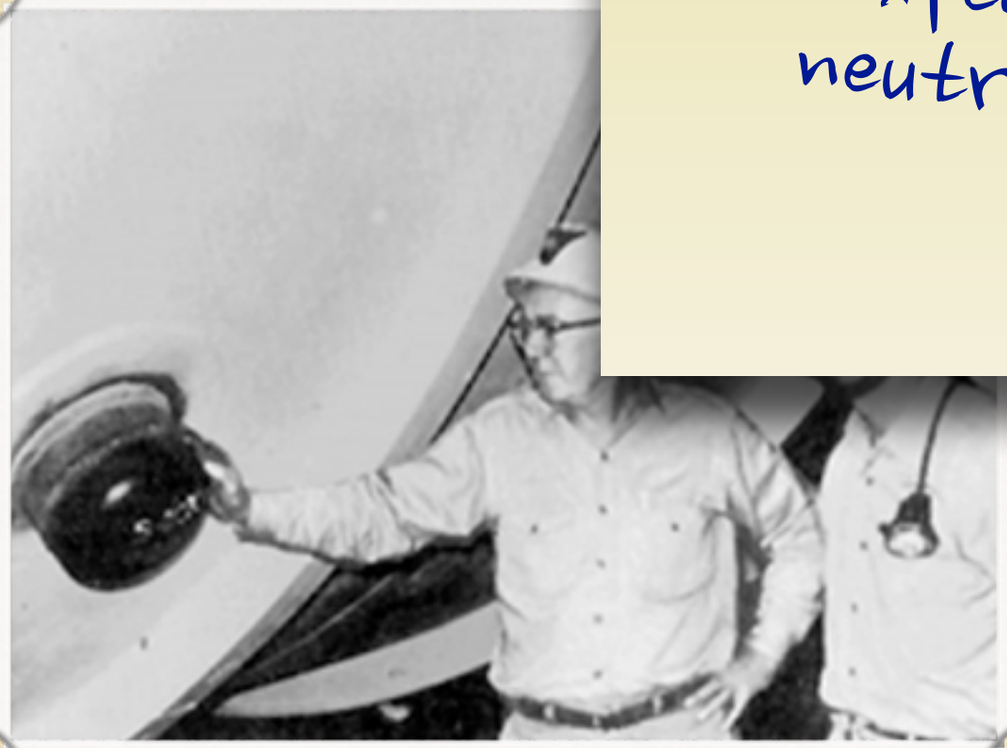
Neutrino detector: a tank of chlorine

Clue: The Homestake Mine Experiment

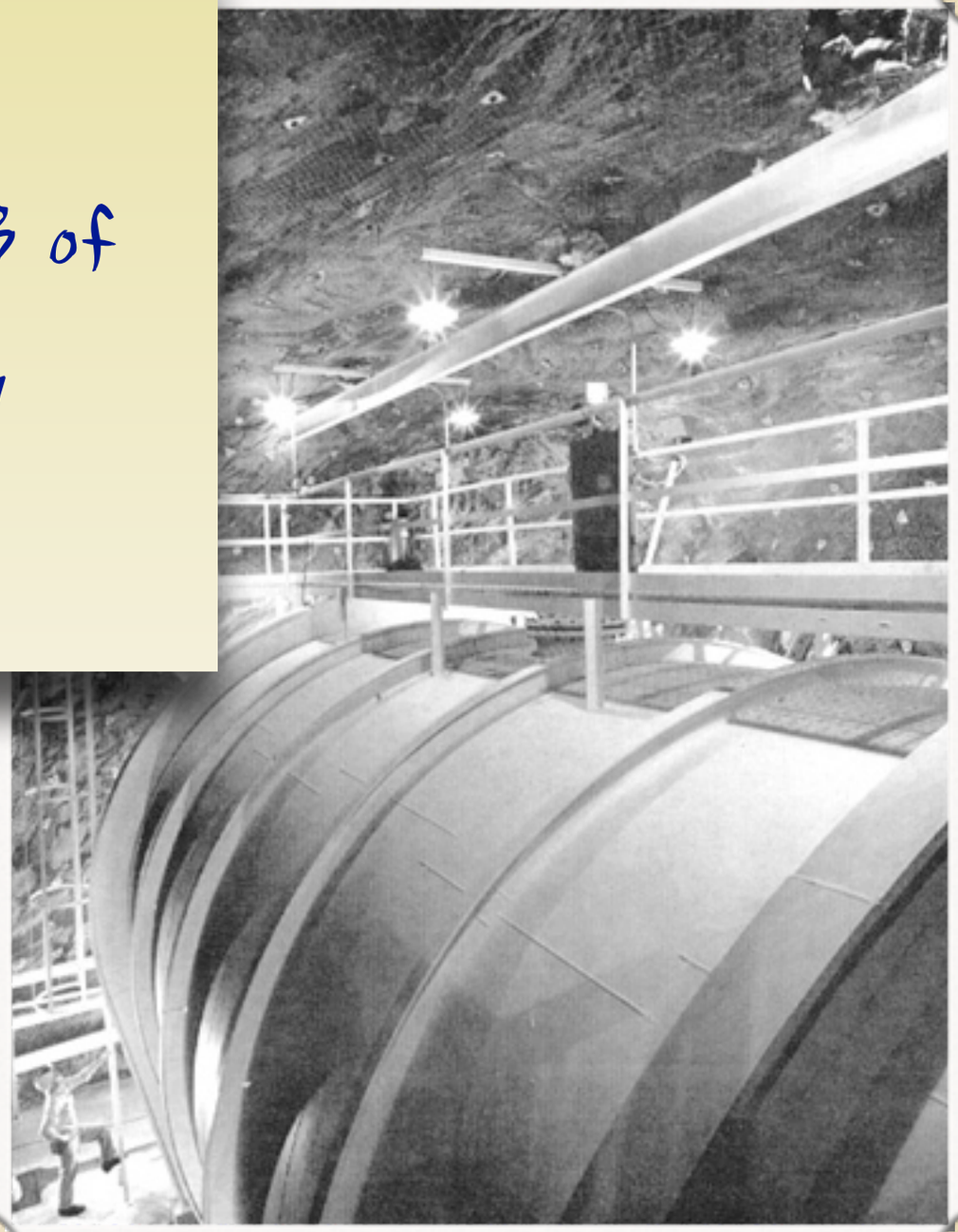
1960's

- Lead, South

Only saw 1/3 of expected neutrinos!



Ray Davis and John Bahcall



Neutrino detector: a tank of chlorine

Final Hypothesis: Travelling Neutrinos Oscillate in Flavor!

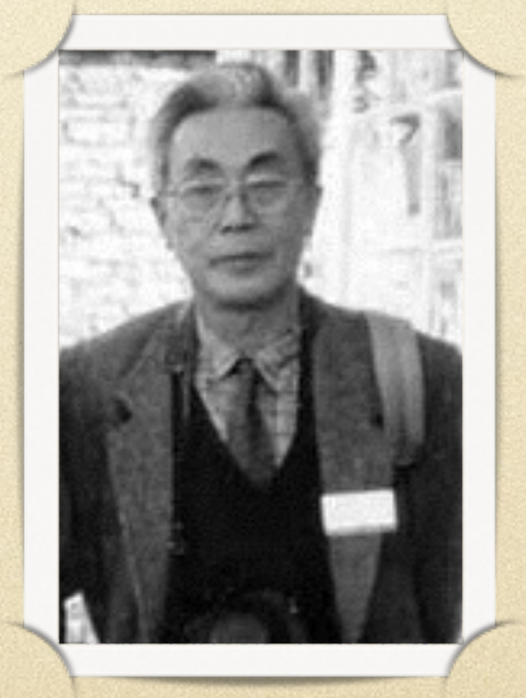
1968

Bruno Pontecorvo

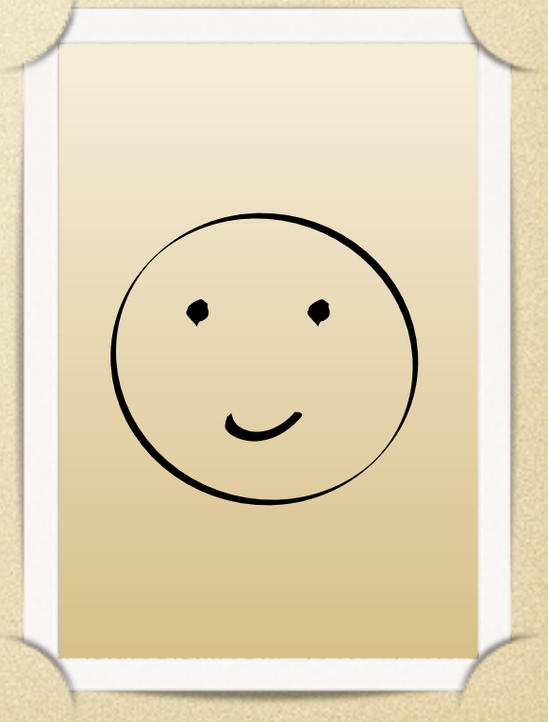
Shoichi Sakata

- Neutrinos don't have a definite mass AND flavor. Only one or the other.

Ziro Maki

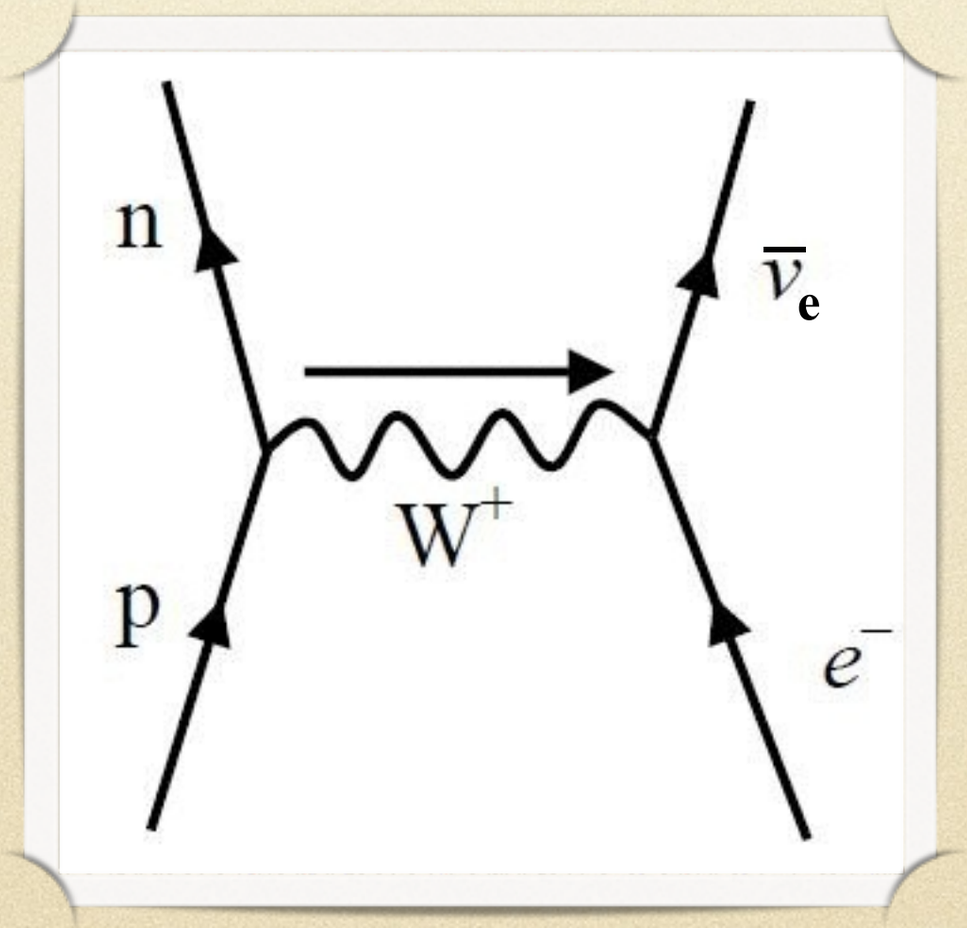


Masami Nakagawa



Neutrino Mixing (short scale)

- Neutrino has definite flavor, but 3 possible masses.



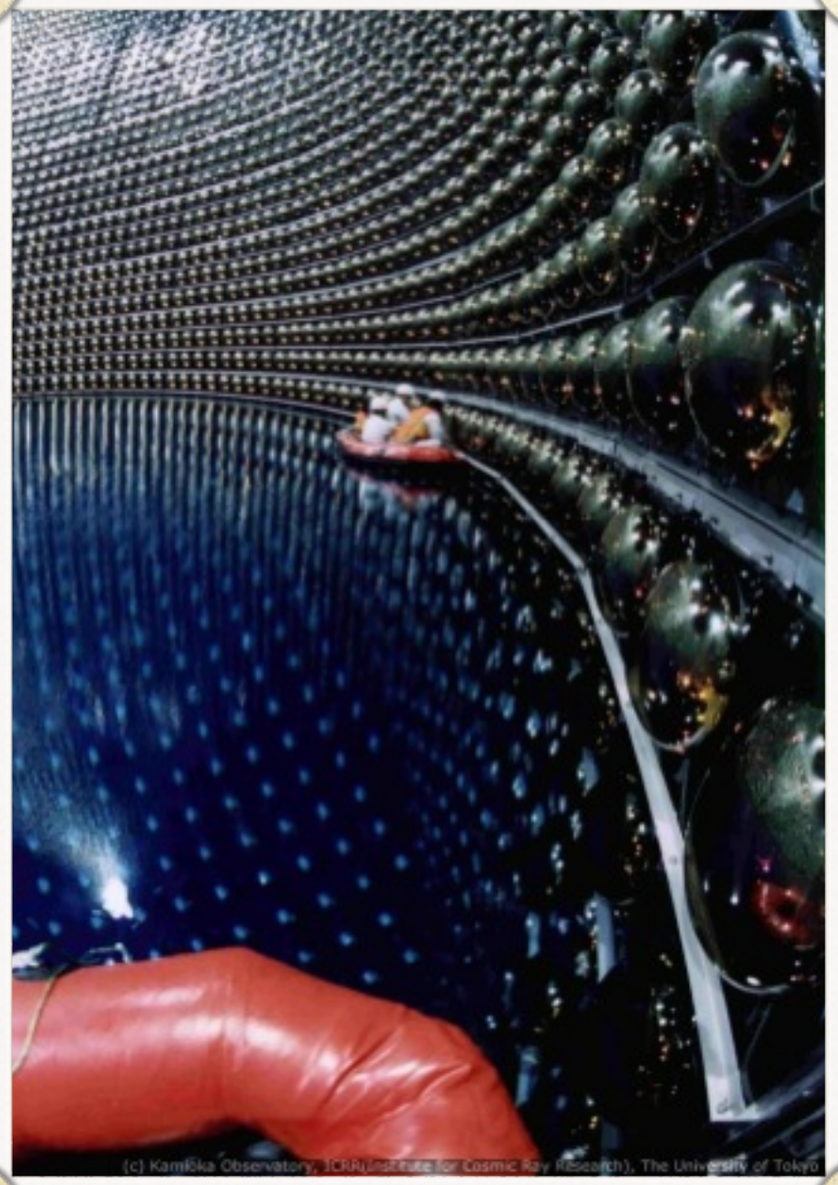
Neutrino Mixing (long scale)

- Neutrino has definite mass, but 3 possible flavors.

$$\nu_e \longrightarrow \nu_\mu \longrightarrow \nu_\tau \longrightarrow \nu_\mu \longrightarrow \nu_\tau \longrightarrow \nu_e$$

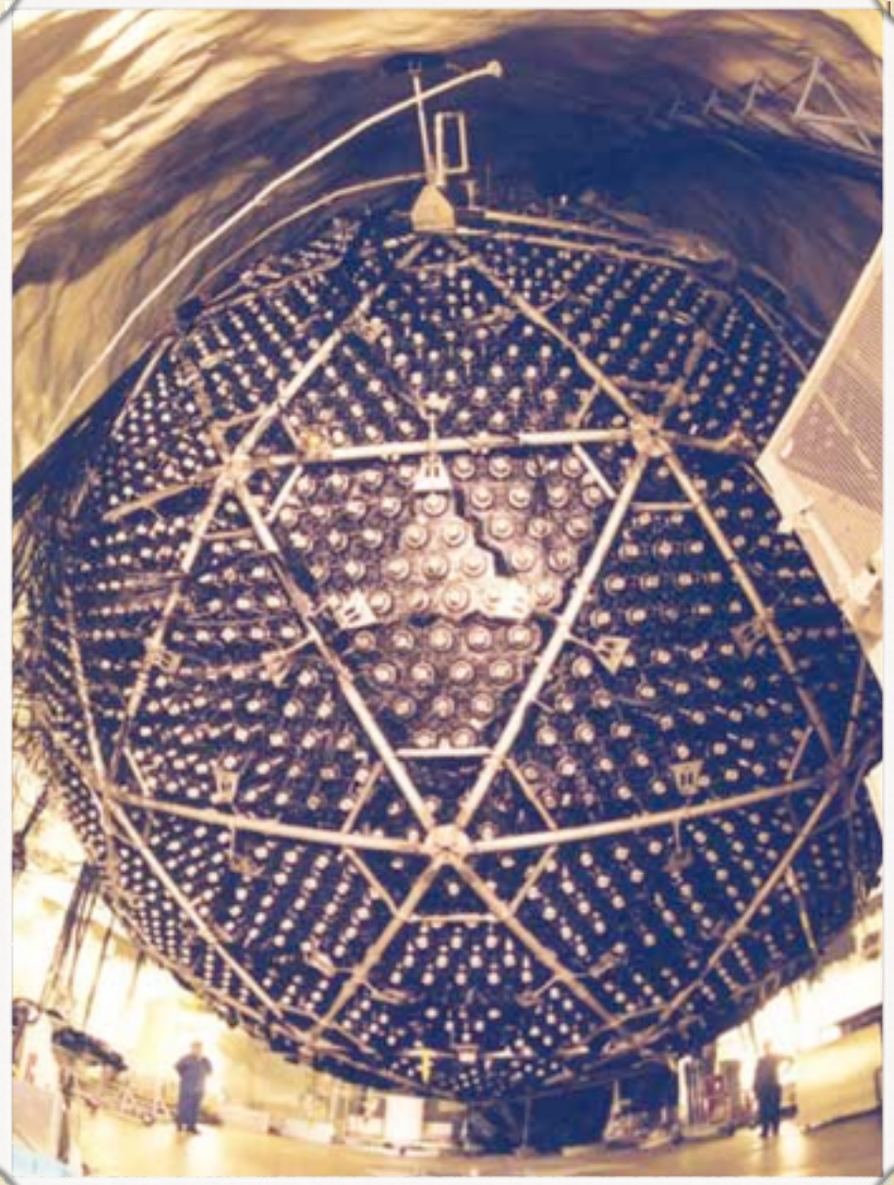
Conclusive Evidence

3 July 1998



Super Kamiokande, Japan.

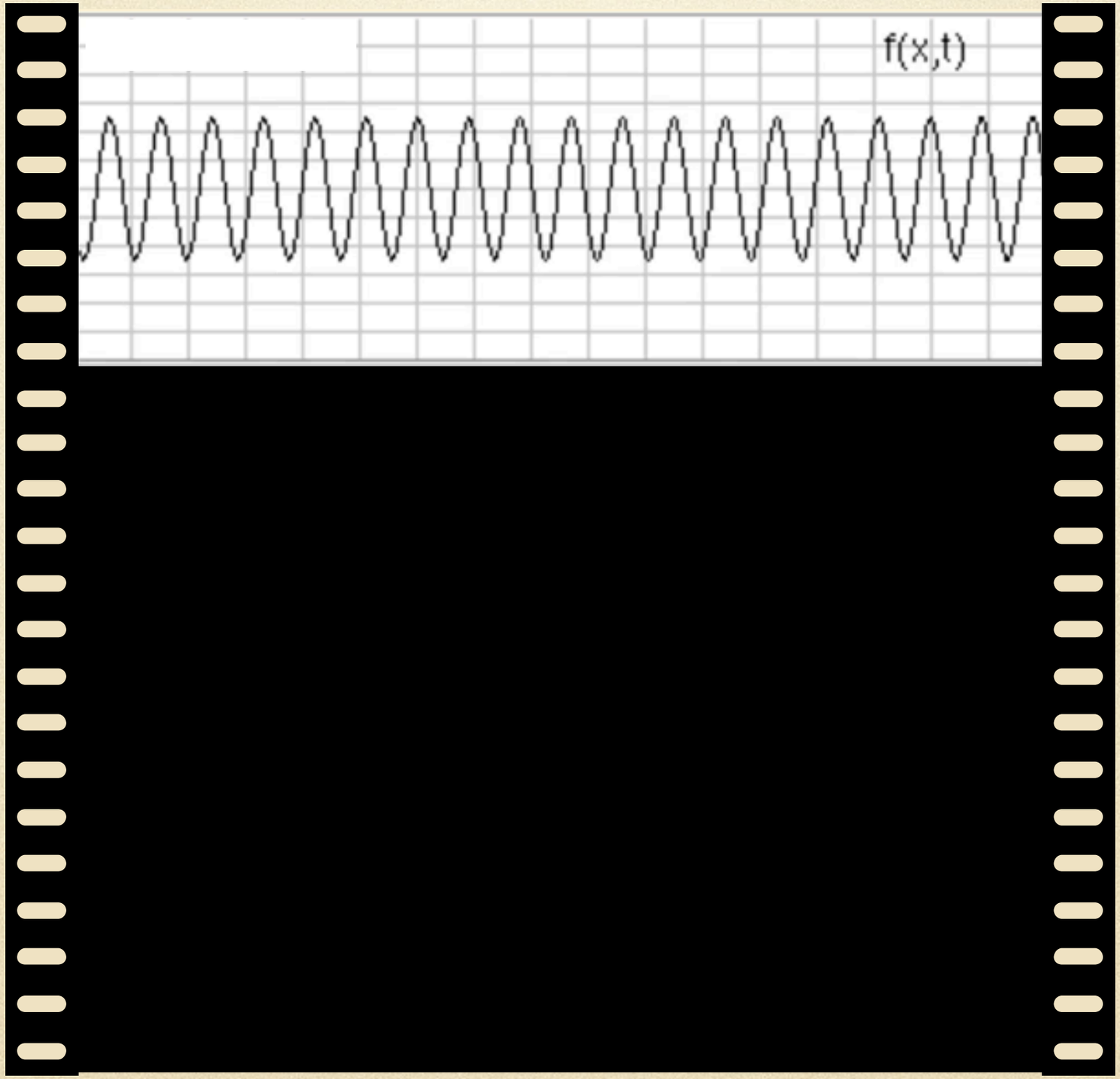
18 June 2001



Sudbury Neutrino Observatory, Canada.

Neutrino Oscillation Probability

$\nu_e \rightarrow \nu_\mu$



440 Hz

Neutrino Oscillation Probability

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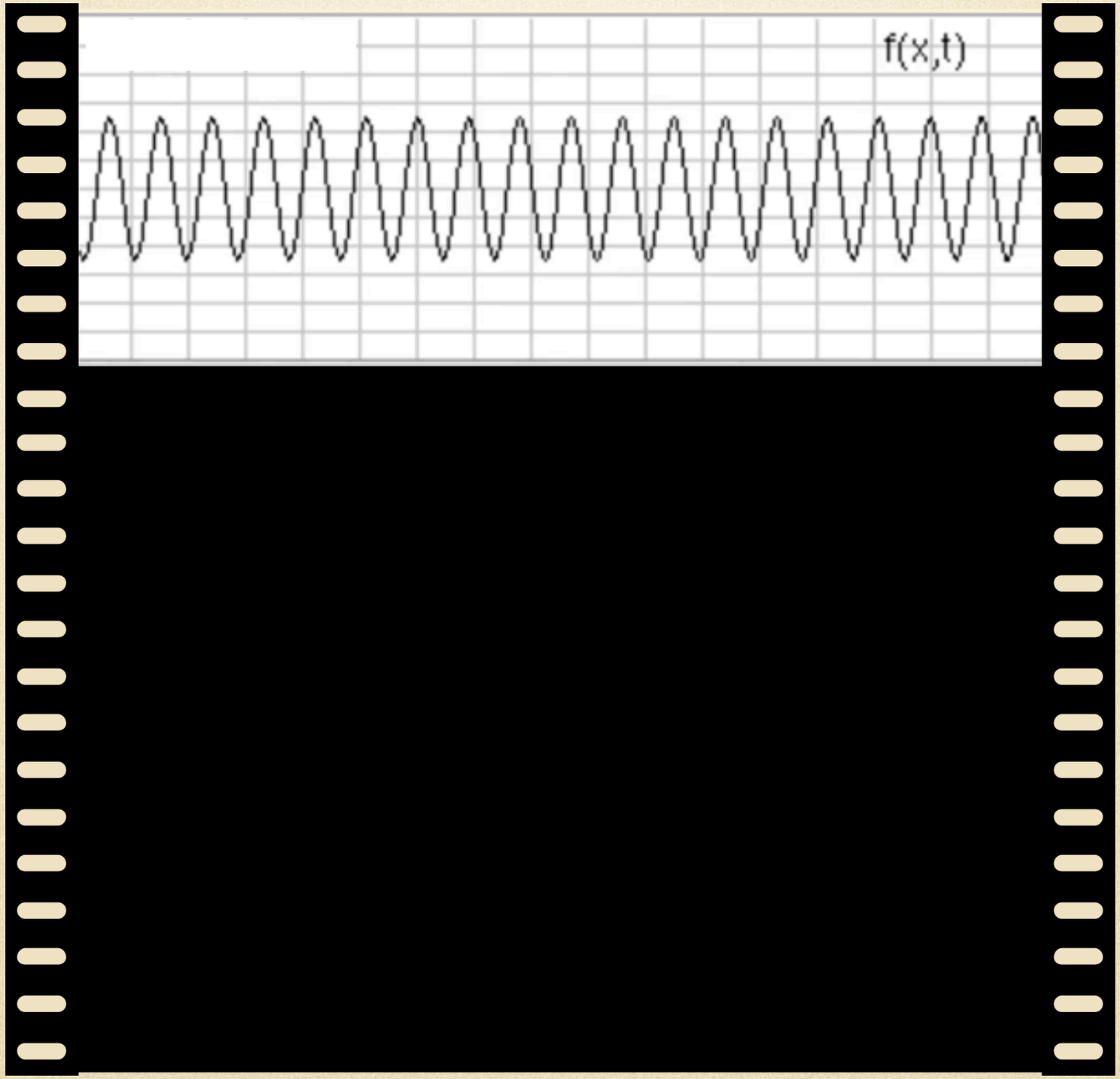

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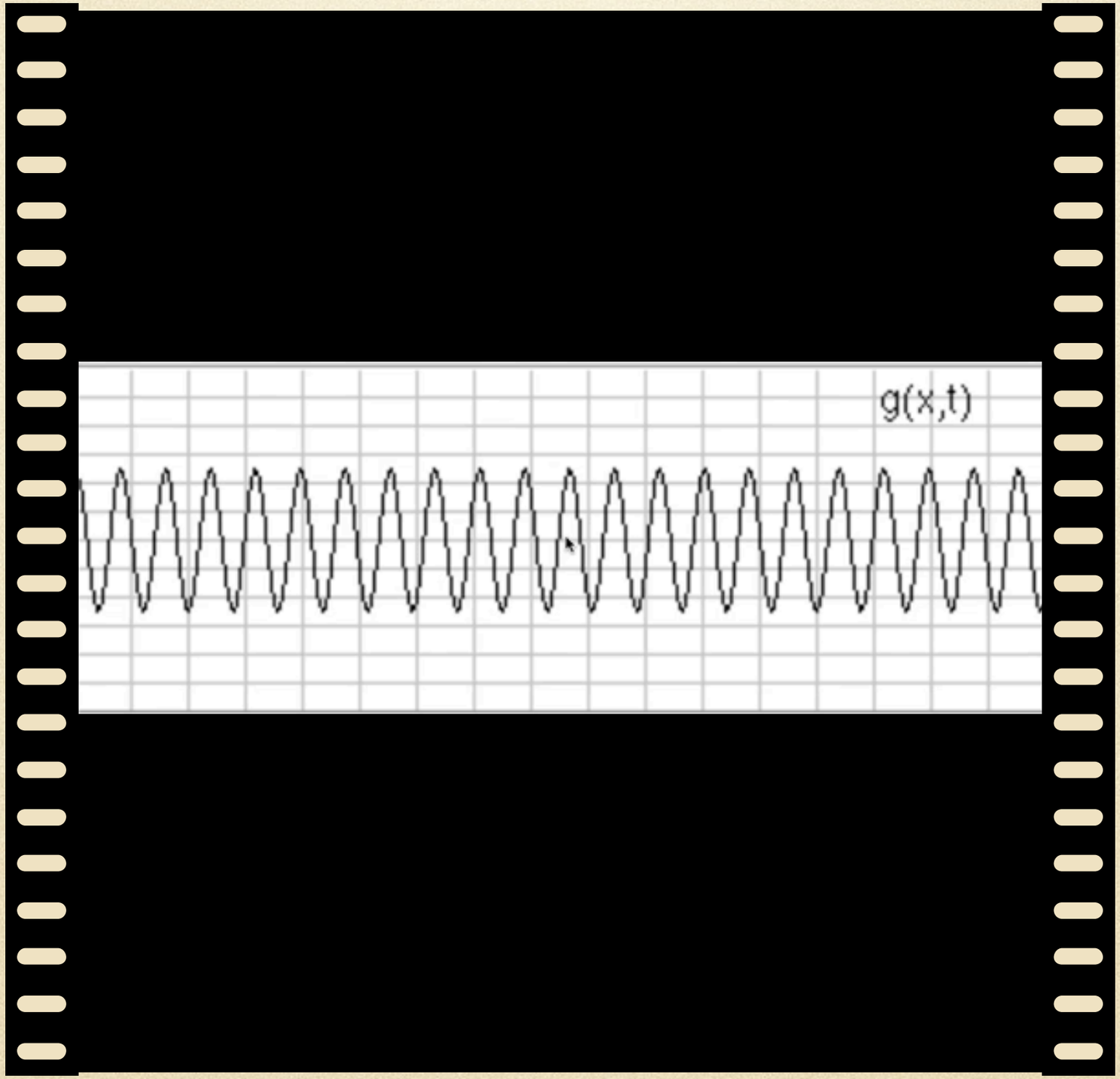

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Neutrino Oscillation Probability

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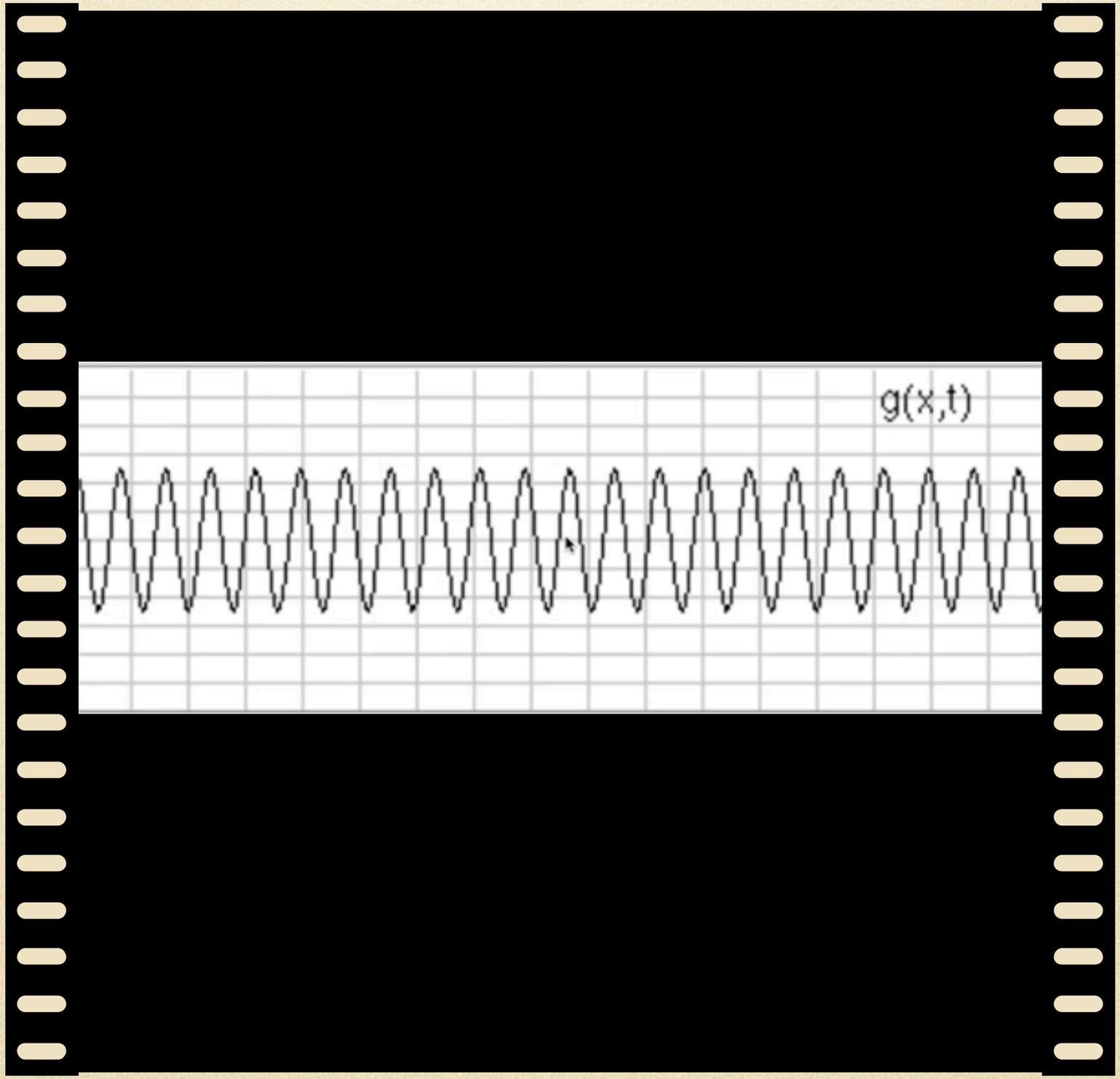

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Neutrino Oscillation Probability

$\nu_e \rightarrow \nu_\mu$

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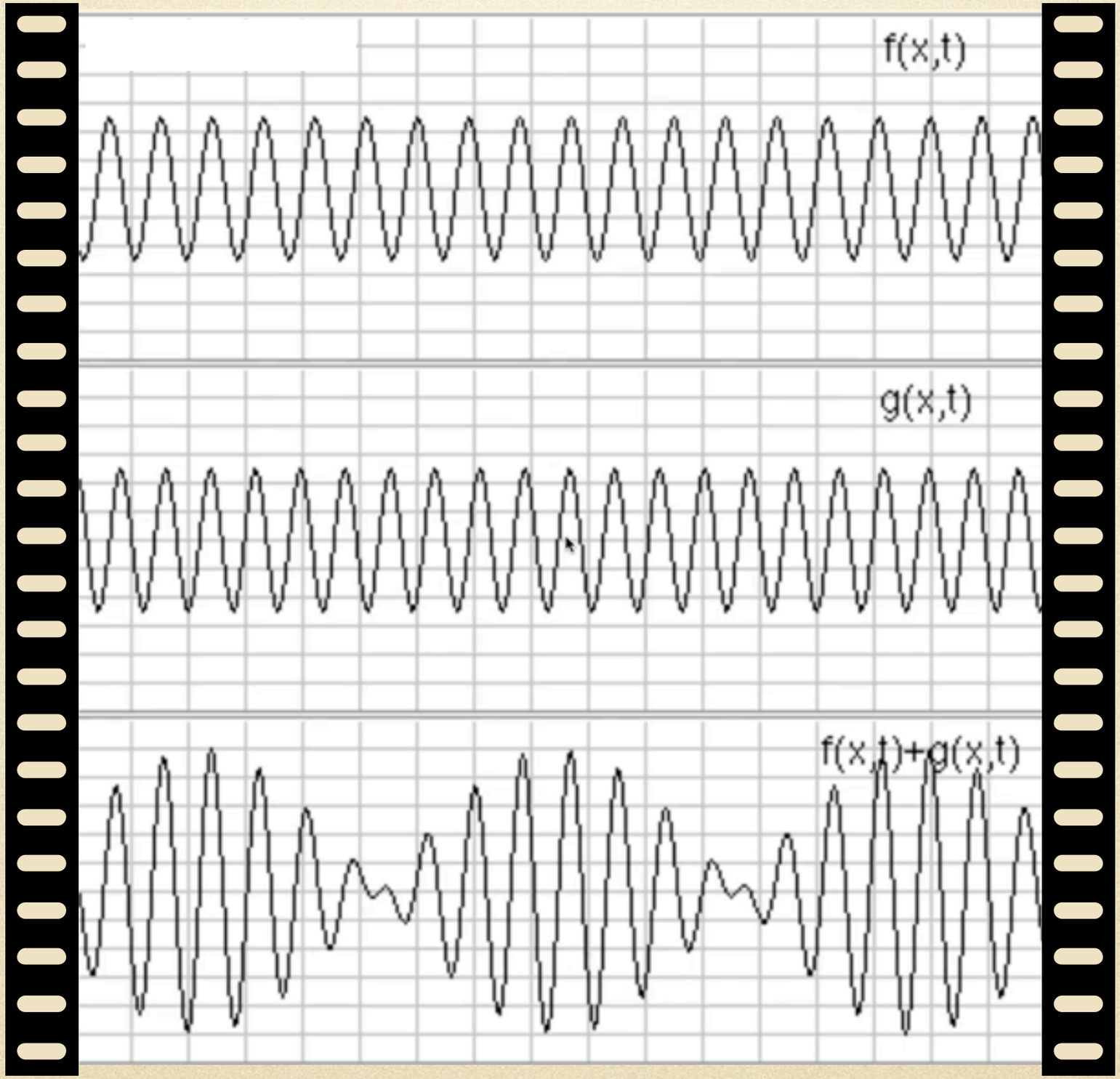
ν_e



440 Hz + 441 Hz

Neutrino Oscillation Probability

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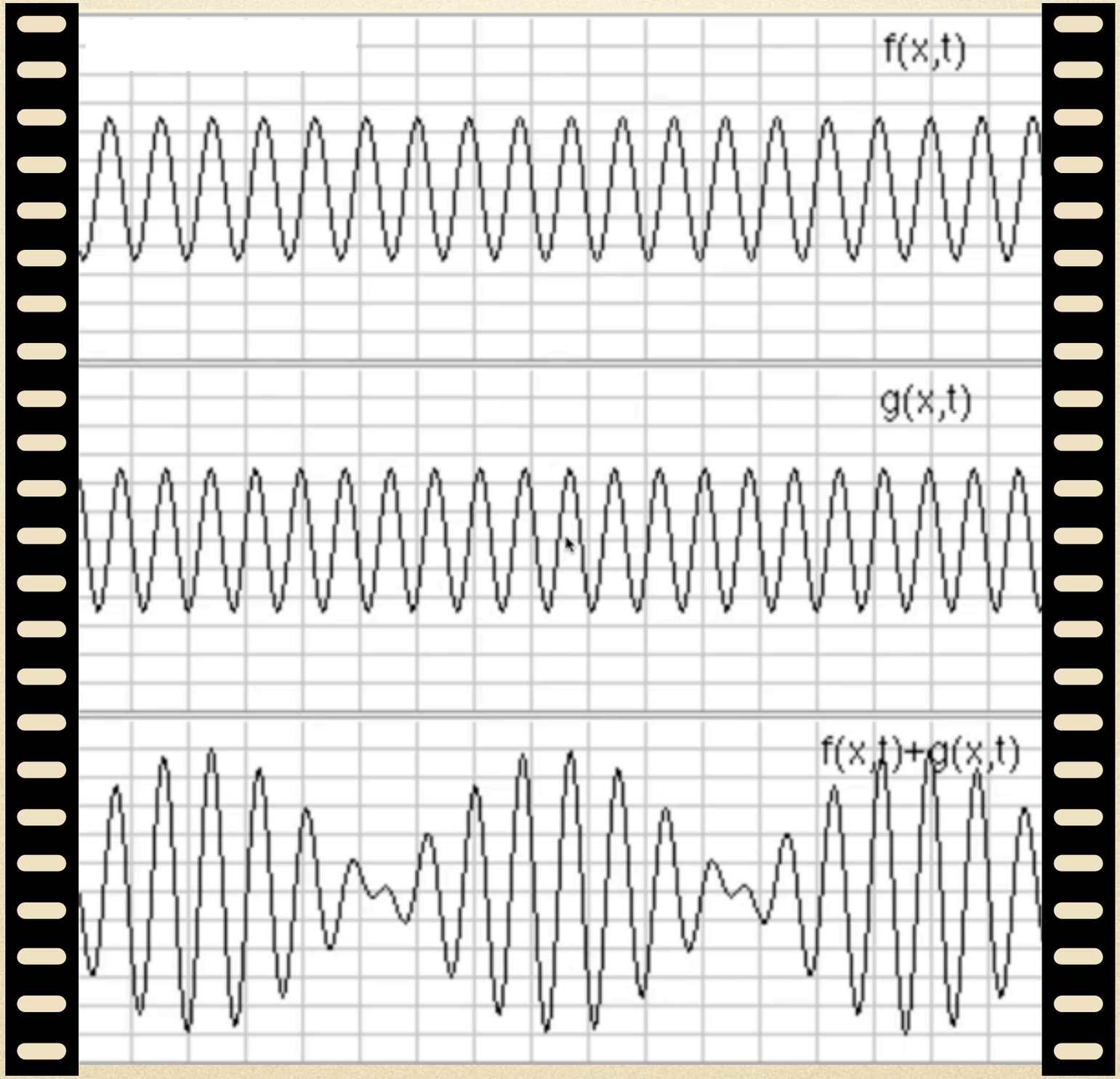
ν_e



440 Hz + 441 Hz

CASE CLOSED

Mystery File 2: The Missing Neutrinos

They are hiding as other flavors!

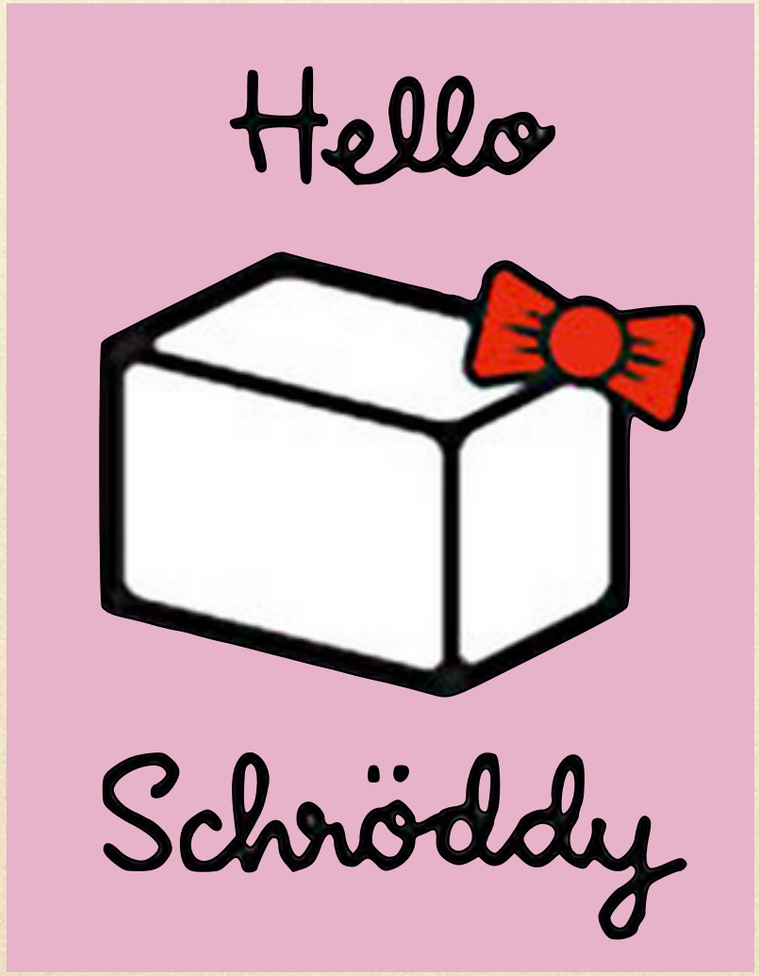
Part 3: Down the Rabbit Hole: Why do neutrinos oscillate?

Why do neutrinos oscillate?

- Review Schrödinger's Equation
- Free particle solution
- Assume some neutrino mixing
- See what happens later

Schrödinger's Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \hat{H}\Psi(\mathbf{r},t)$$



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Free Particle Solution

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  - V(r,t) = 0

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$$\Psi(x,t) = Ae^{ax}e^{bt}$$

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- Use: $E = \frac{p^2}{2m}$ since we only have kinetic energy

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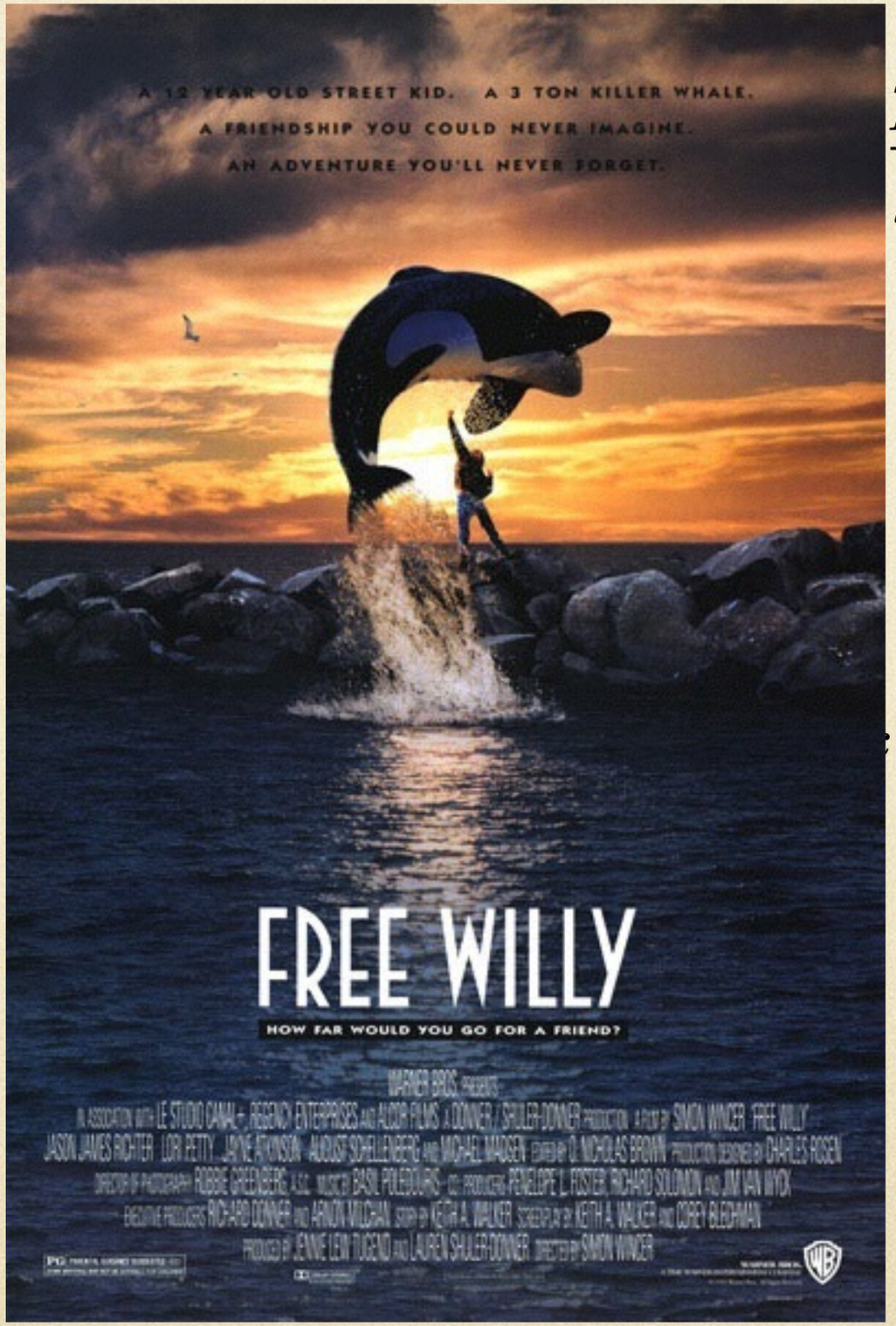

$$\frac{p}{\hbar}$$

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Free Neutrino

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- In bra-ket notation: $\alpha = e, \mu, \tau$

$$|\nu_\alpha(x,t) >= \Psi(x,t)$$

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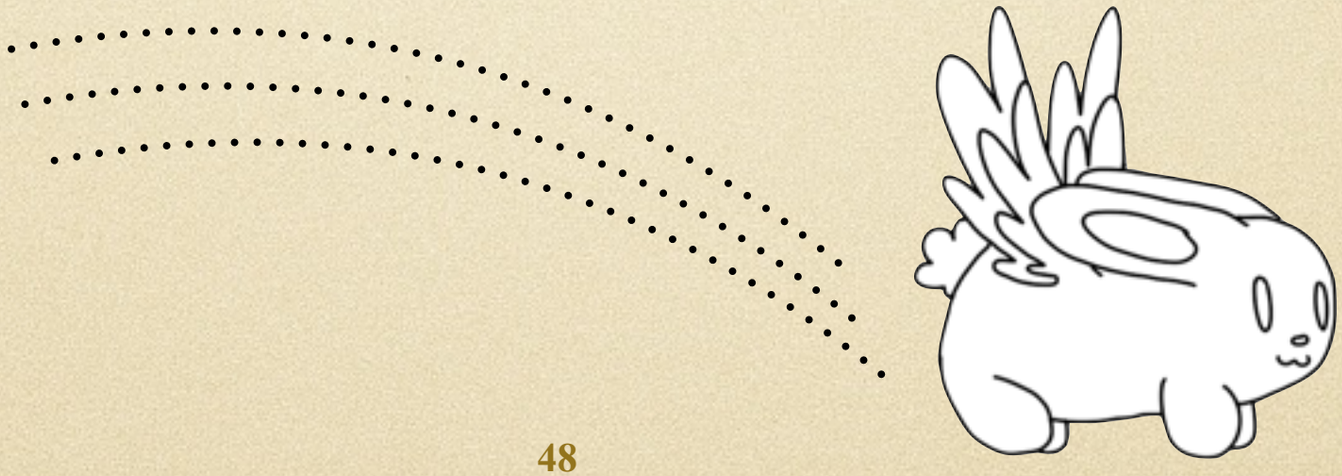

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Assume flavor eigenstates can be written as a linear combination of mass eigenstates.

θ is the "mixing angle"

Seeing what happens

- Let's compute probability of ν_e oscillating some distance later to ν_μ.

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$$\cdot (\cos\theta|\nu_1(0,0) > + \sin\theta|\nu_2(0,0) >)$$

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Seeing what happens

$$< \nu_\mu(x,t)|\nu_e(0,0) > \; = -\sin\theta\cos\theta e^{i\phi_1} + \sin\theta\cos\theta e^{i\phi_2}$$

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$$= -sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2}$$

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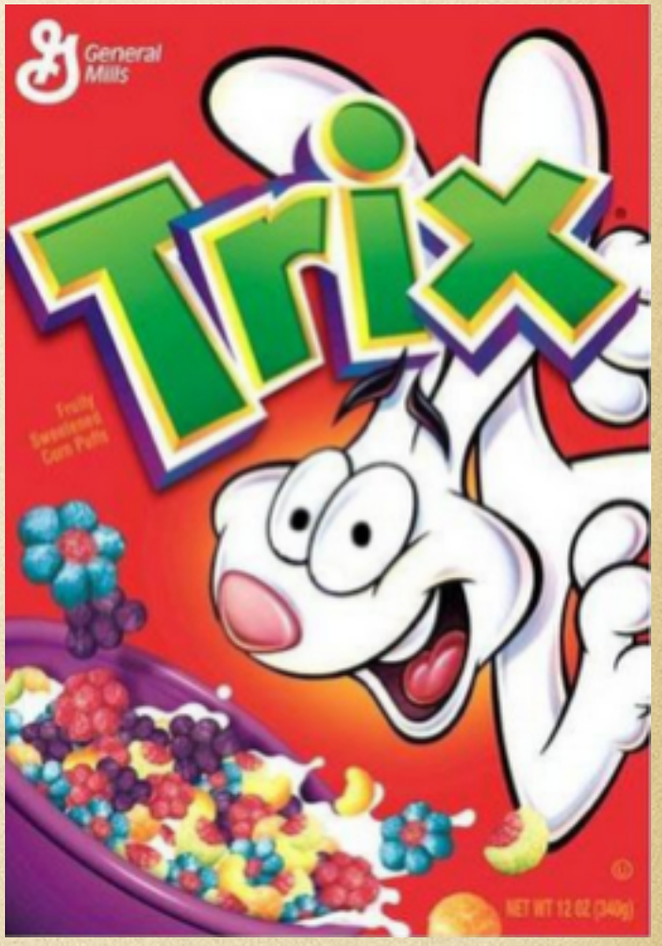

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- Plug in $p_1 = p_2 = E$ and $E_1 = E + \frac{m_1^2}{2E}$ from last page

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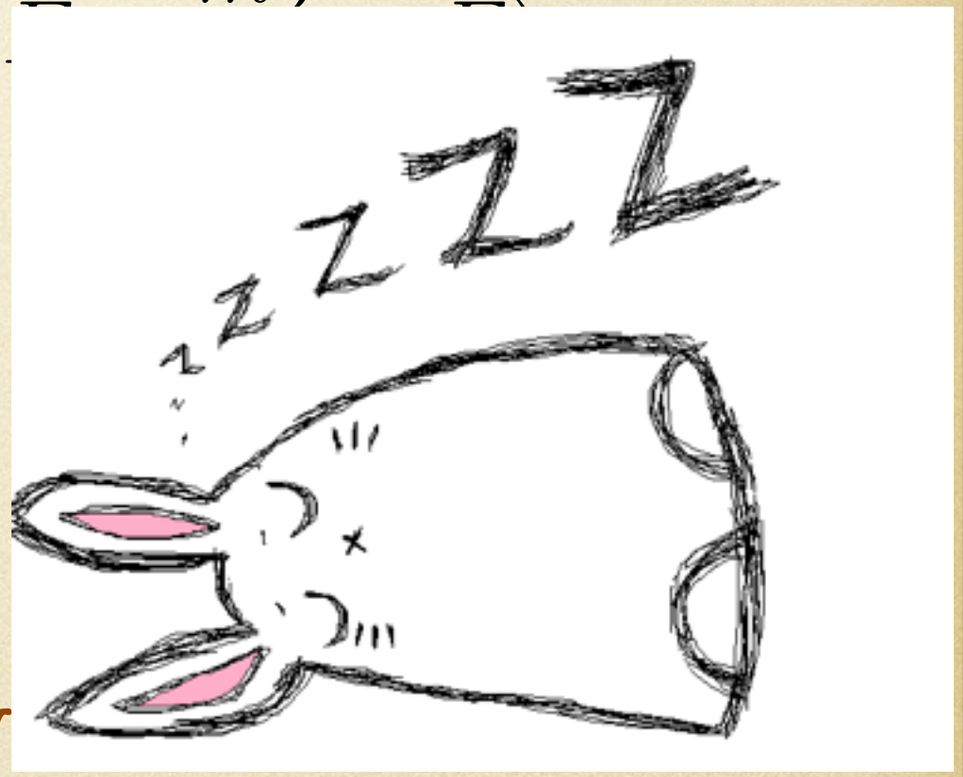

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Oscillation Probability

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where Δm_{12}^2 is in units of eV^2, L is km, E is GeV

So what!?

gawd, I wish the speaker used a few more cartoon bunnies to make whole this thing easier

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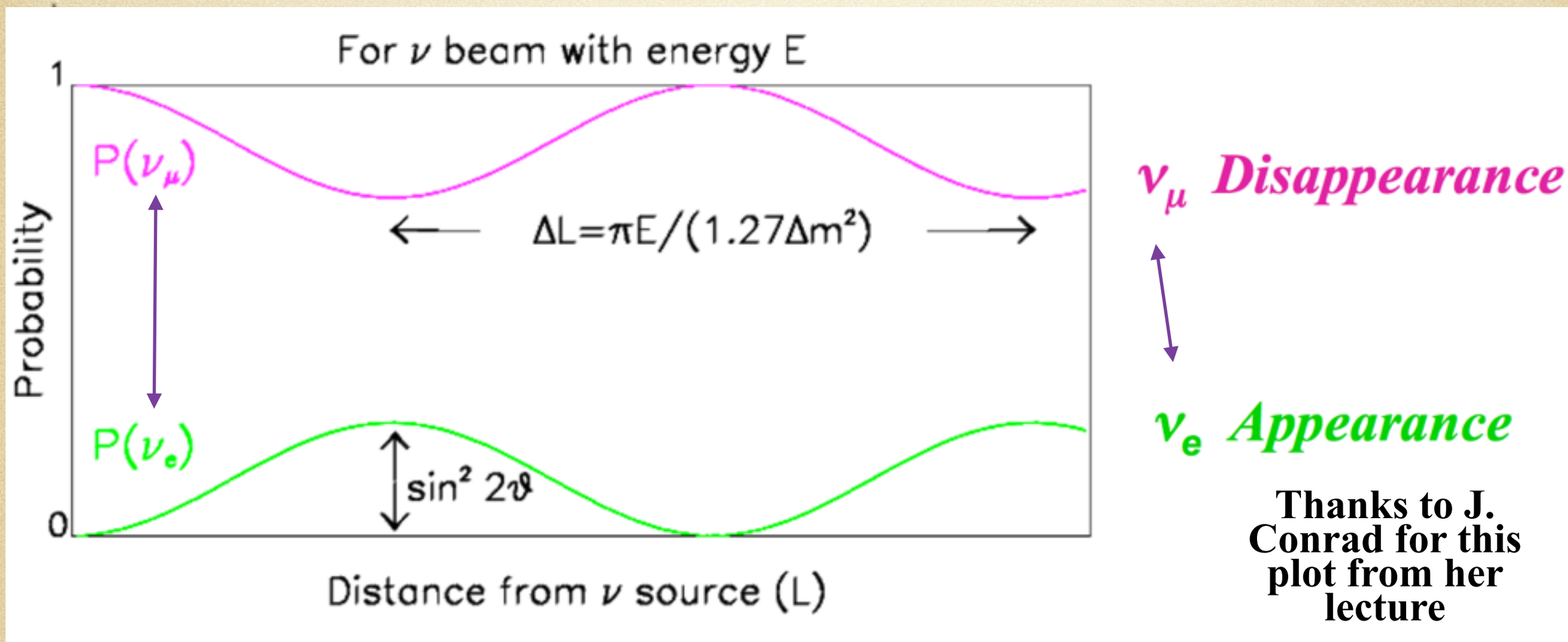
deduce ν mass > 0

Finally

- It is left as an exercise to the reader to expand the 2 neutrino mixing case to the 3 neutrino case.

Thanks for your attention!

- Lots of cool things I didn't talk about:
  - current experiments
  - sterile neutrinos
  - CP-violation
  - bunnies
  - technological advances



Thanks to J. Conrad for this plot from her lecture

$$P(\nu_e \rightarrow \nu_\mu) = sin^2(2\theta)\sin^2\left(1.27\Delta m^2_{12}\frac{L}{E}\right)$$

We observe this happening in experiments

deduce ν mass > 0

Unsolved

File 3: Symmetry Violations

Fact: Universe is Made of Matter (Not Antimatter)

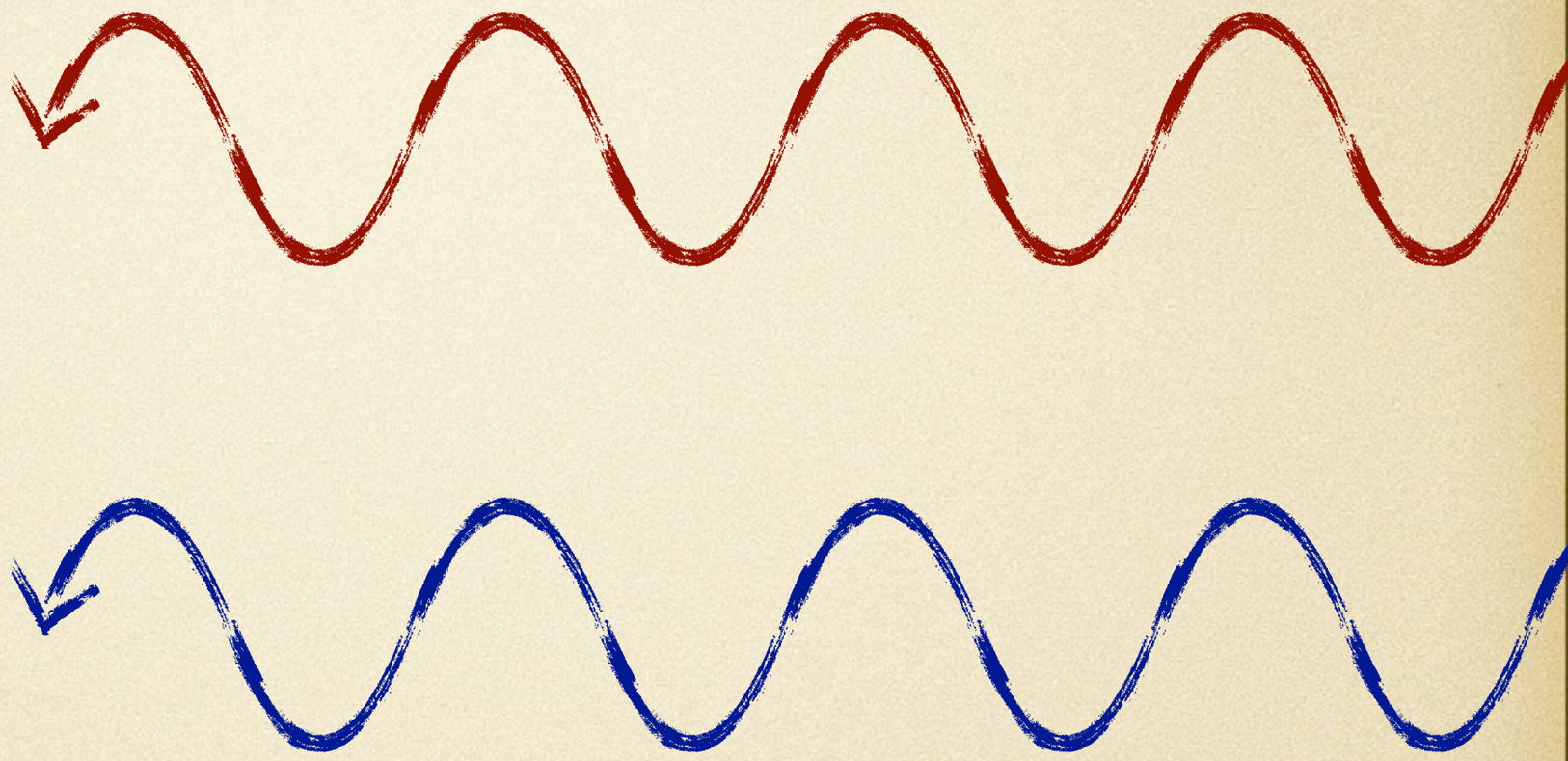
Fact: Universe is Made of Matter (Not Antimatter)

Can the neutrino provide answers?

Neutrino

vs.

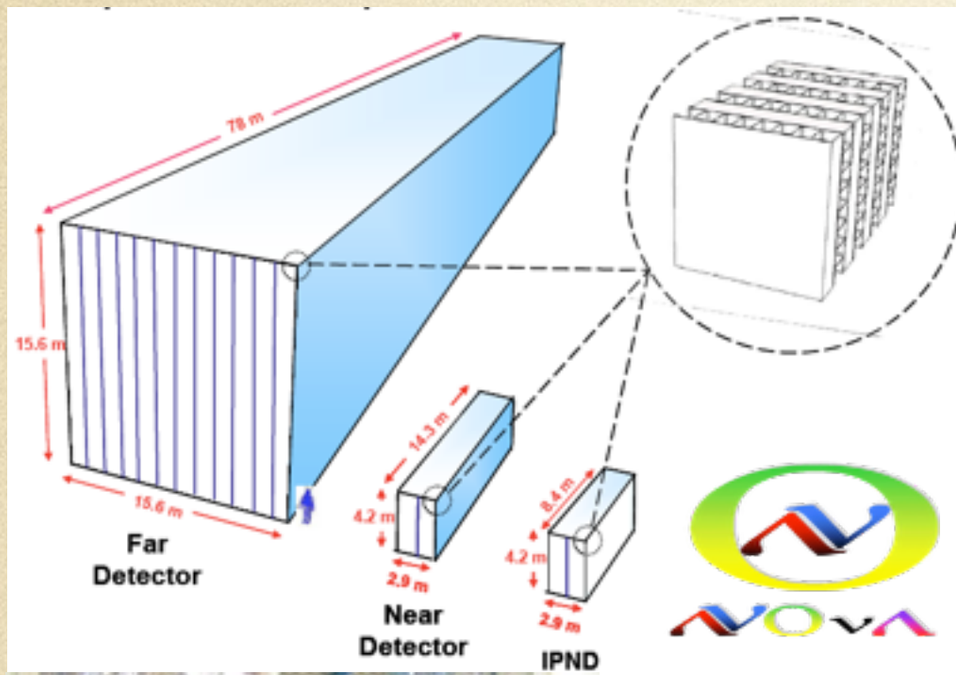
Antineutrino

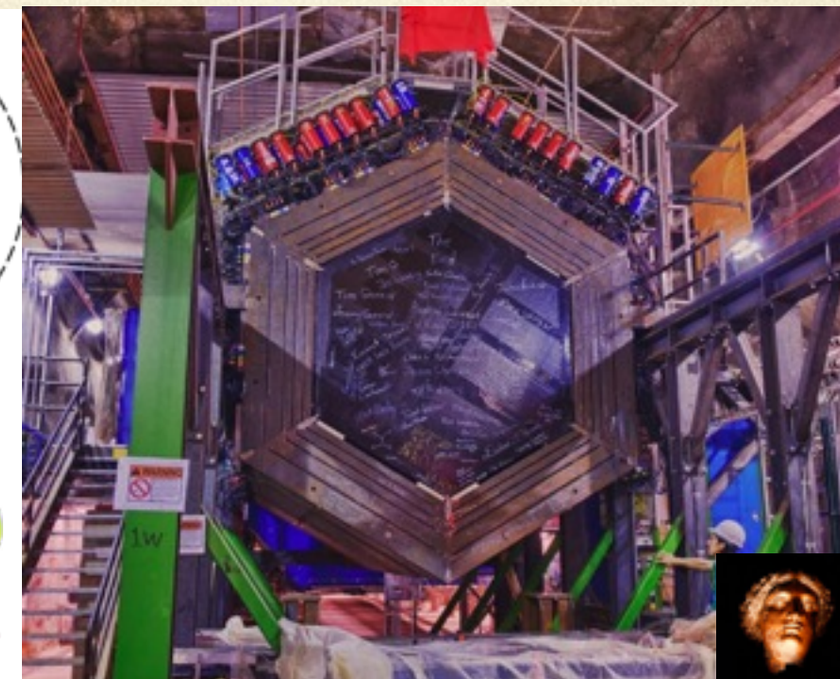


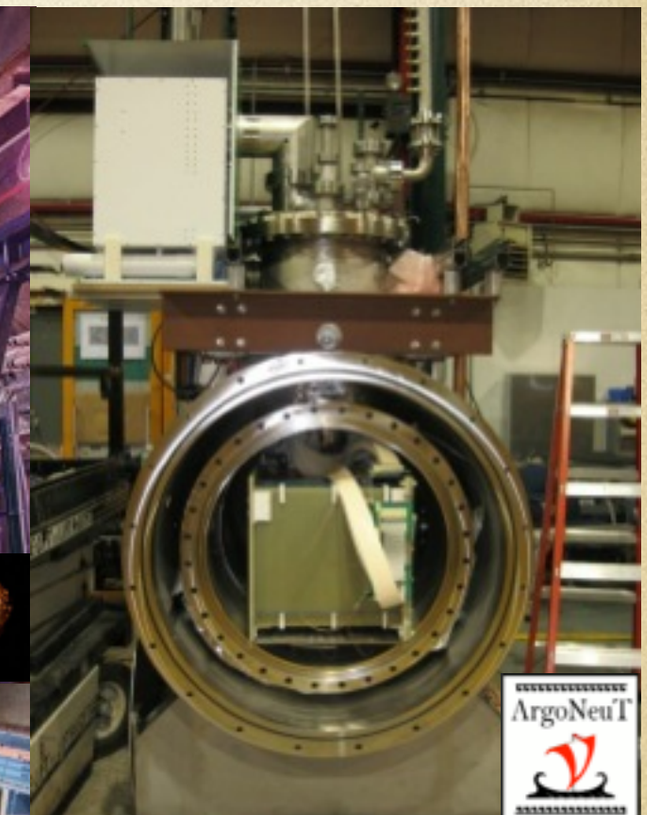
- Might explain matter vs. antimatter asymmetry.
- Might be it's own antiparticle (called Majorana).
- Might be new, heavy ones.

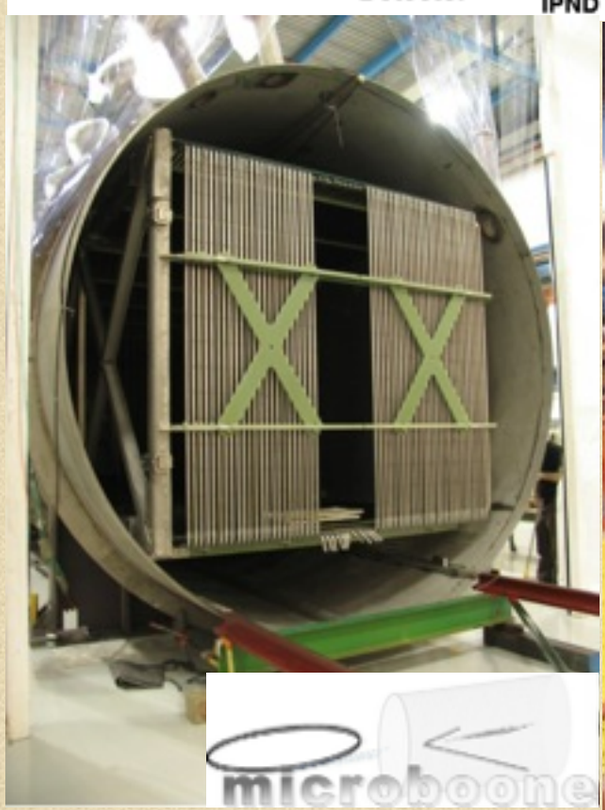
Part 3: Current Experiments

At Fermilab


78 m
15.6 m
15.6 m
Far
Detector
14.3 m
4.2 m
2.9 m
Near
Detector
8.4 m
4.2 m
2.9 m
IPND
NOvA




ArgoNeuT


microboone

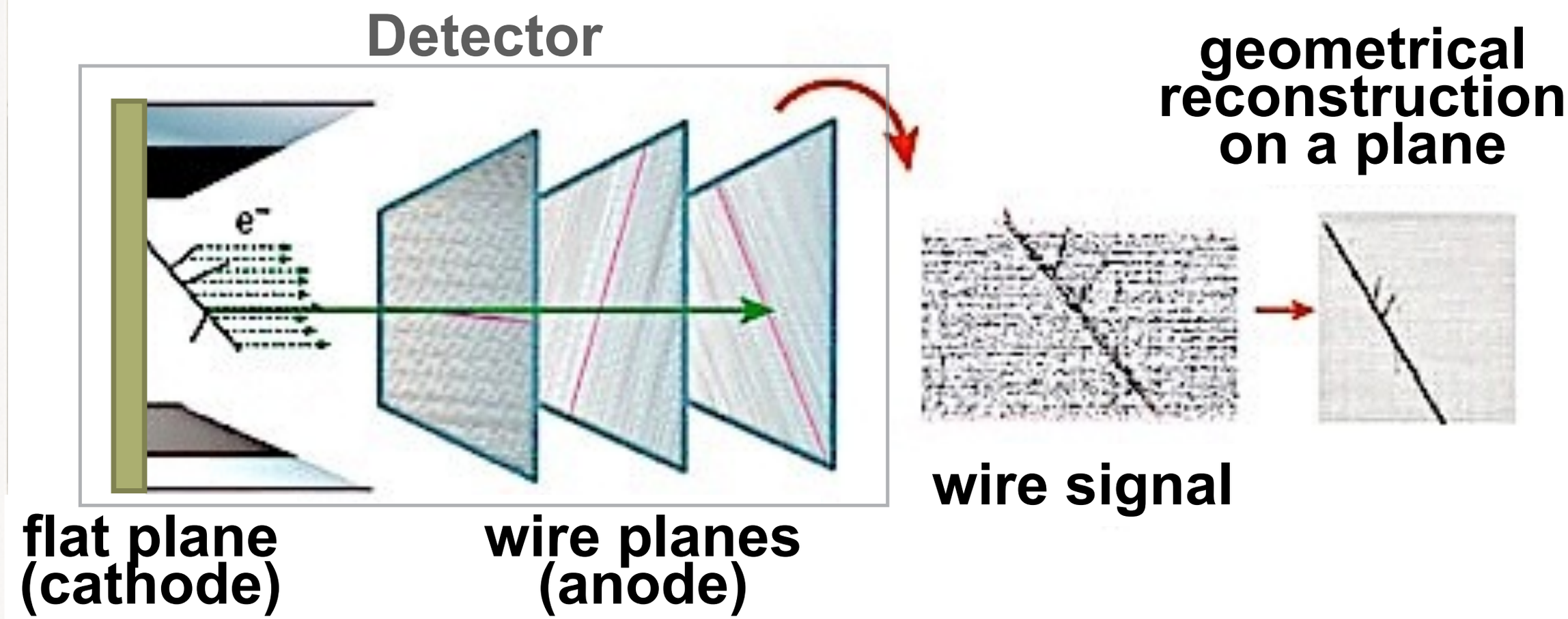
485

MiniBooNE

Time Projection Chamber

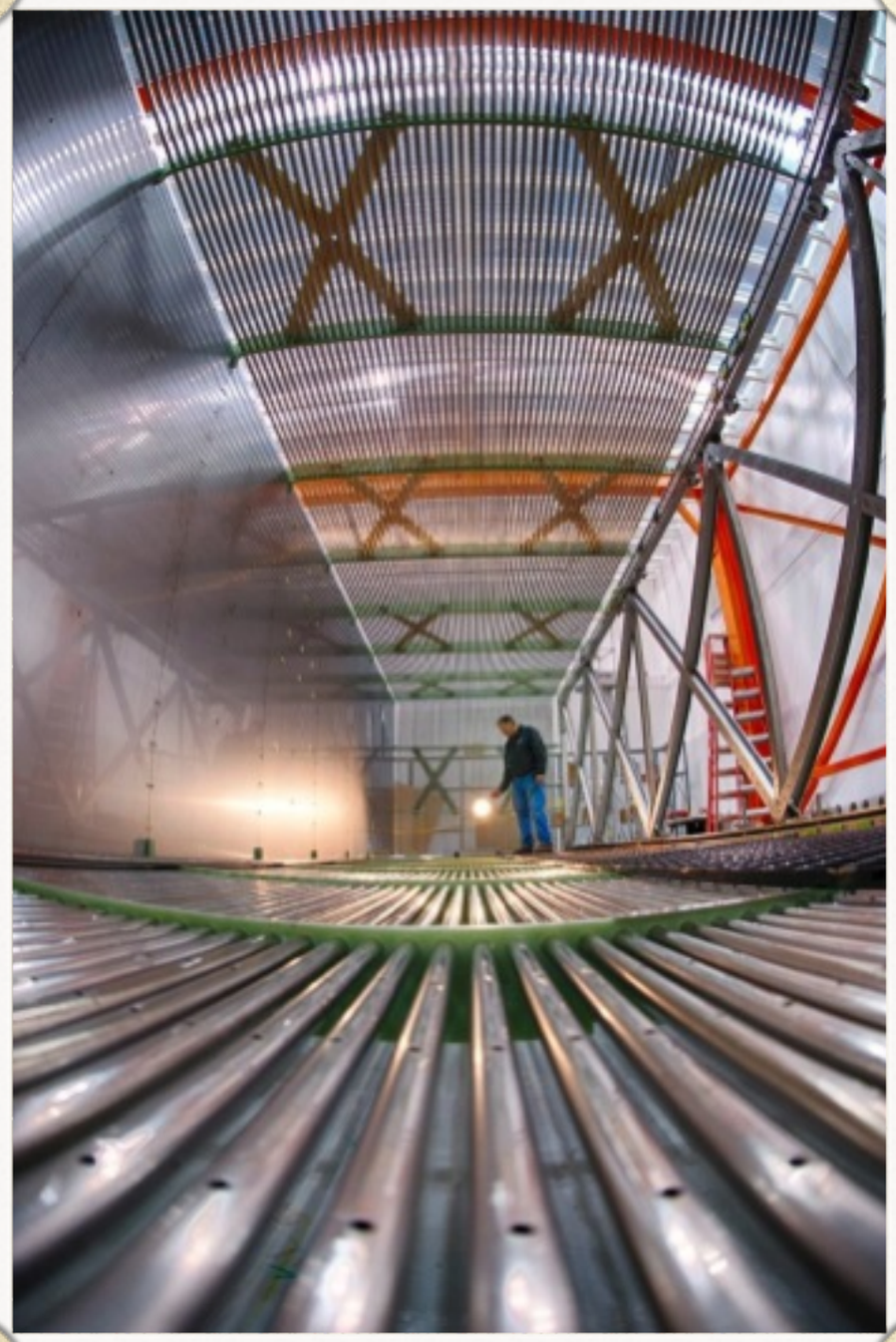
Time Projection Chamber

Time Projection Chamber



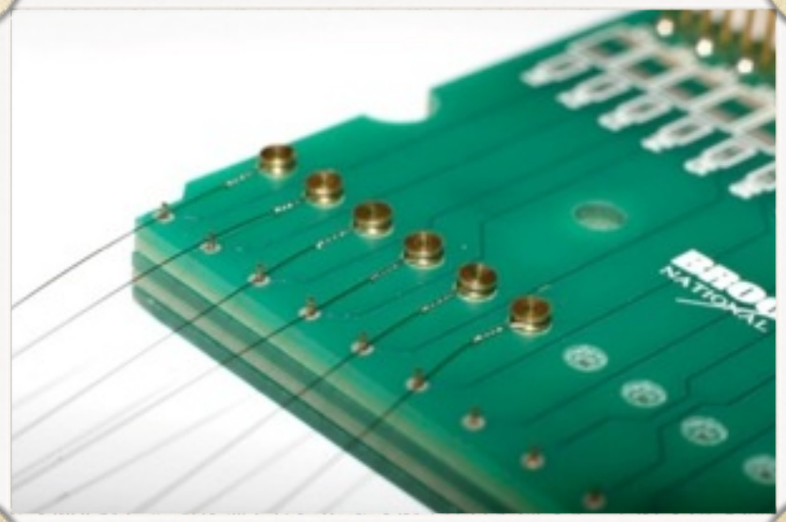
- Time that electrons drift are projected to give the horizontal spacial component.

MicroBooNE

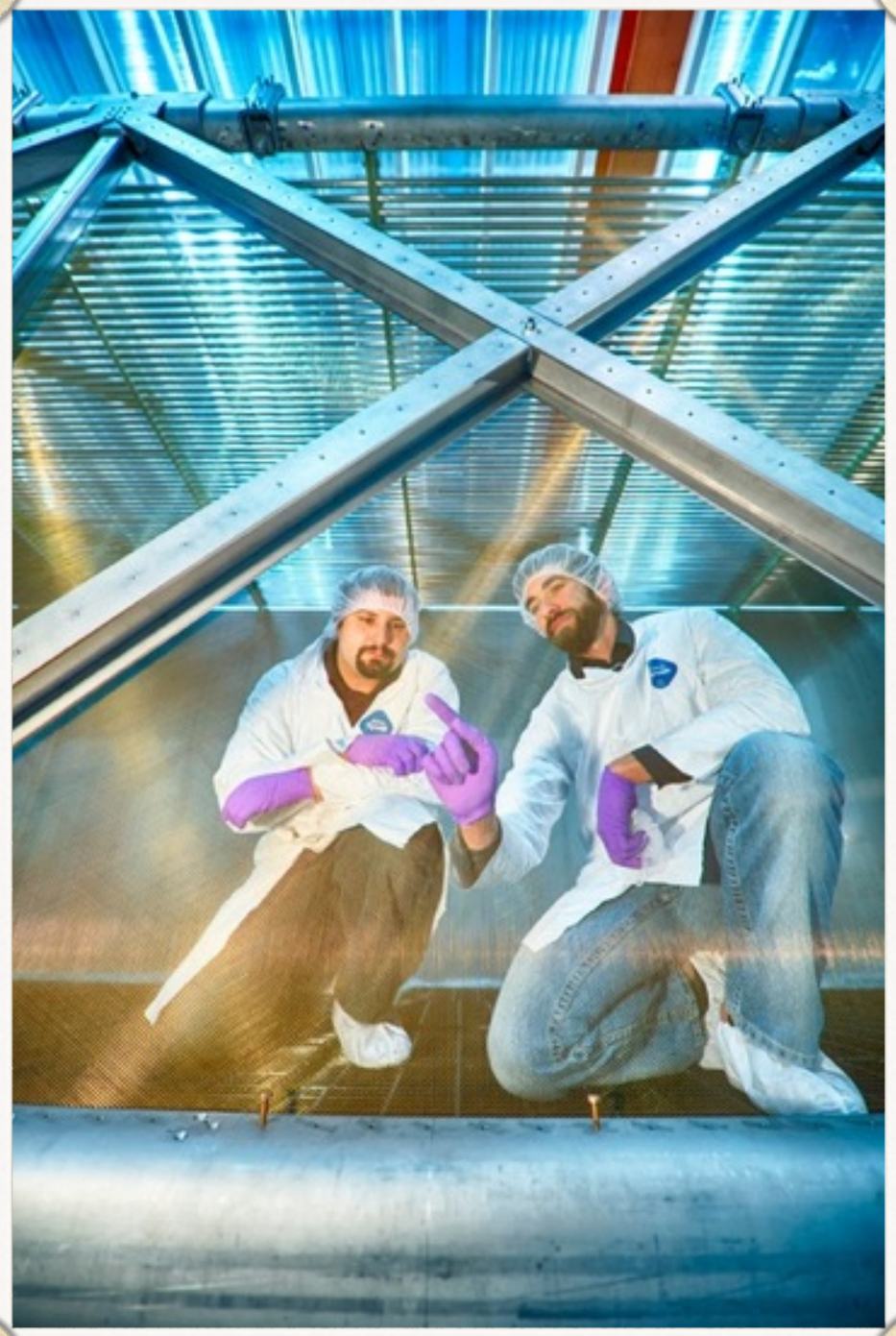


- Drift region

- Anode wires



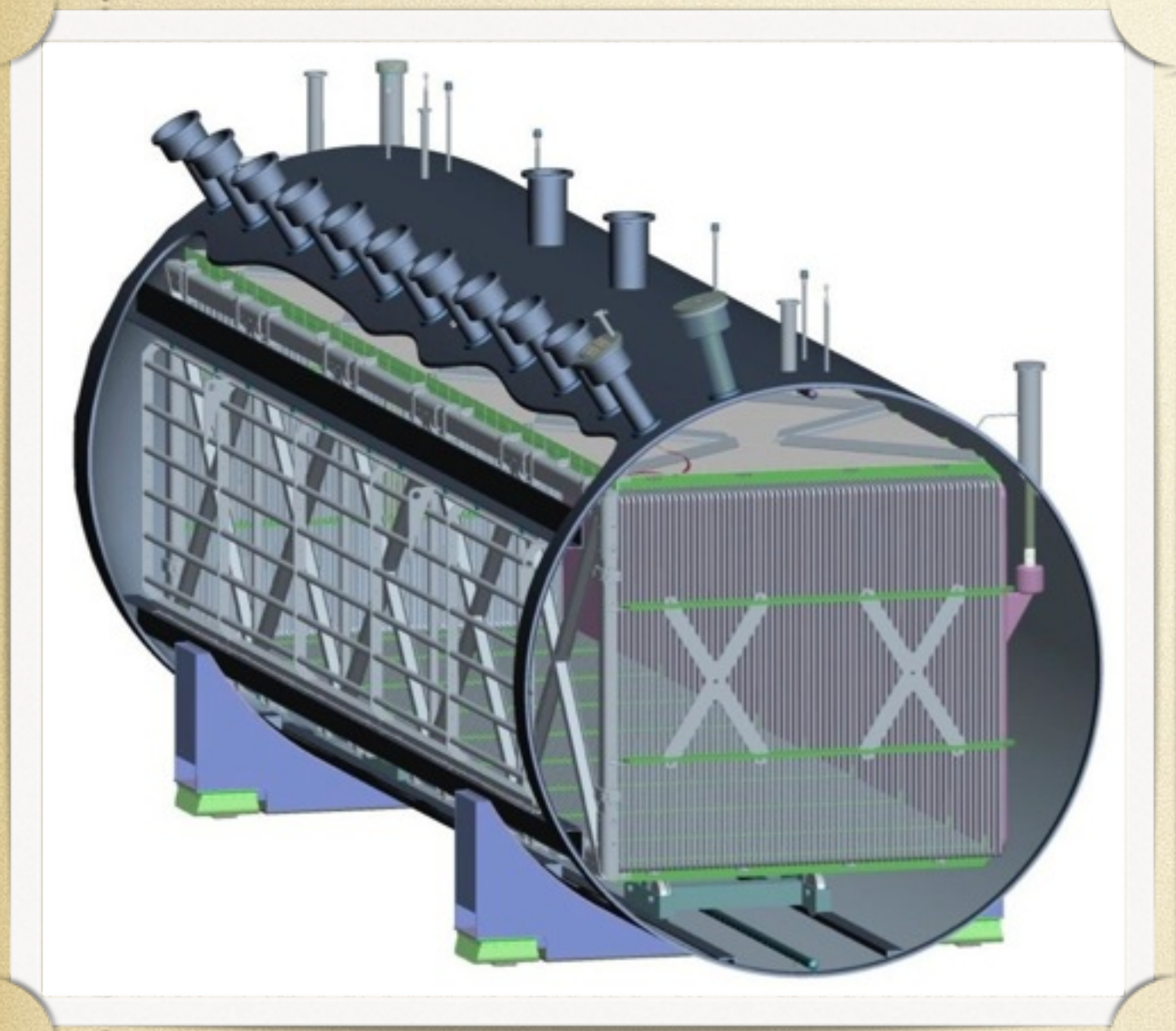
MicroBooNE

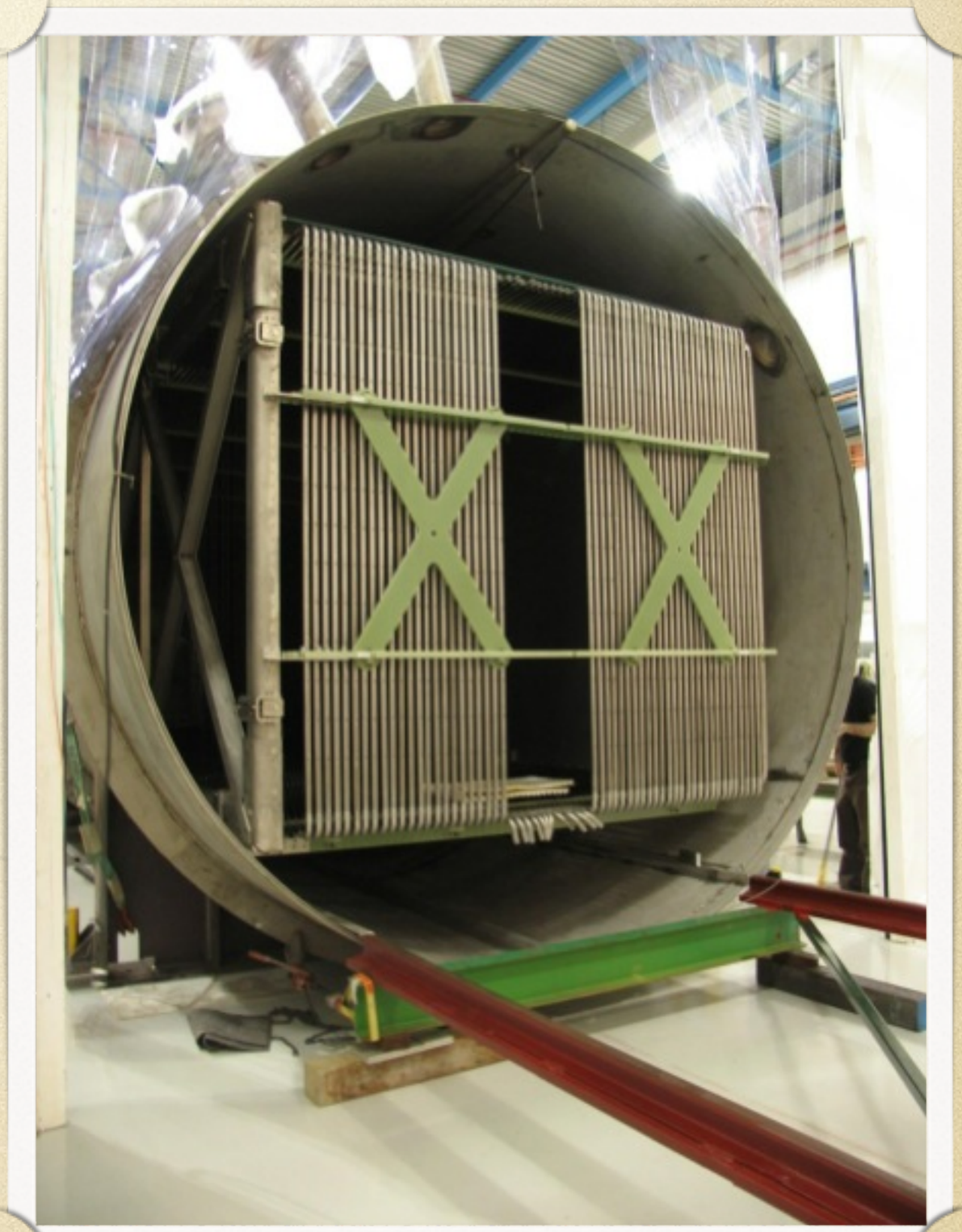


- Whole TPC

- Anode wires

MicroBooNE



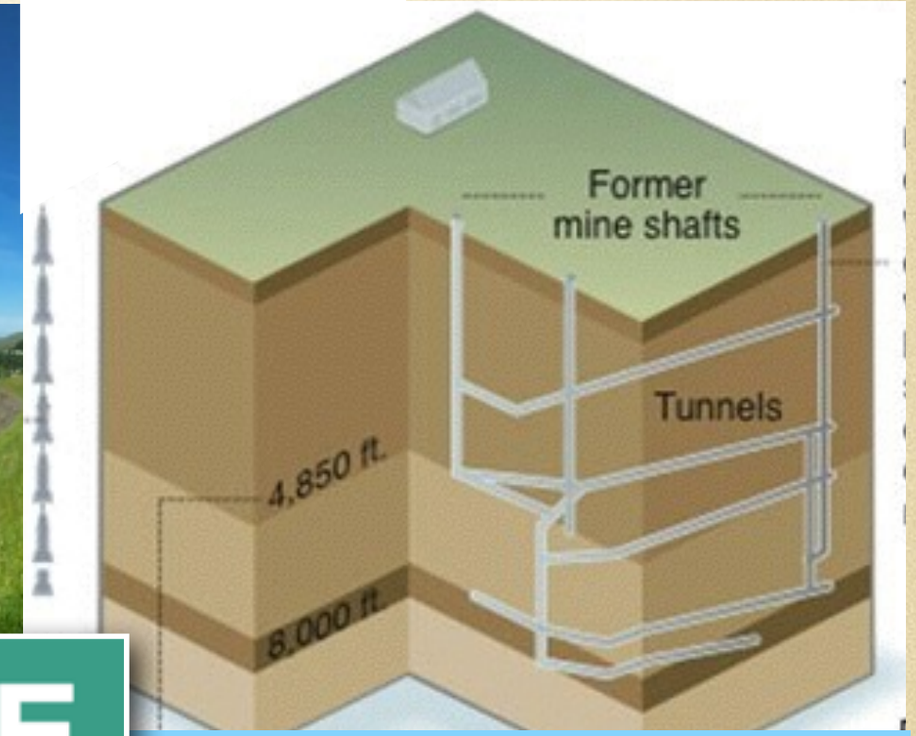
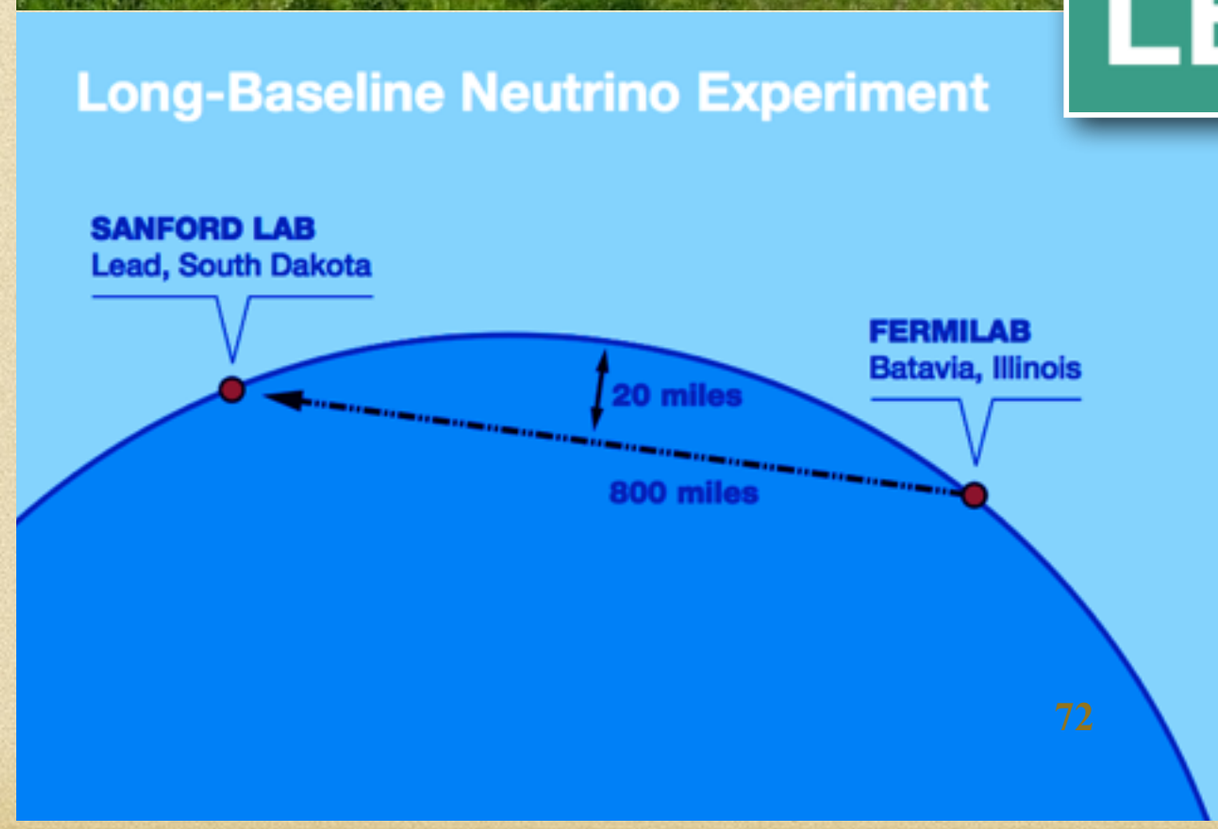


- Inserting the TPC into the cryostat cylinder

MicroBooNE

- Final installations are happening now!
- Commissioning this spring and summer!
- First data expected this summer!

Beyond MicroBooNE



LBNE