Neutrinos (part II): The Ghosts of the Universe

Tia Miceli Ph.D.
New Mexico State University

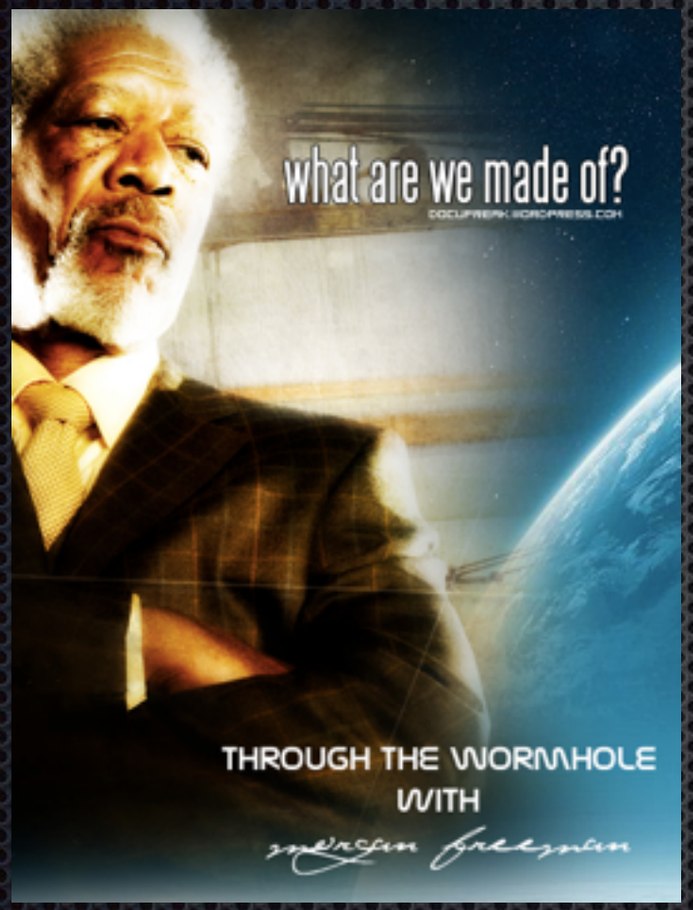
30 June 2015
Fermilab Undergraduate Lecture Series

- What's particle physics? (translated into "Tia Speak")
- Where are we in neutrino physics? (Story time!)
  - FUN MATH for neutrino masses!
- Where are we going in neutrino physics? (A.K.A why Tia researches neutrinos)

- What's particle physics? (translated into "Tia Speak")
- Where are we in neutrino physics? (Story time!)
  - FUN MATH for neutrino masses!
- Where are we going in neutrino physics? (A.K.A why Tia researches neutrinos)

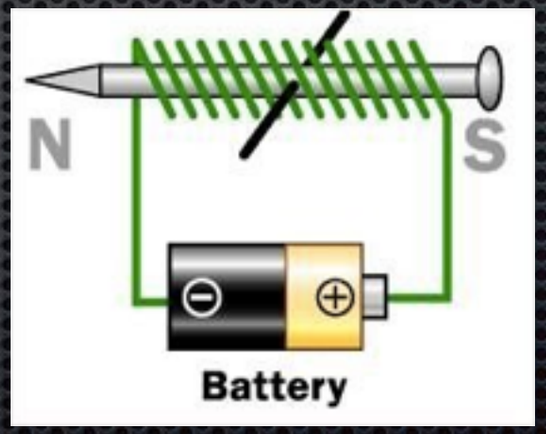
Particle physics questions

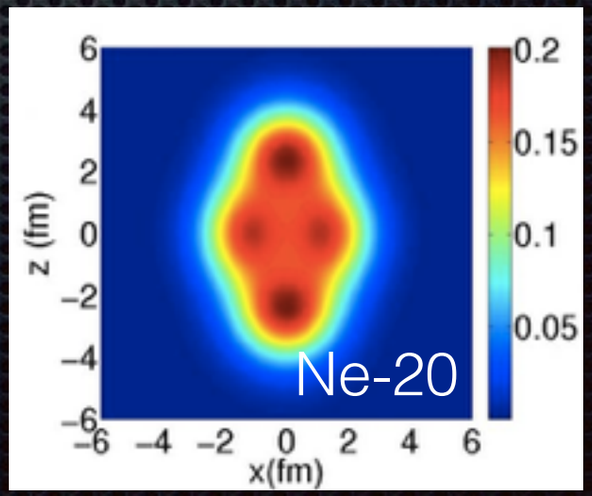
What are we made of?



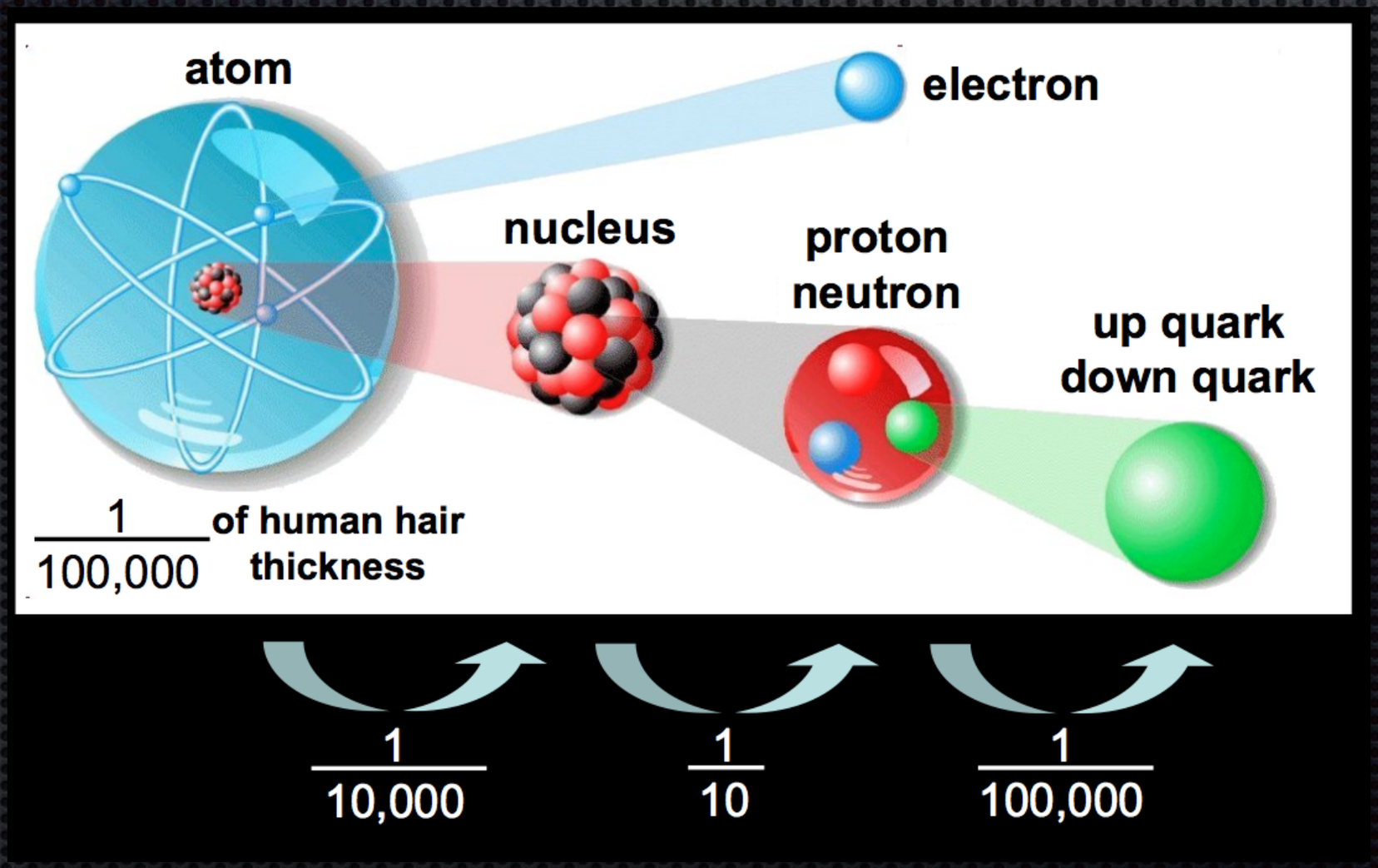
How does it work?

How sure are we?

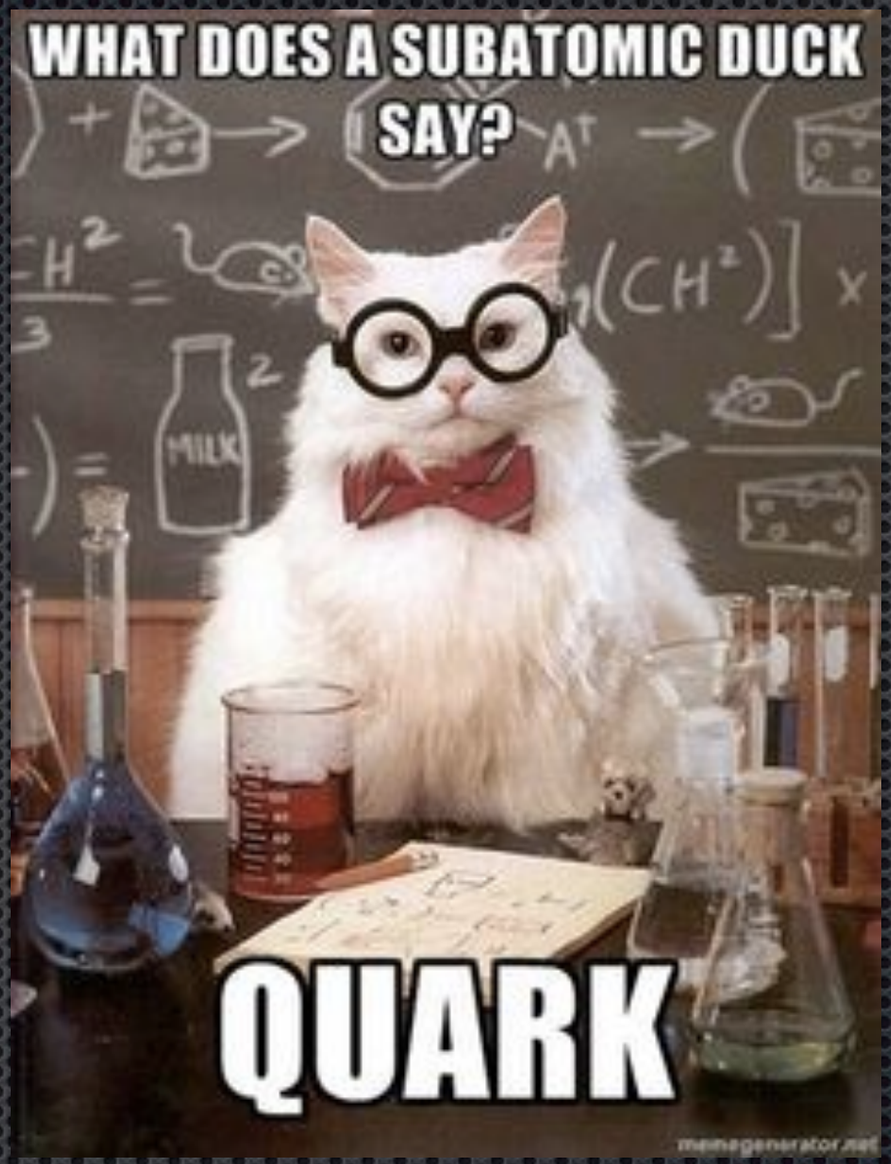




Particle physics is everywhere

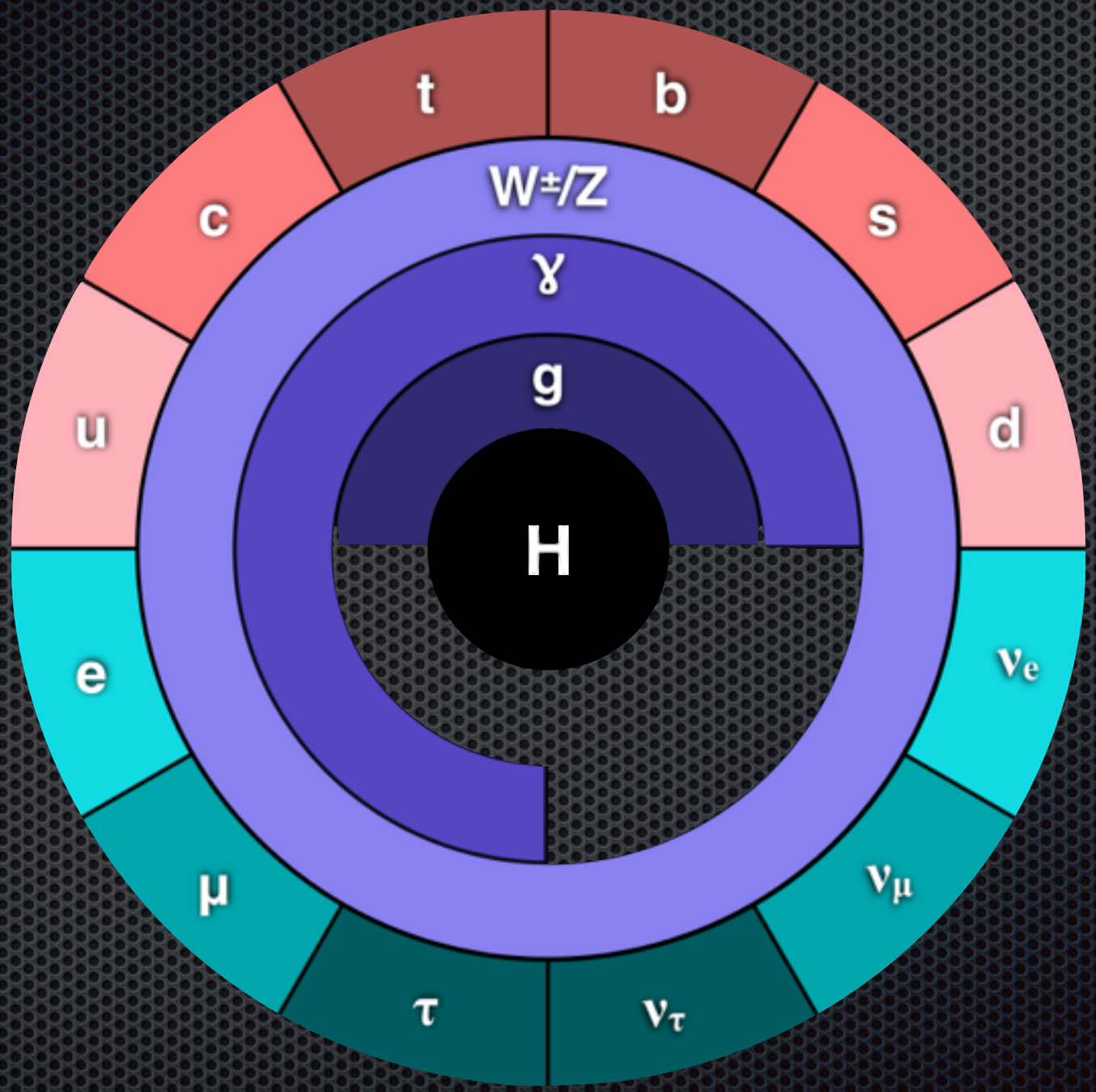


Particle physics is everywhere

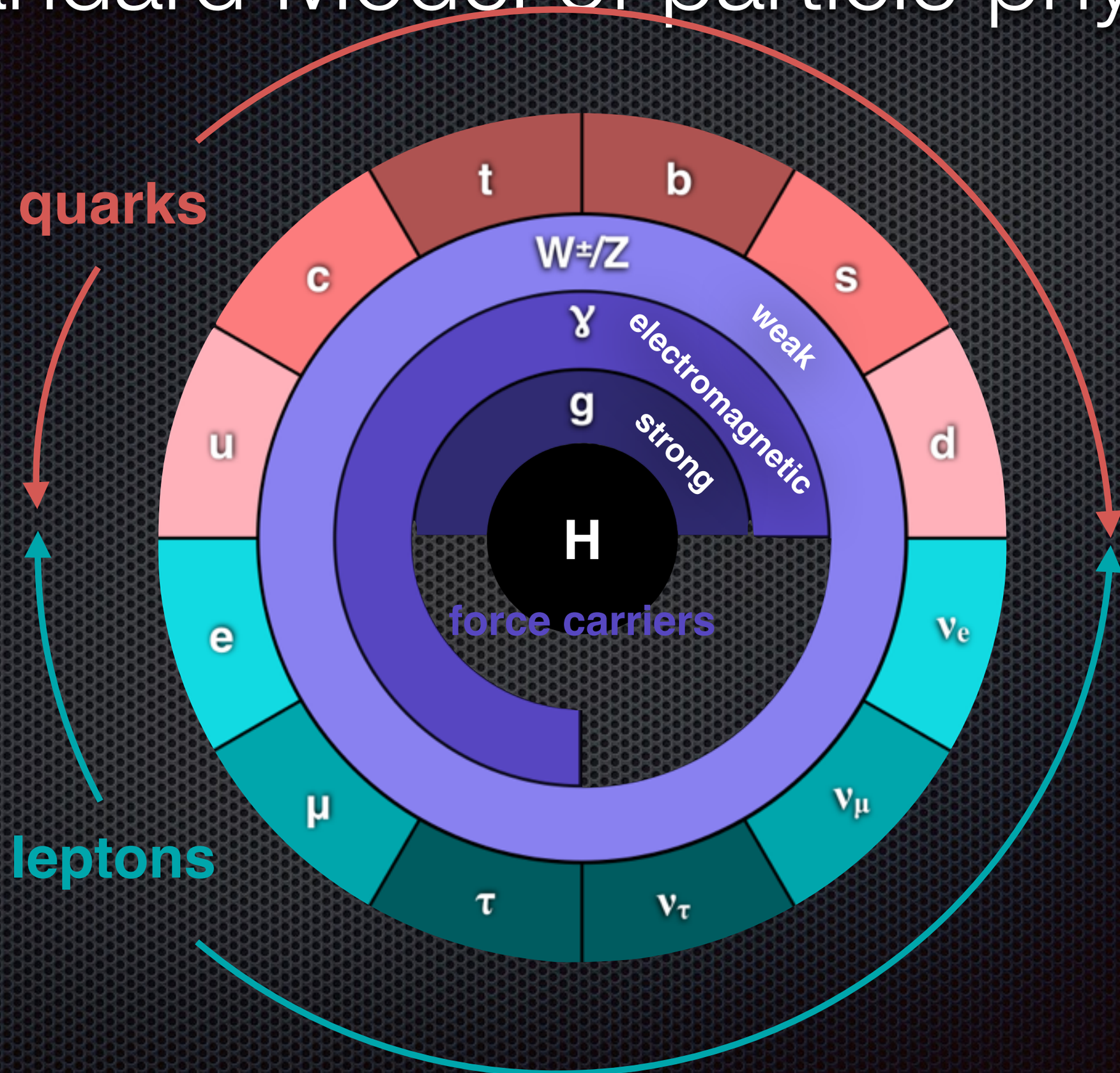


Up quarks, down quarks, and electrons… oh my!
What else?

Standard Model of particle physics



Standard Model of particle physics



Standard Model of particle physics

Note: spin is intrinsic value of angular momentum

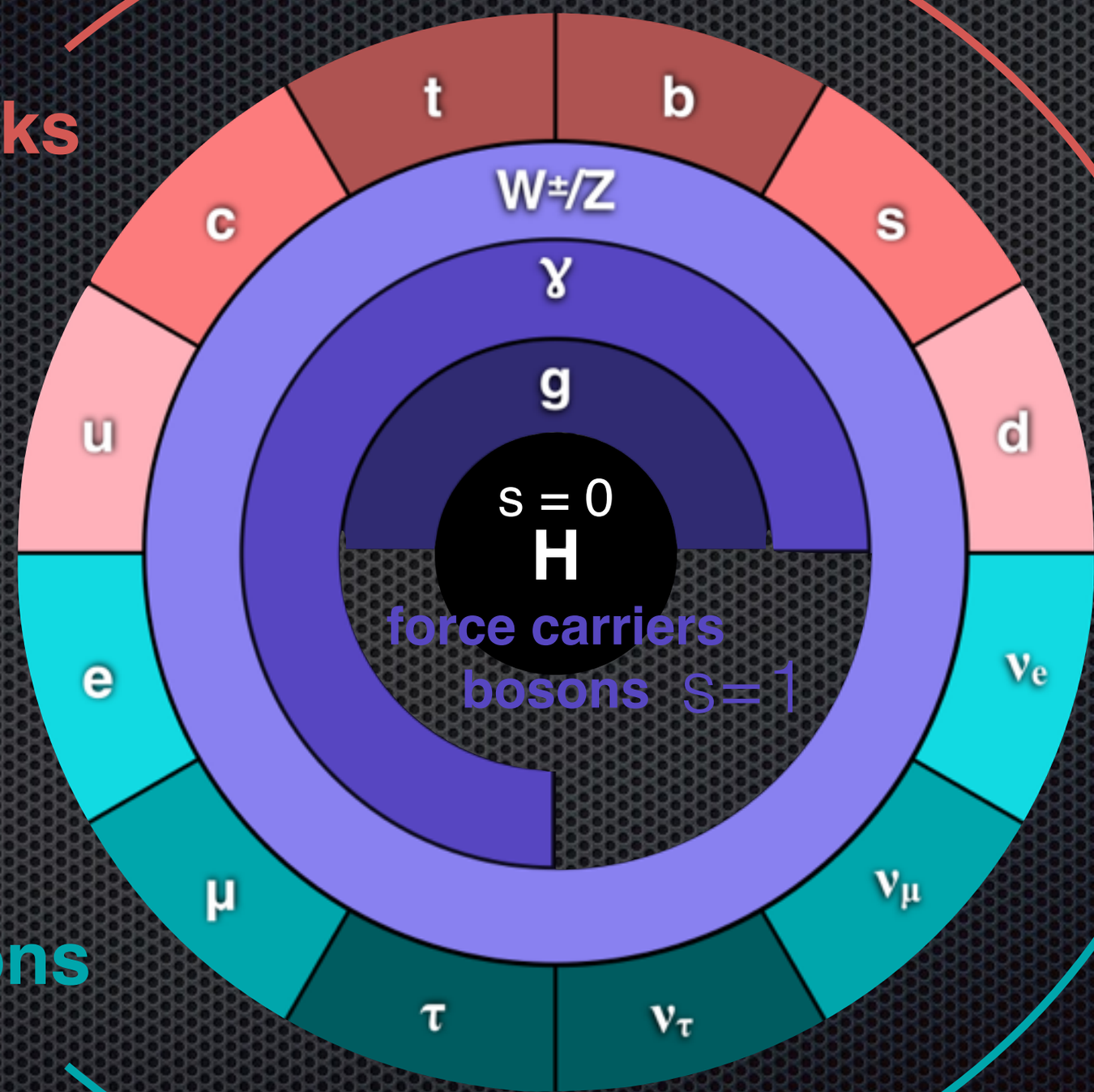
fermions

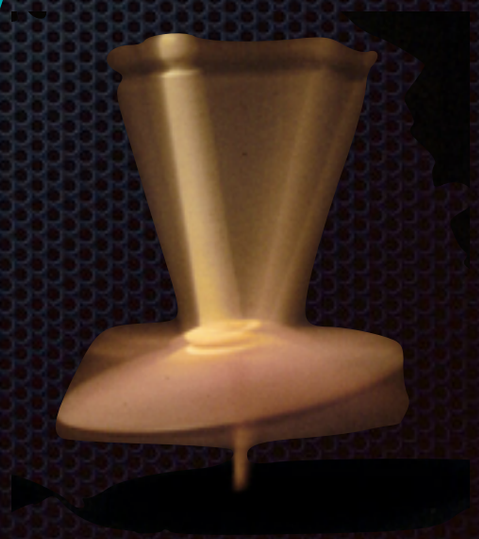
s = 1/2

s = 1/2

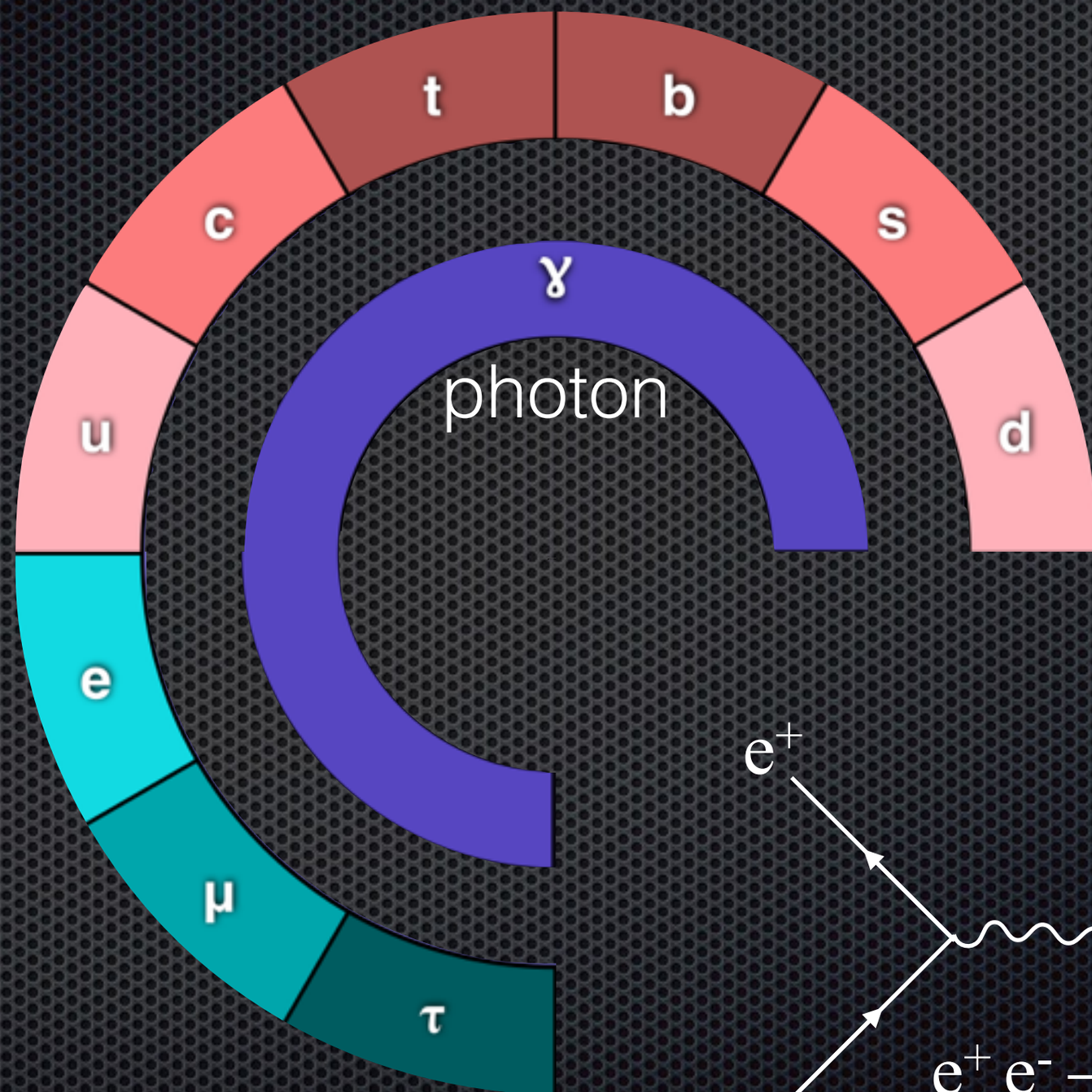
quarks

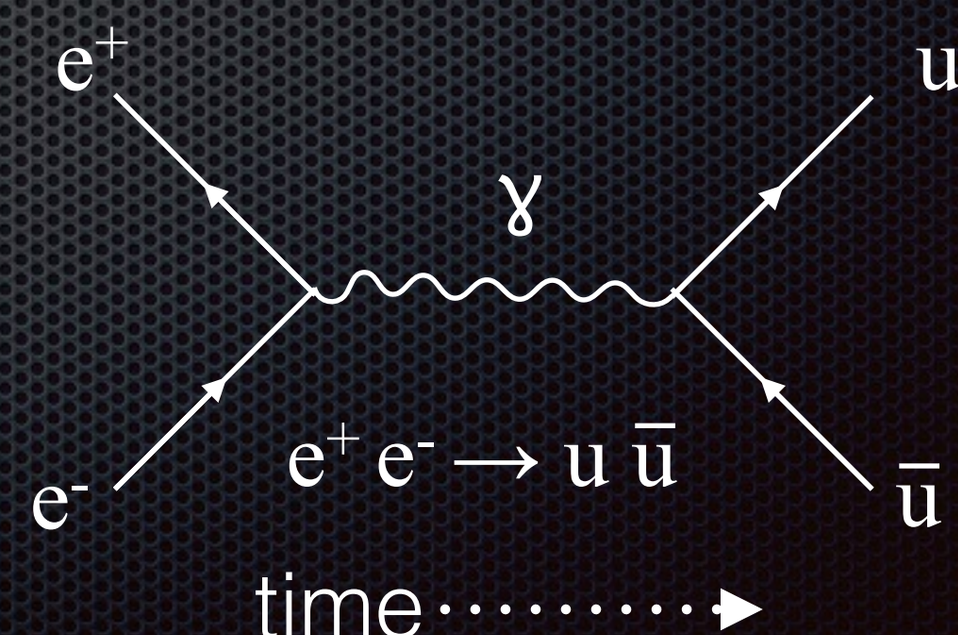
leptons





Standard Model of particle physics

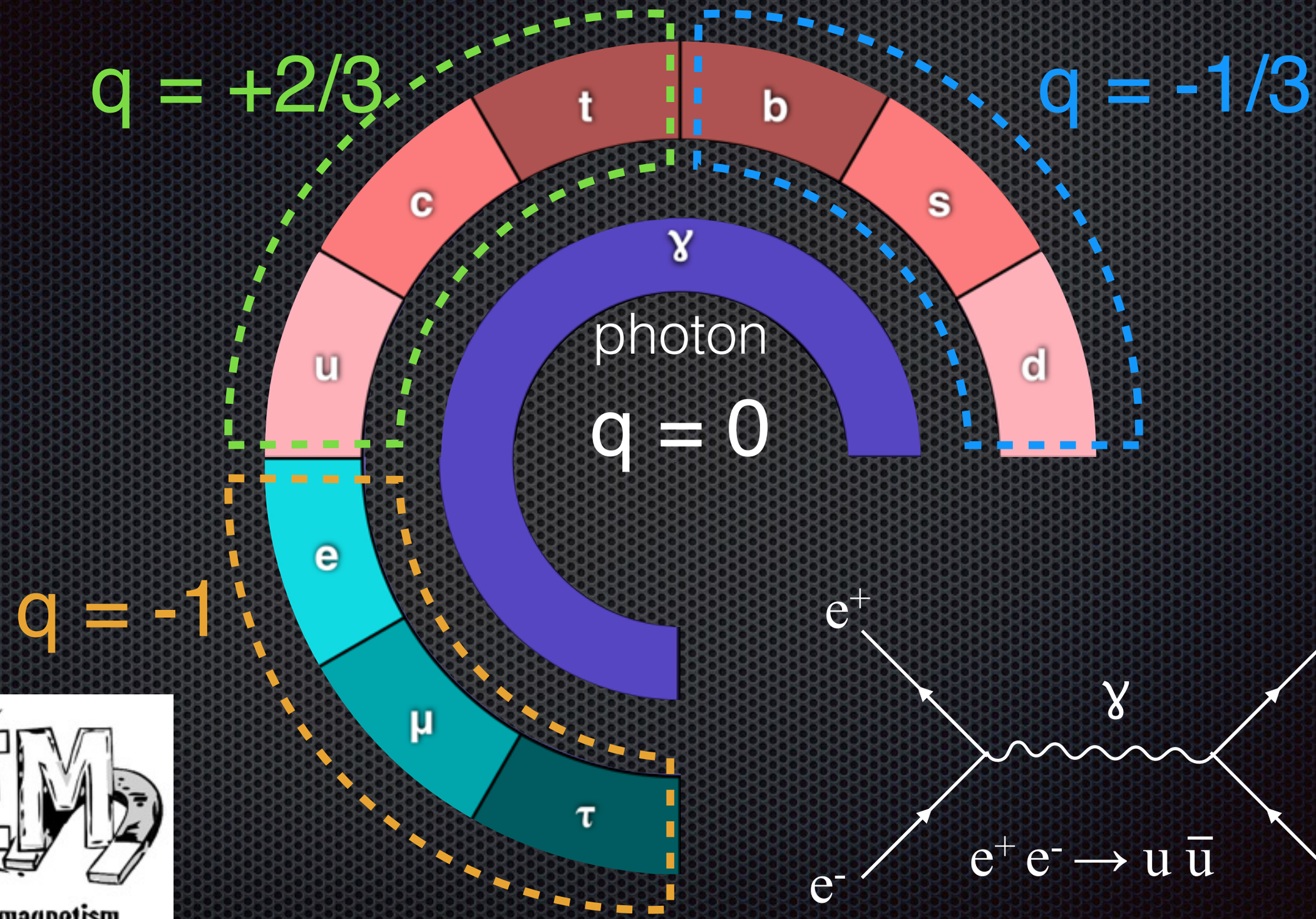


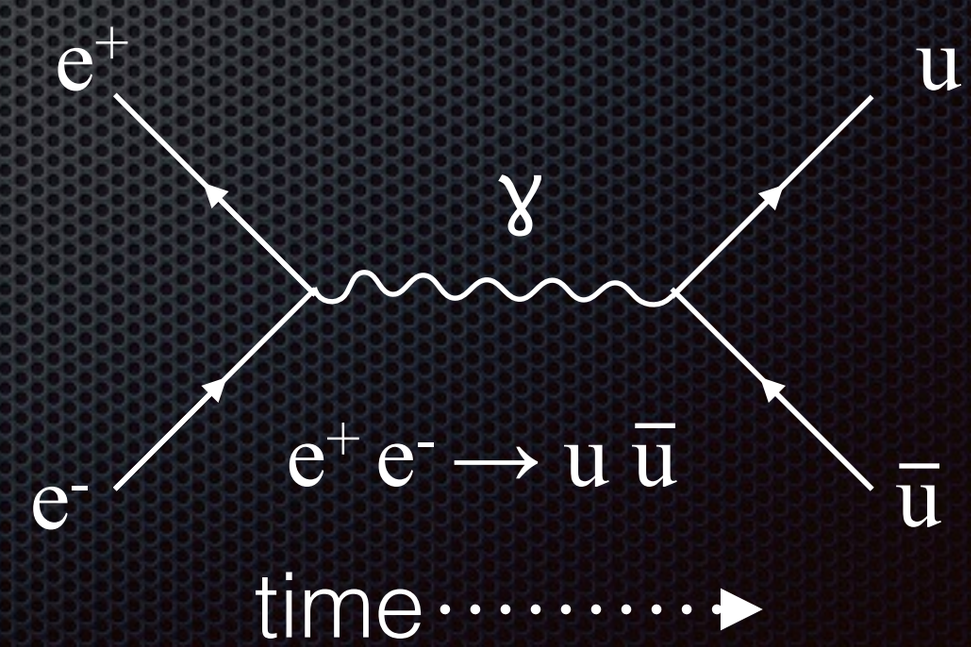


electromagnetism

Tia Miceli

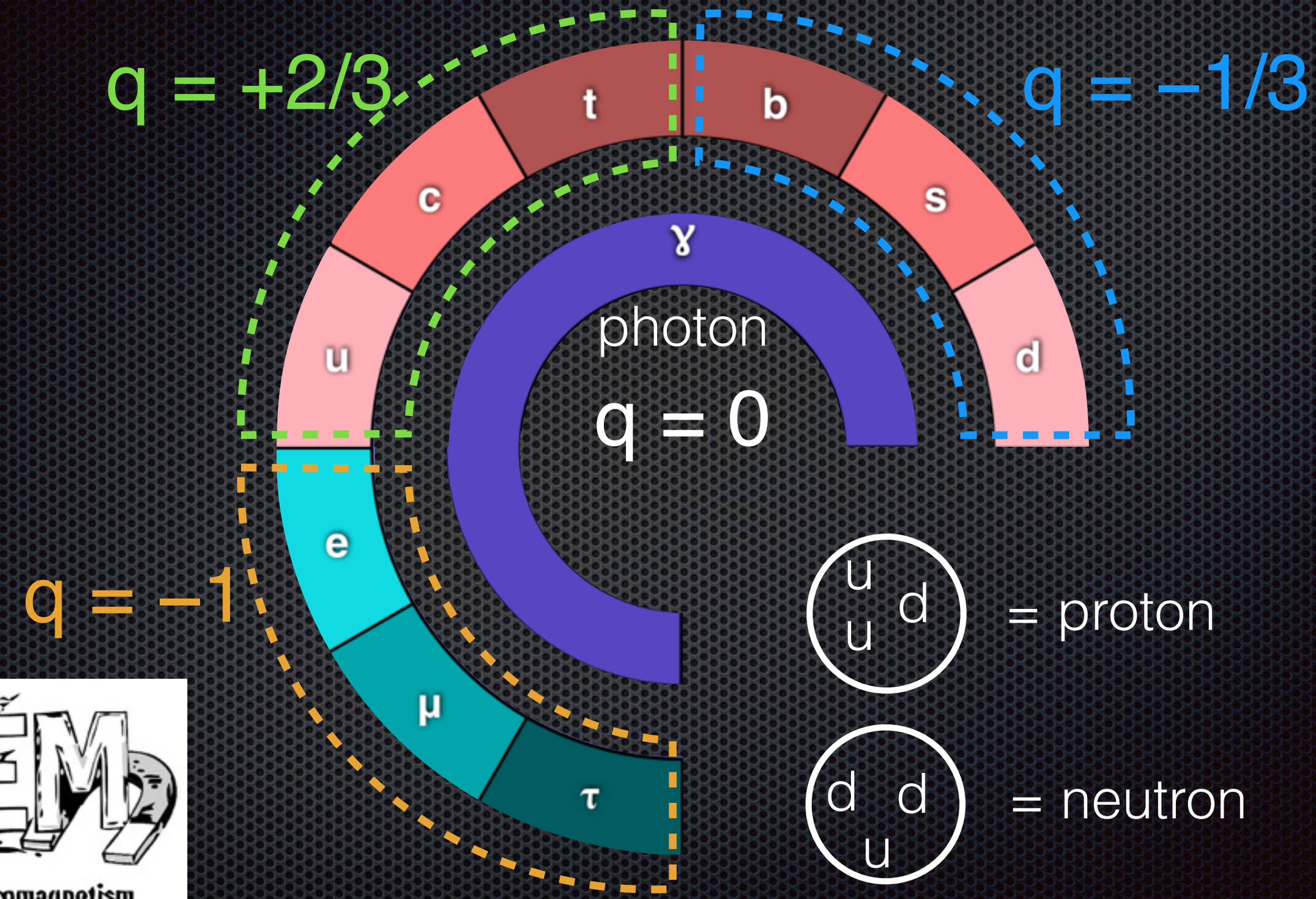
Standard Model of particle physics



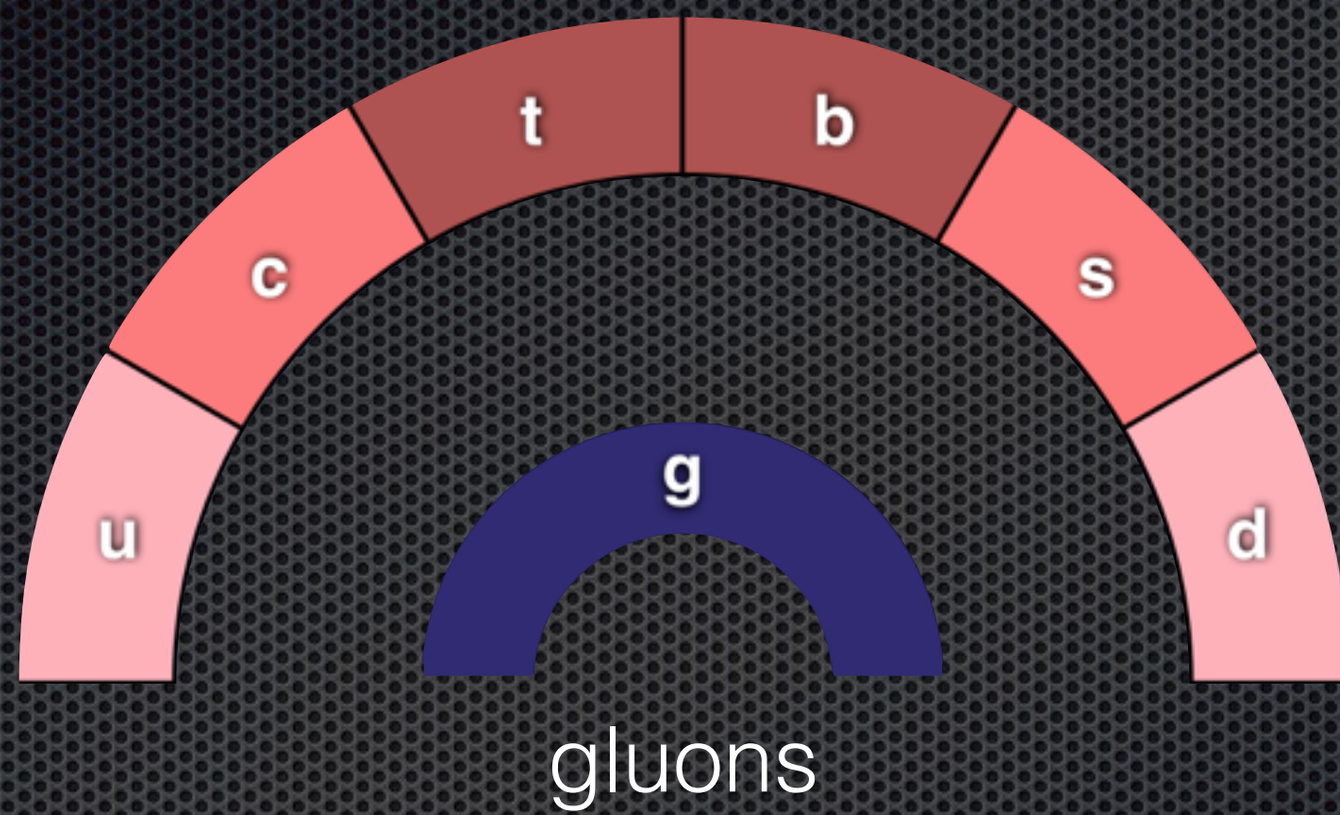


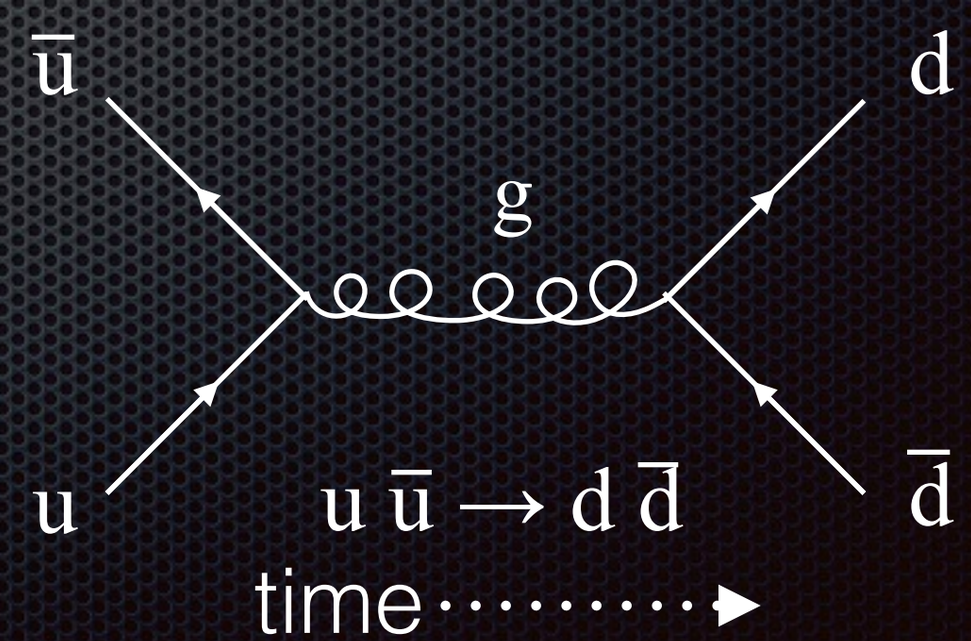
Tia Miceli

Standard Model of particle physics

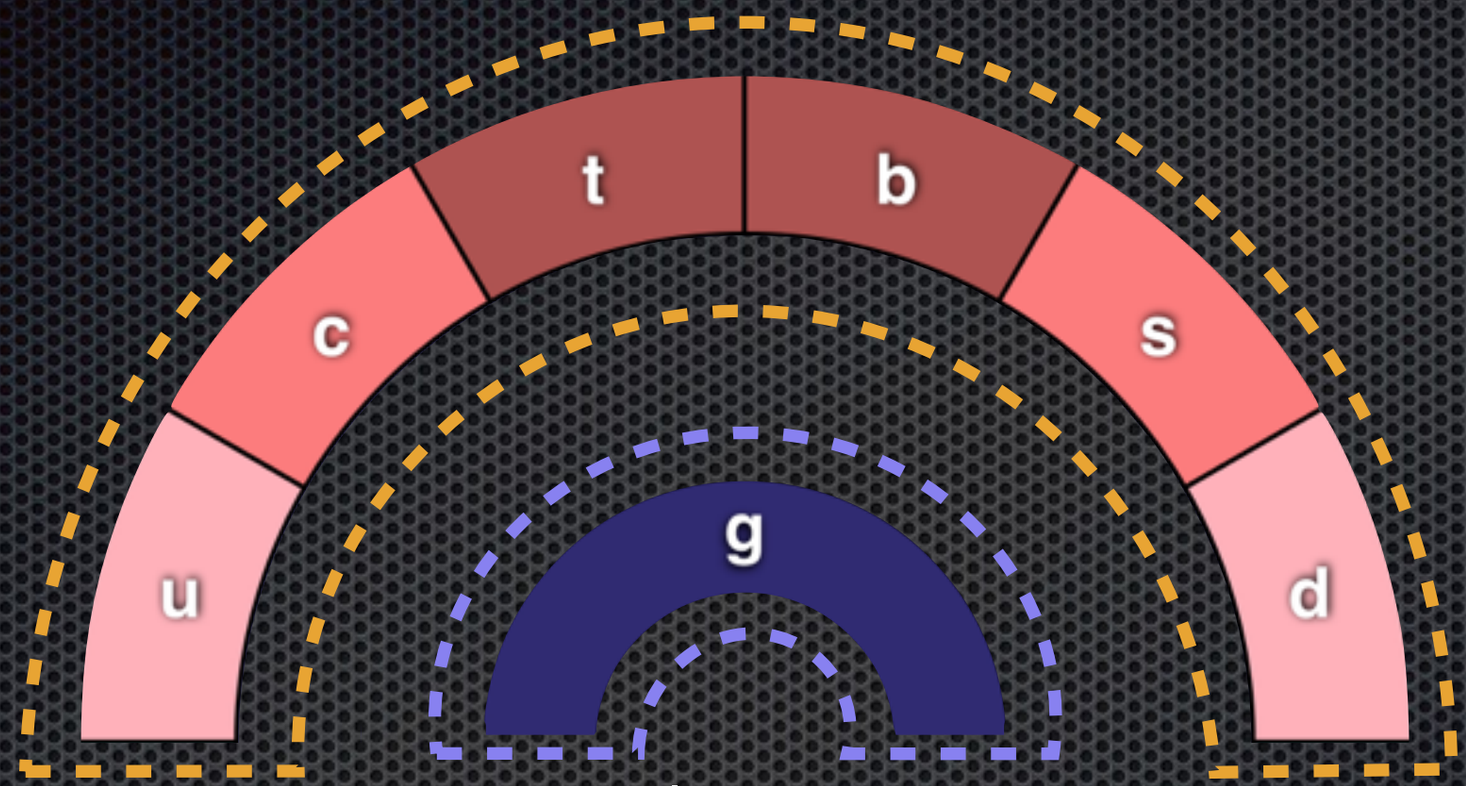


Standard Model of particle physics





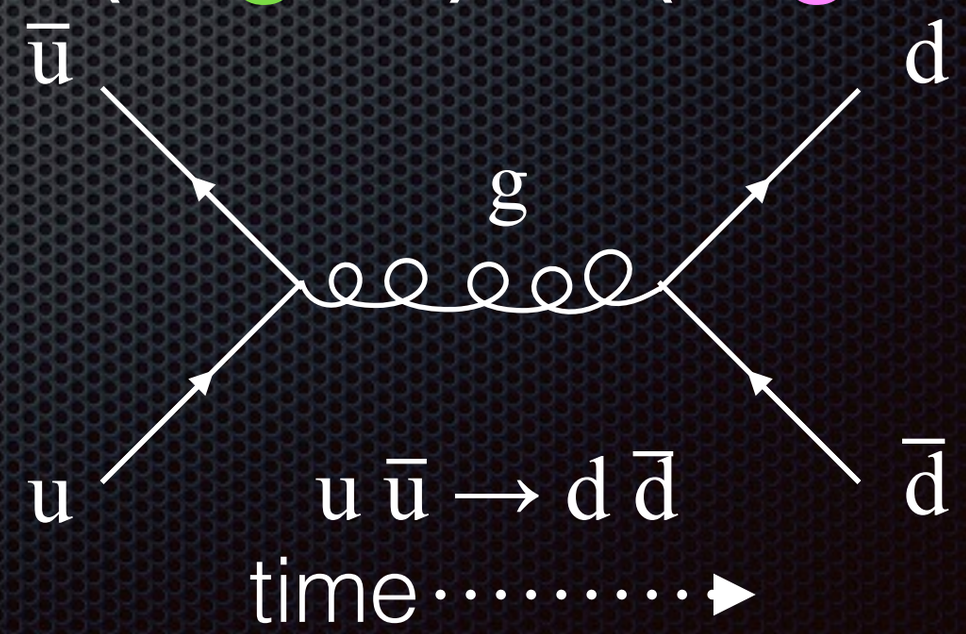
Standard Model of particle physics



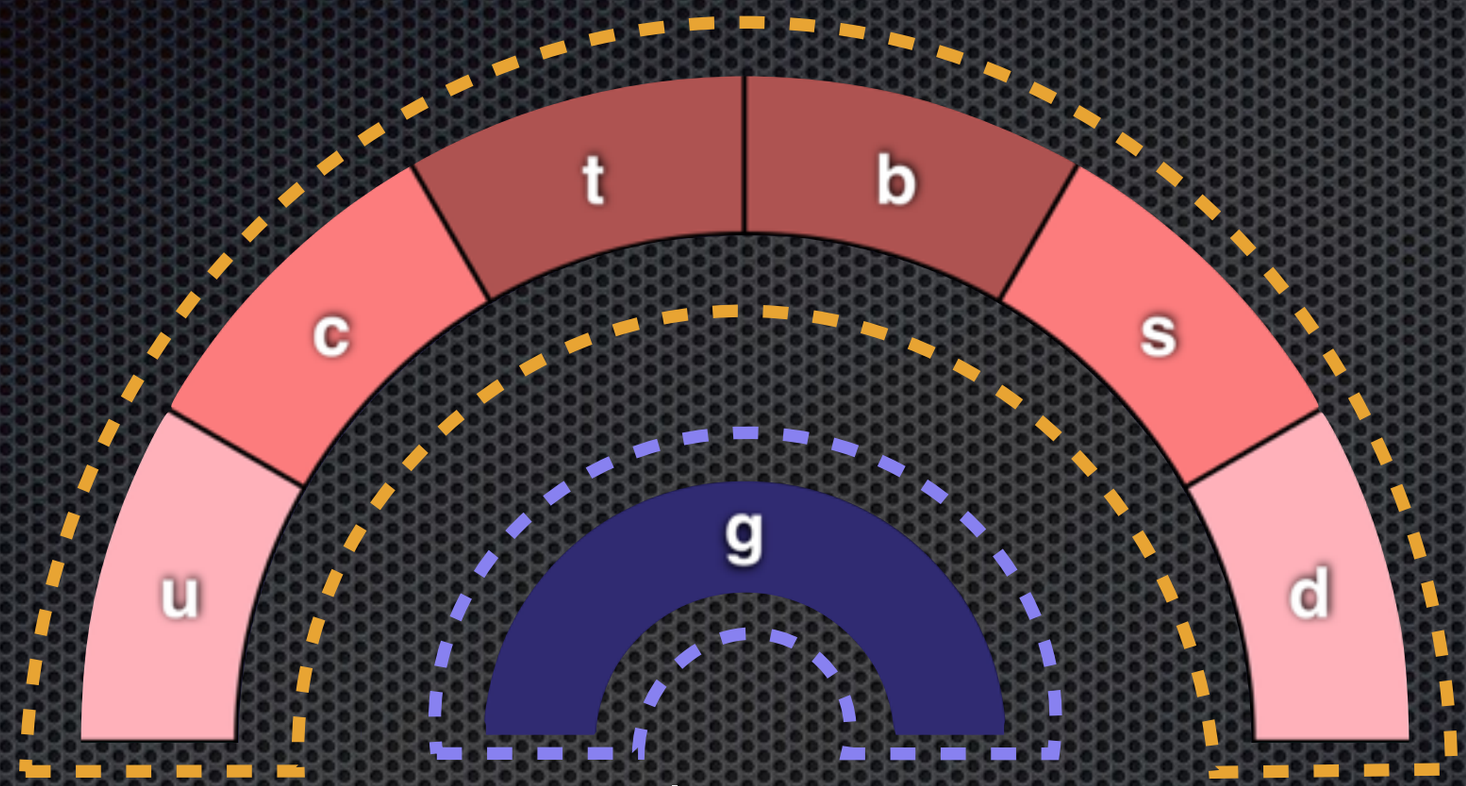
color = r, g, b

gluons

color = (r, g, b) + (r, g, b)



Standard Model of particle physics



gluons

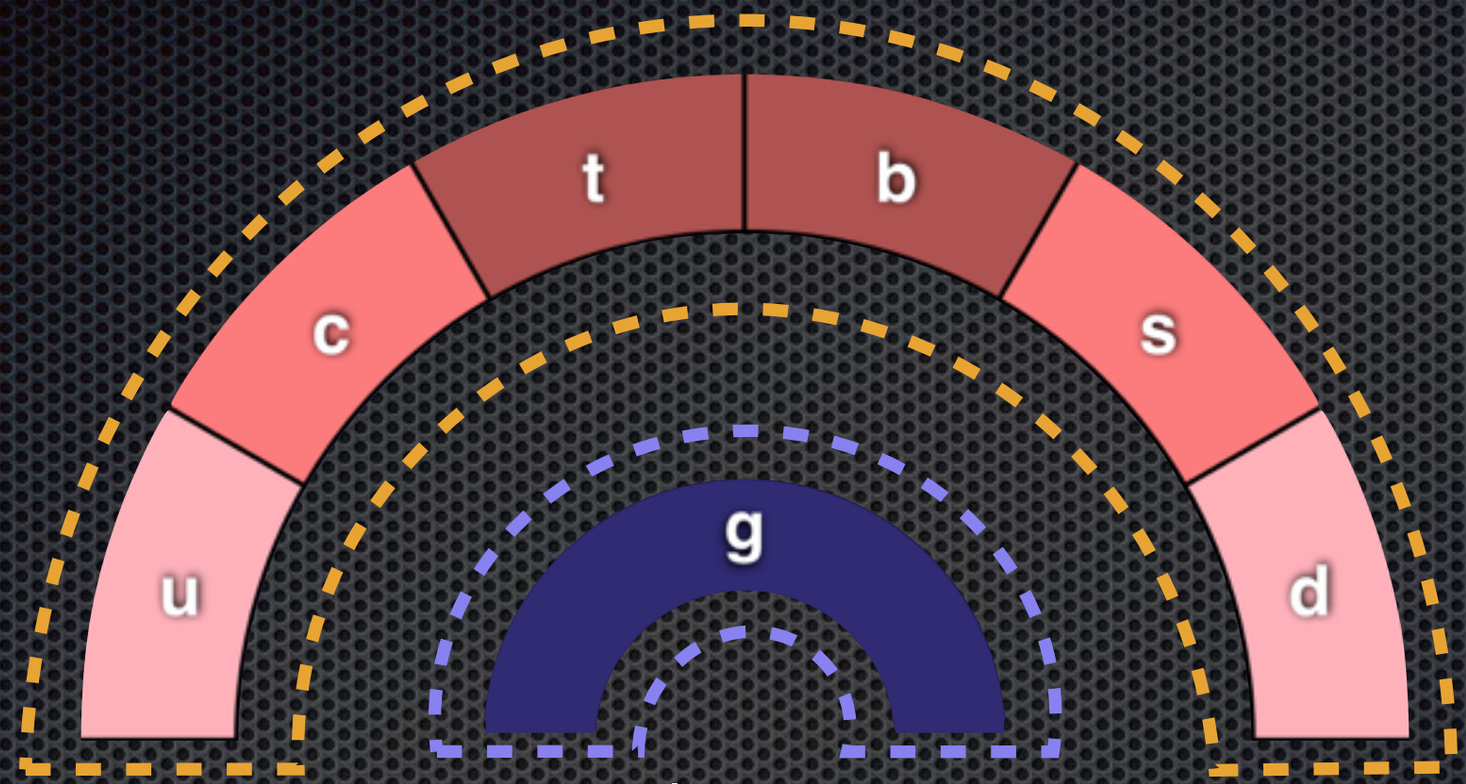
color = r, g, b

color = (r, g, b) + (r, g, b)

= proton

= neutron

Standard Model of particle physics



color = r, g, b

gluons

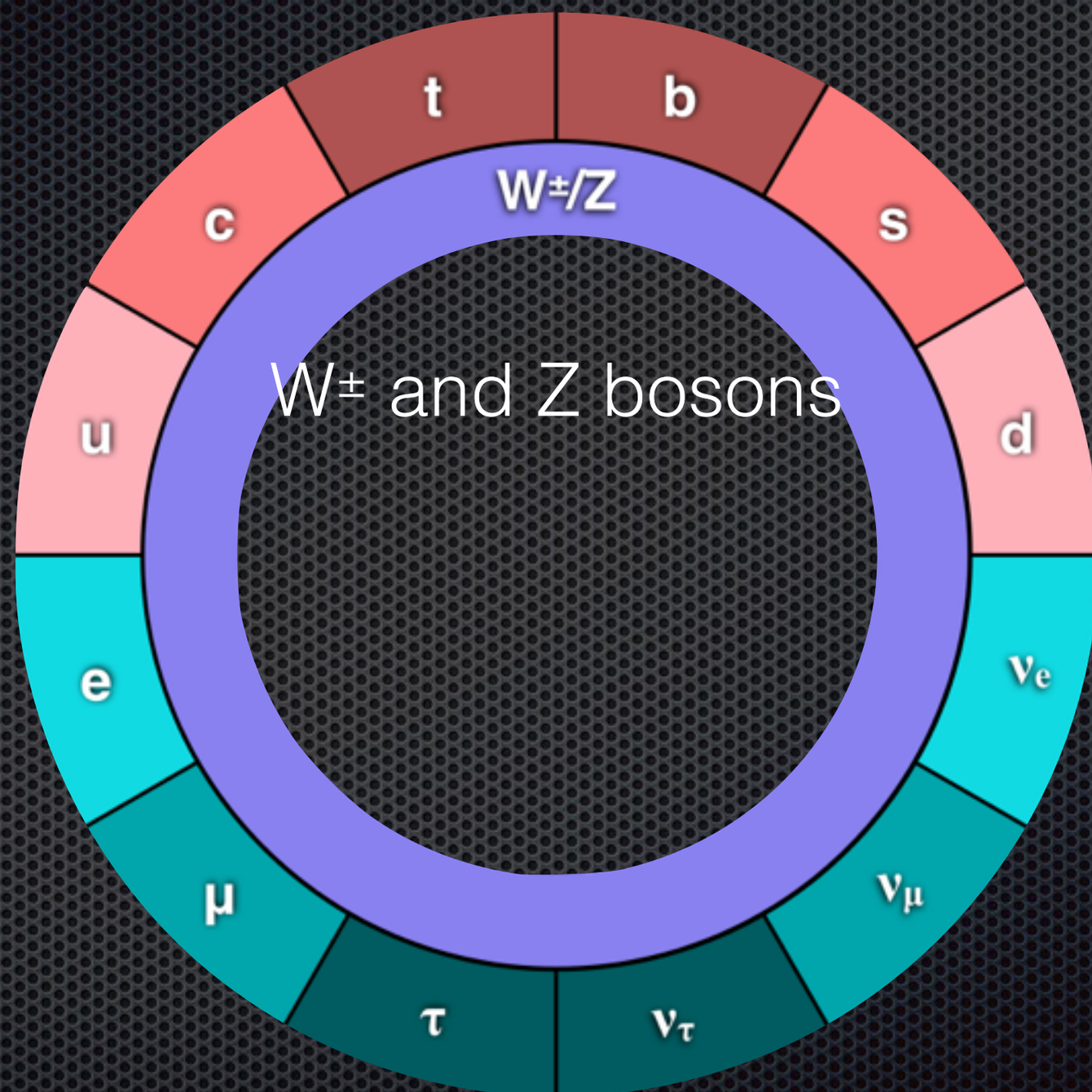
color = (r, g, b) + (r, g, b)

= proton

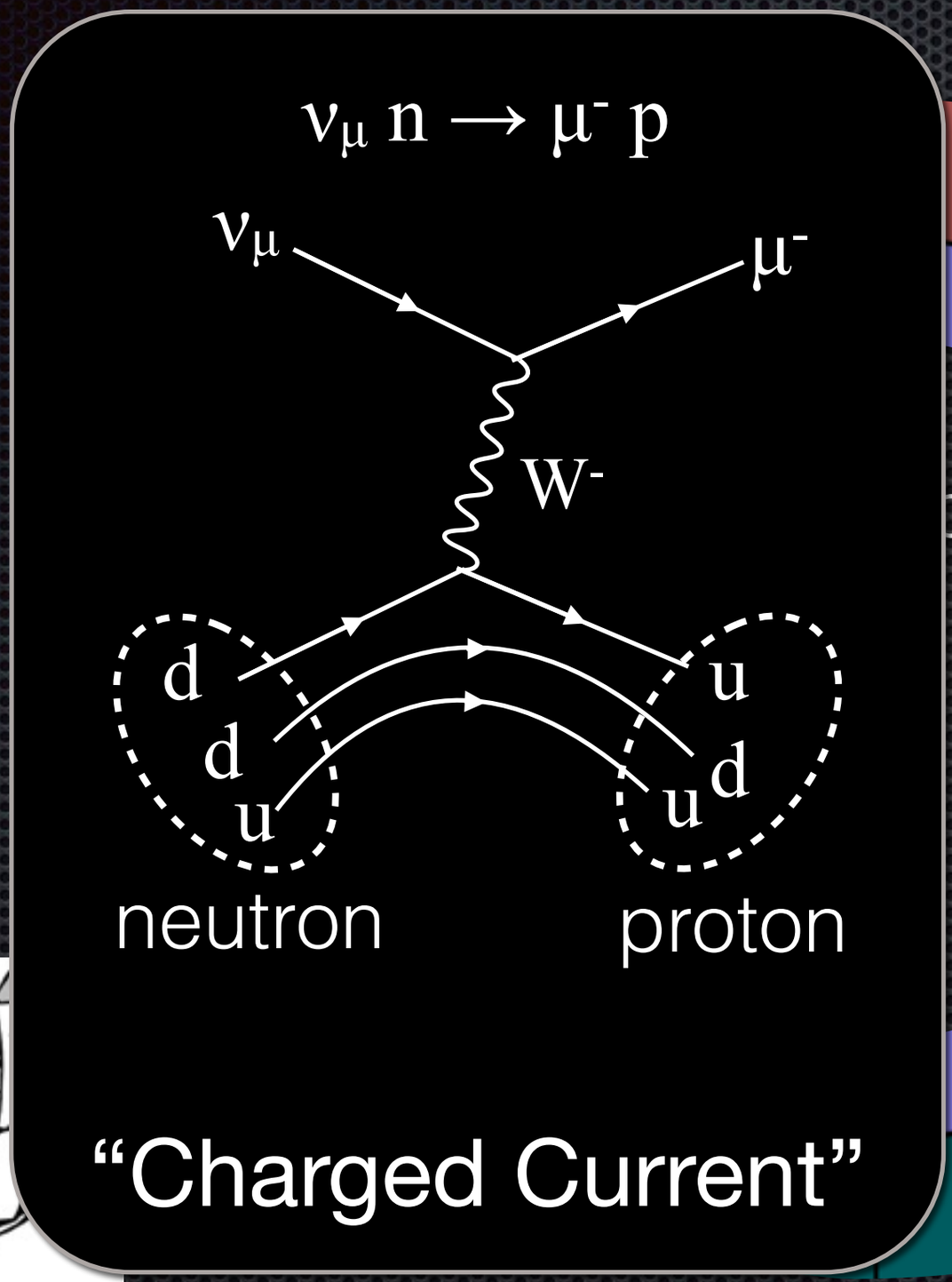
= neutron

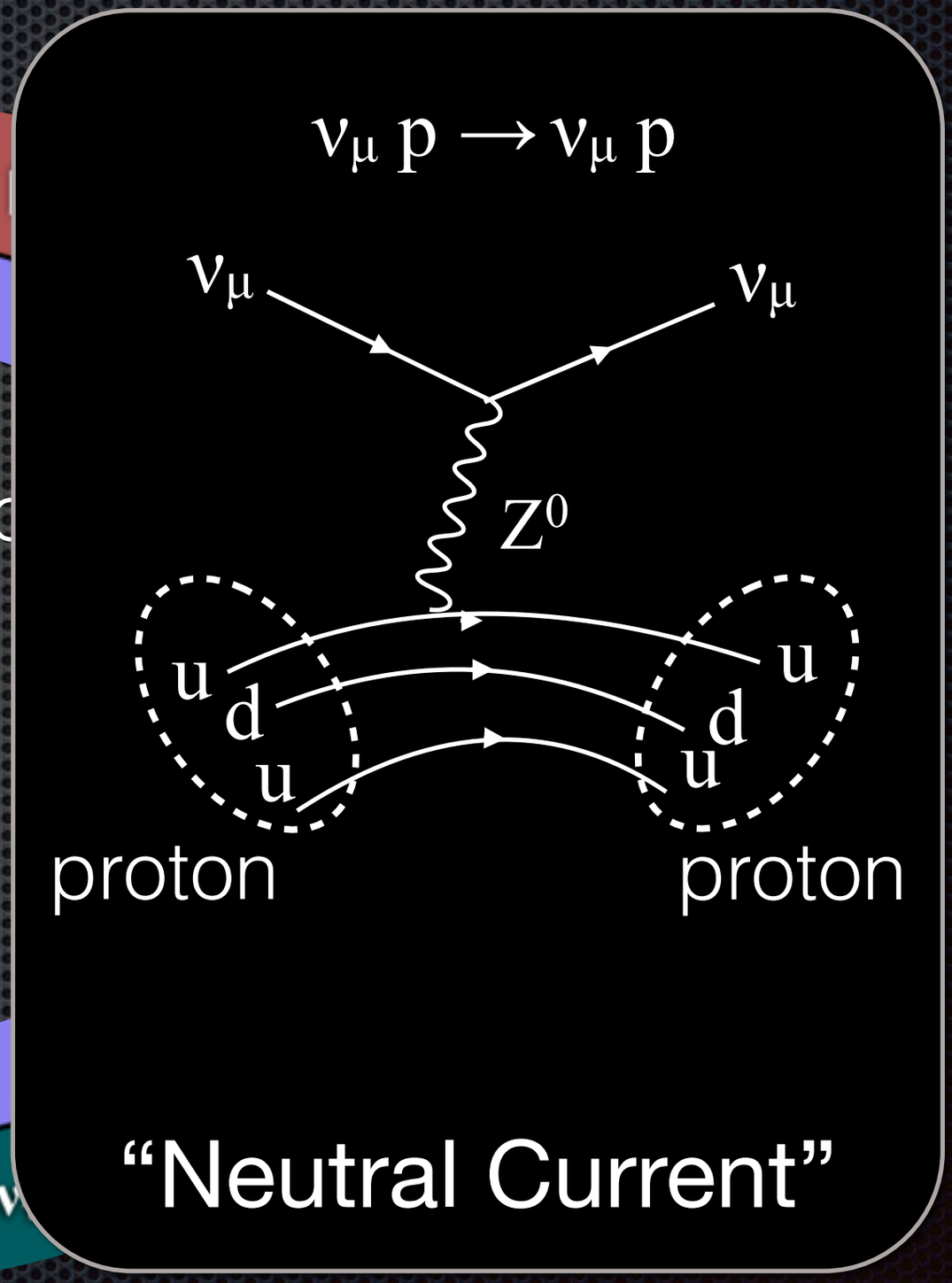
= pion

Standard Model of particle physics



Standard Model of particle physics

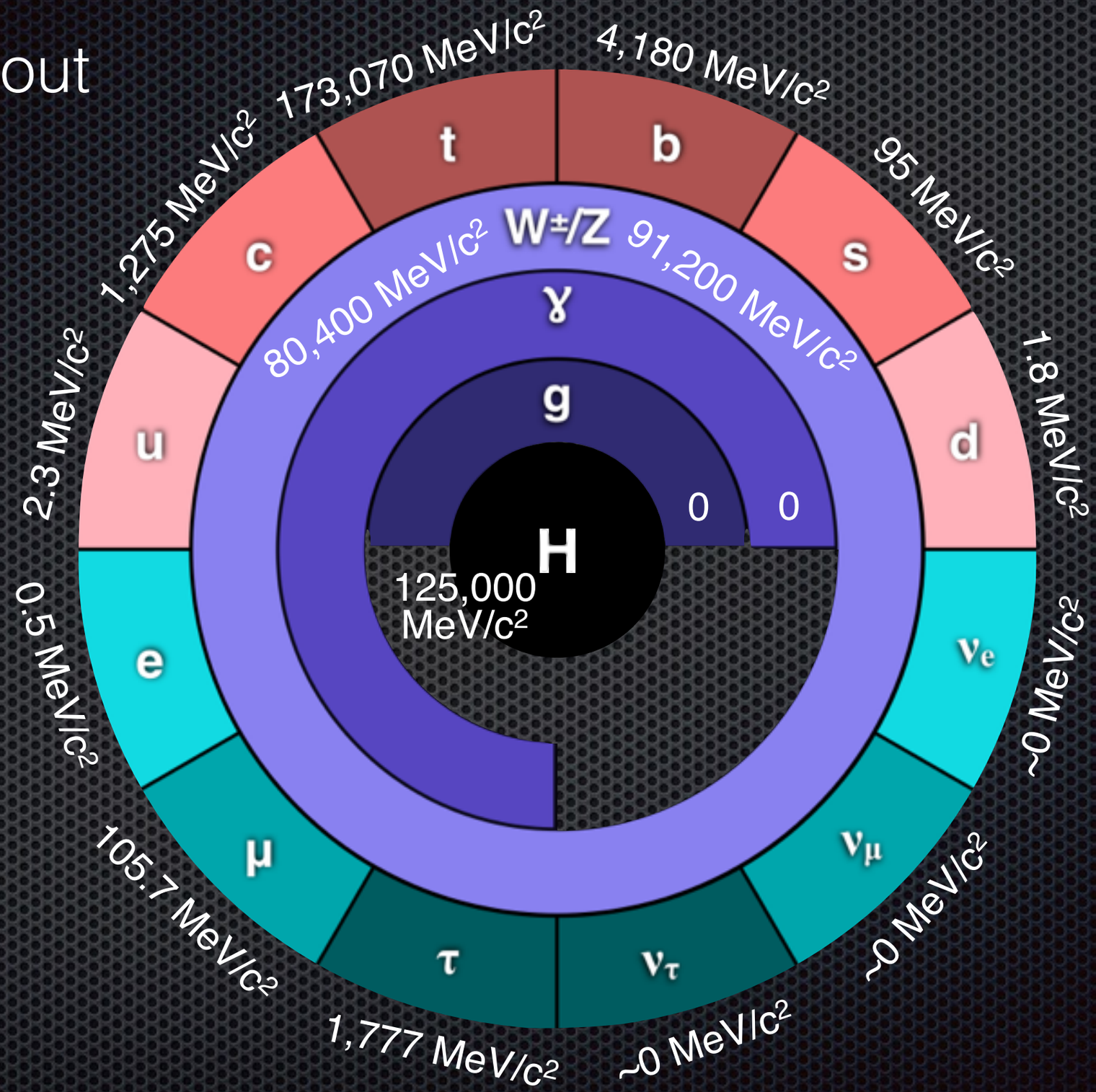




weak hypercharge governs

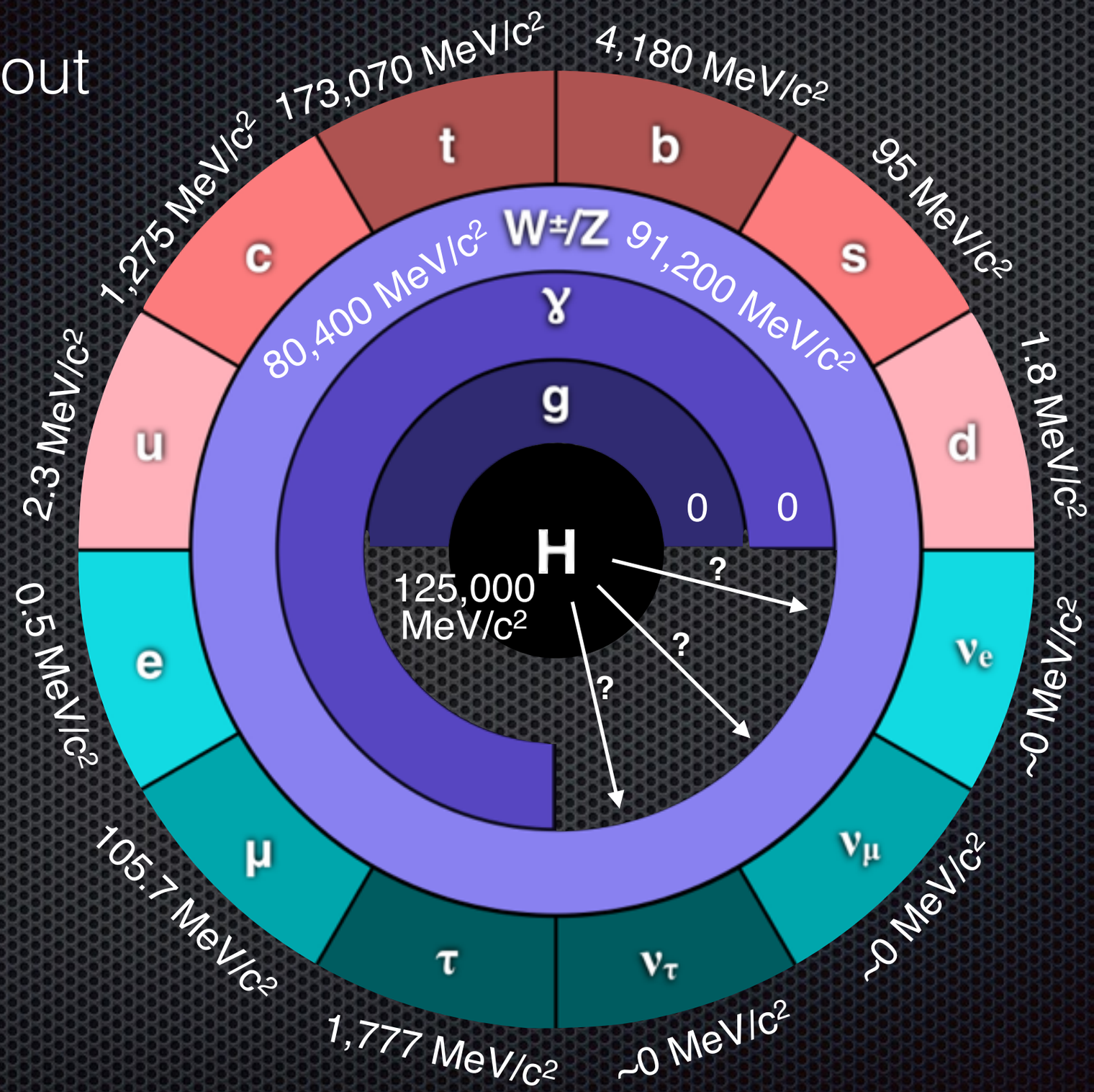
Standard Model of particle physics

A word about mass…



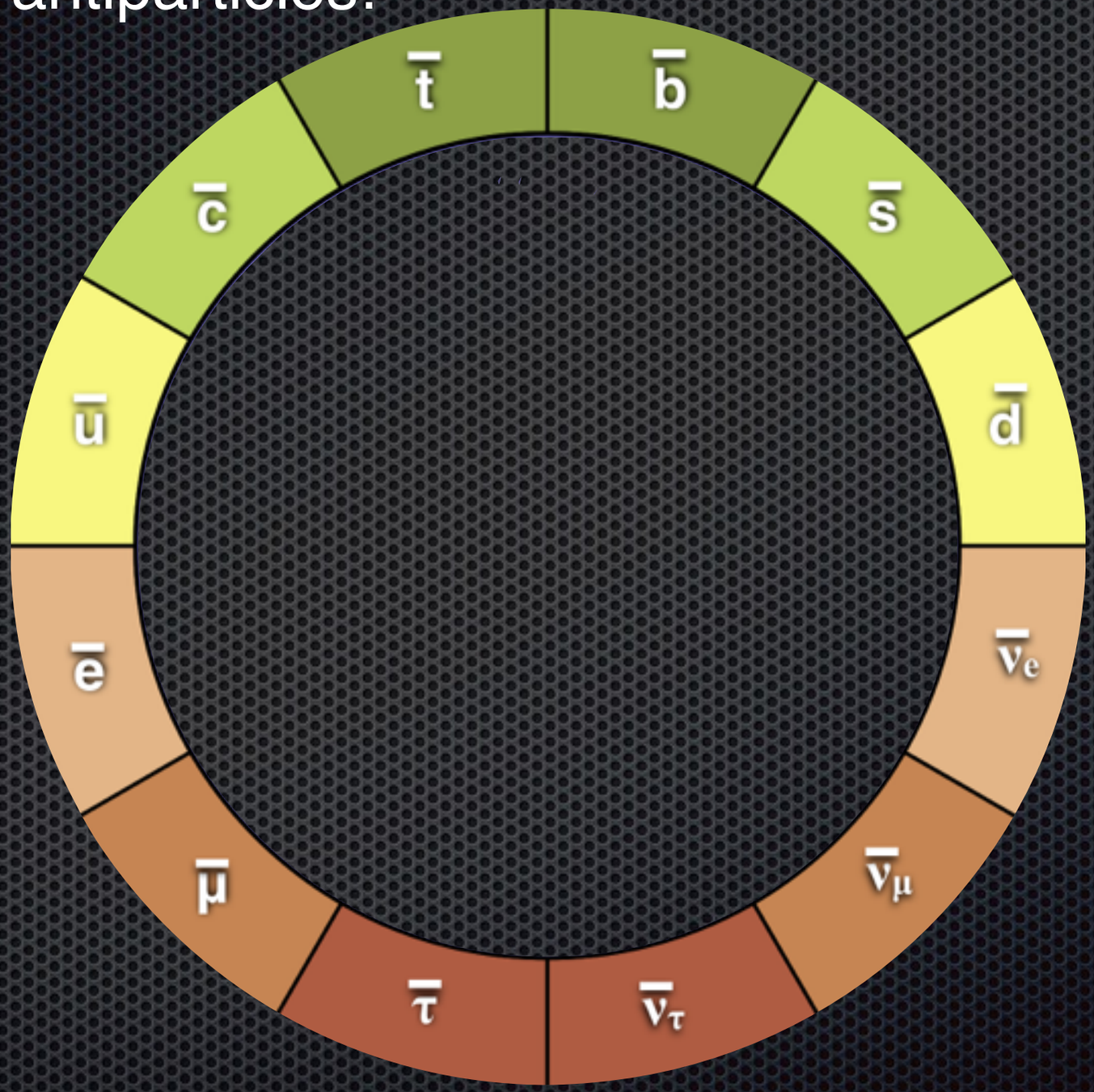
Standard Model of particle physics

A word about mass…



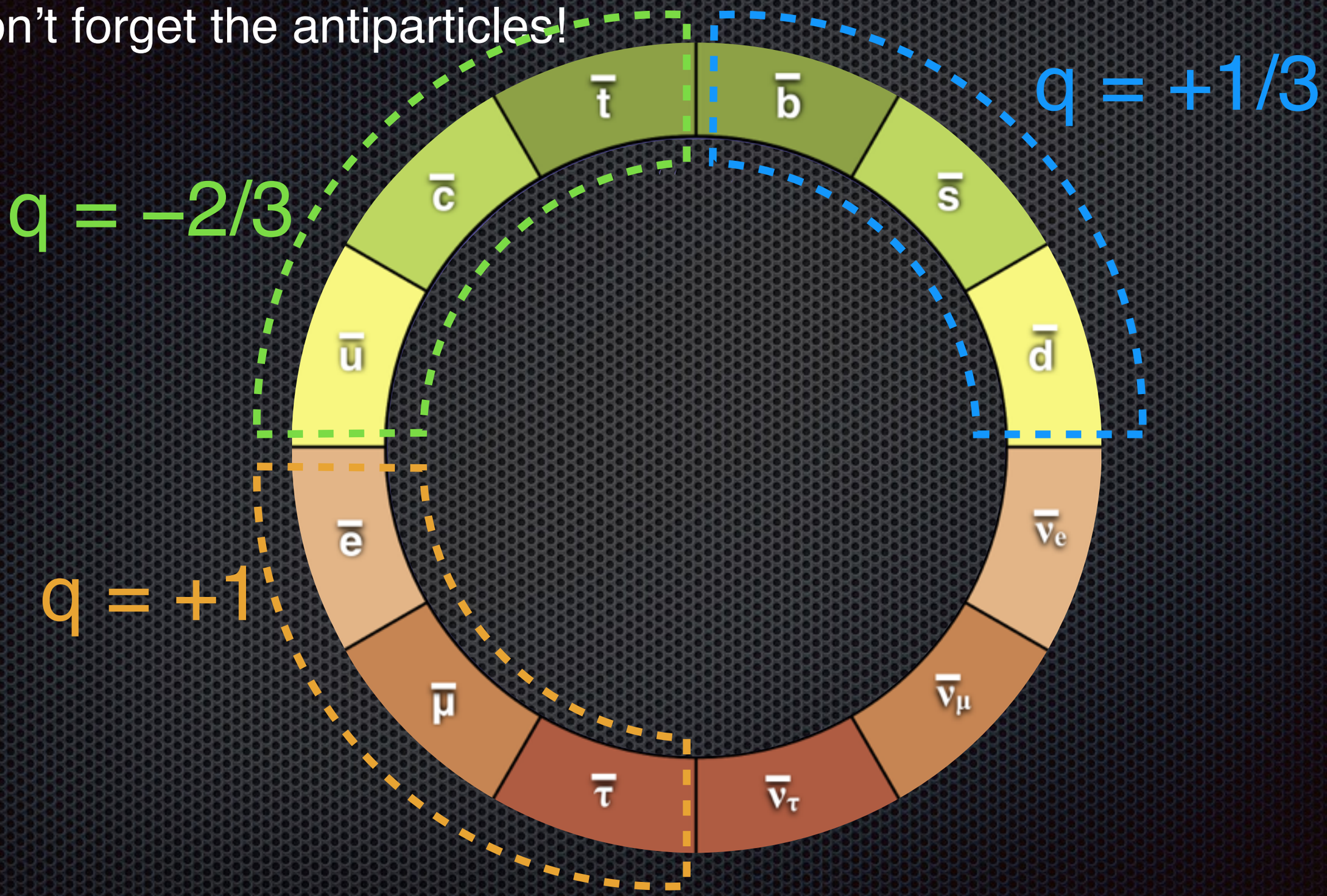
Standard Model of particle physics

Don’t forget the antiparticles!



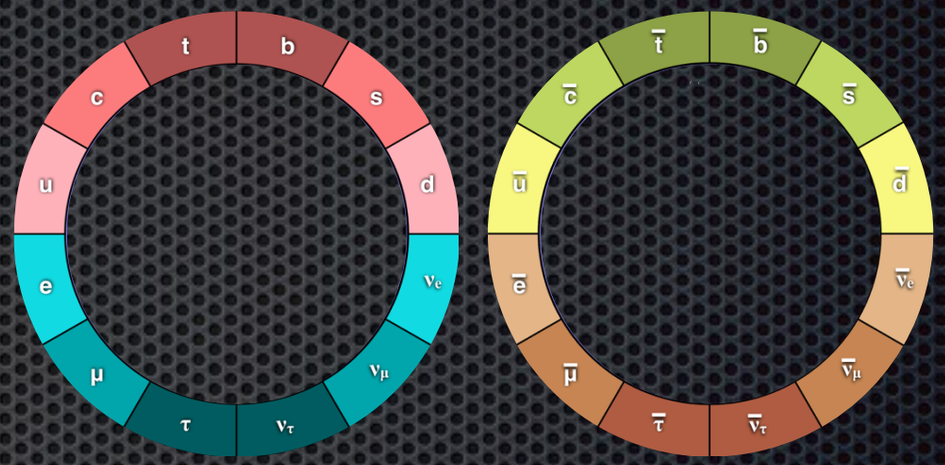
Standard Model of particle physics

Don't forget the antiparticles!



Standard Model of particle physics

- These discovered fermions →



- Whose behavior are described by interactions with these discovered bosons (“force carriers”) using quantum field theory. →

Standard Model of particle physics

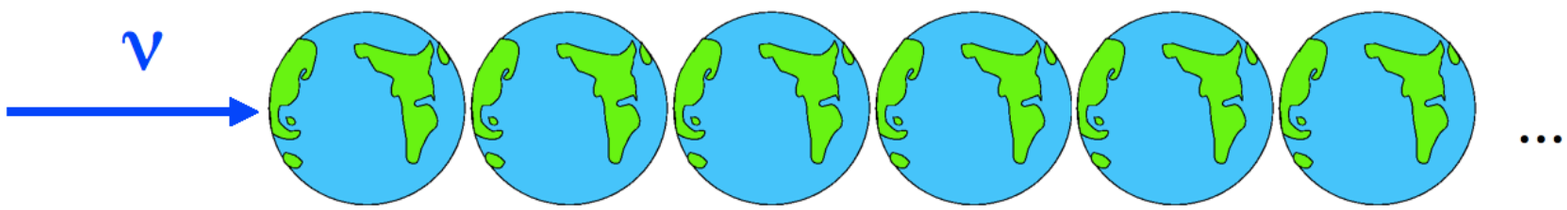
A word about strength of forces…

P.S. Gravity is not included in the Standard Model, but is very weak. Strength ~10^{-40}.

force	strength (low E)	distance
strong	1	10^{-15} m (nucleus size)
e&m	$\frac{1}{137}$	∞
weak	10^{-6}	10^{-18} m (0.1% nucleus)

Neutrinos are hard to detect!

the weak force is weak!
neutrinos interact
100,000,000,000
times less often than quarks

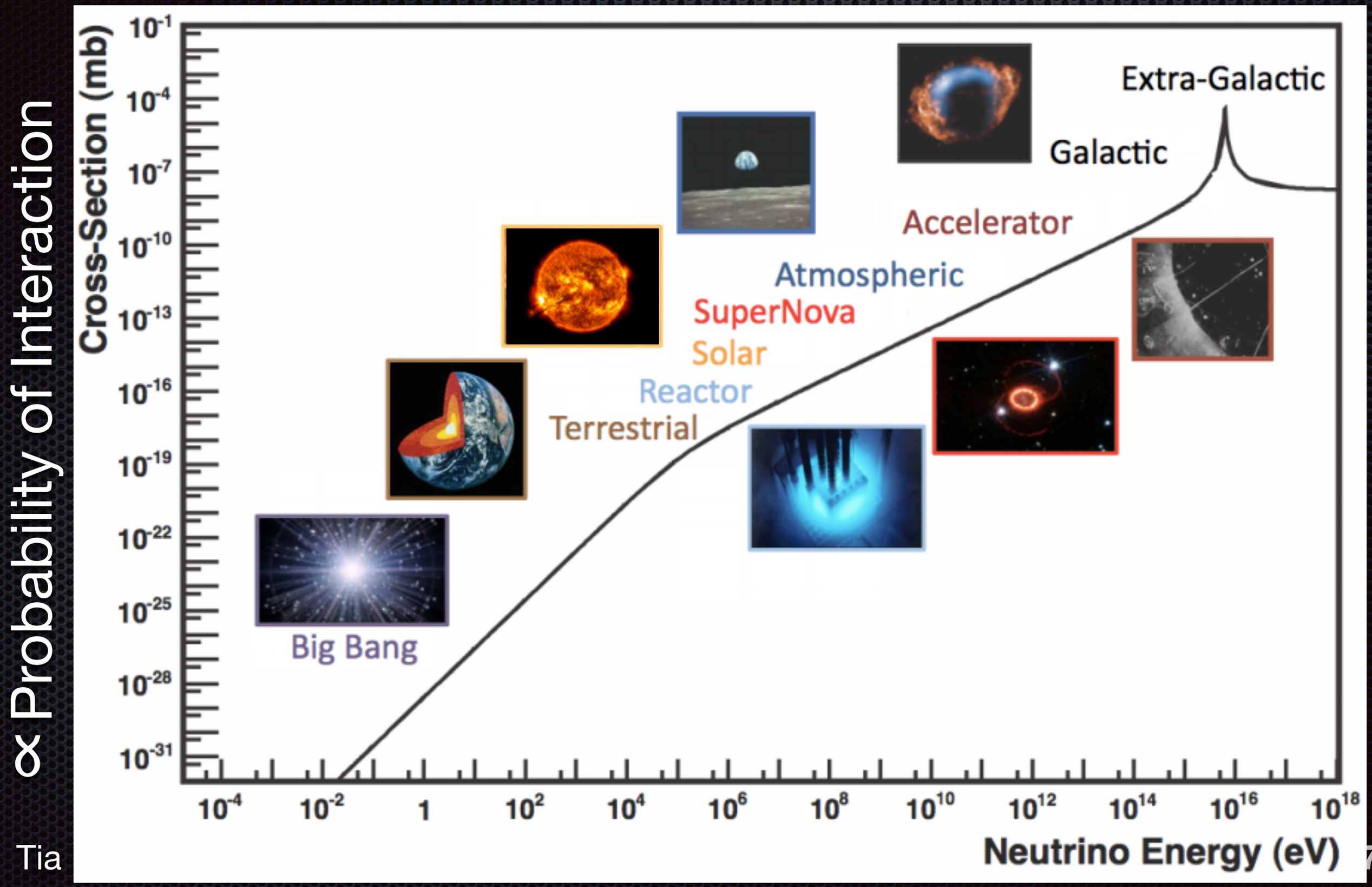


A neutrino has a good chance of traveling through
200 earths before interacting at all

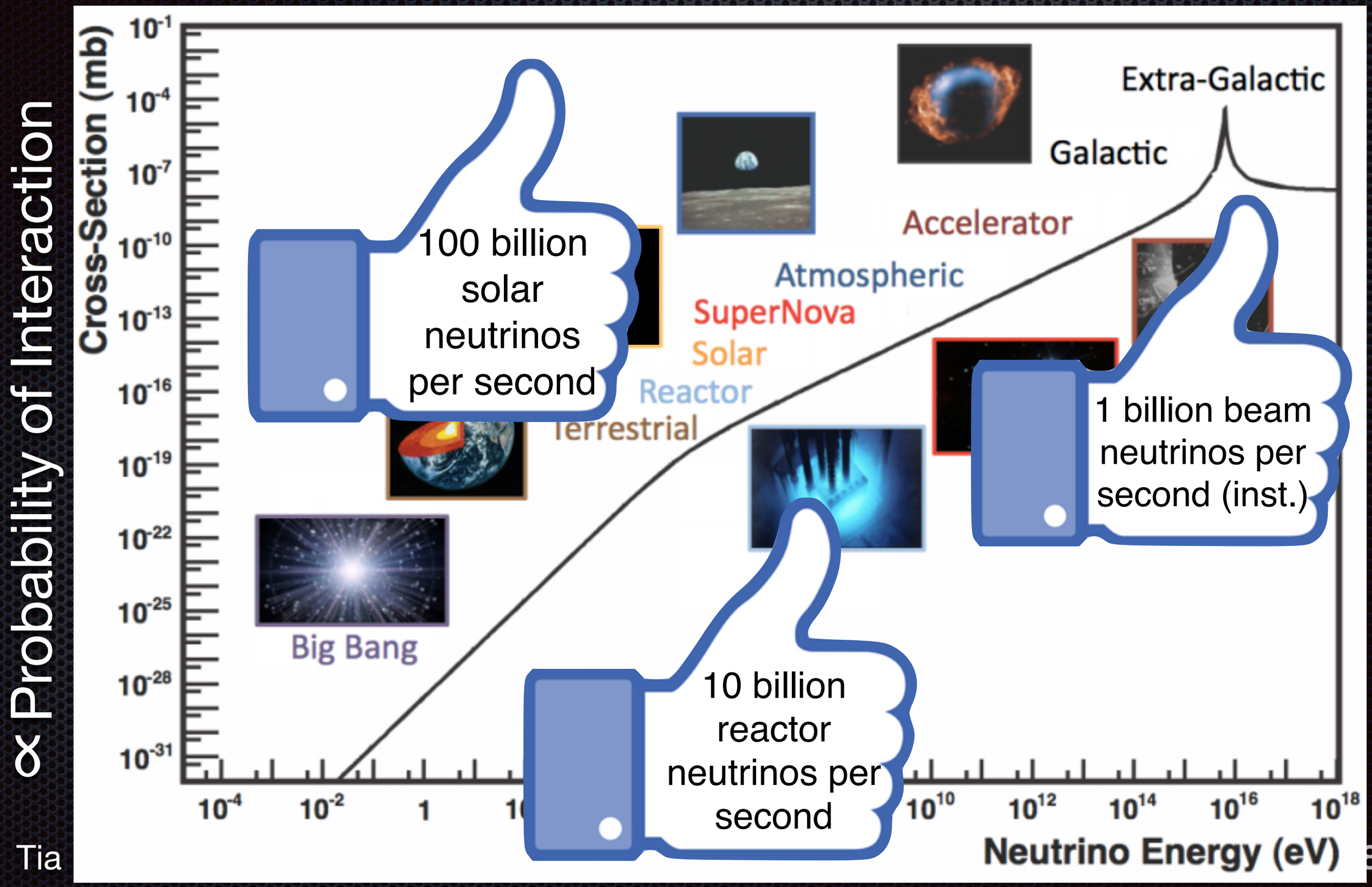
Because they interact rarely…

- We increase our chances of observation by studying very prolific neutrino sources with gigantic detectors!

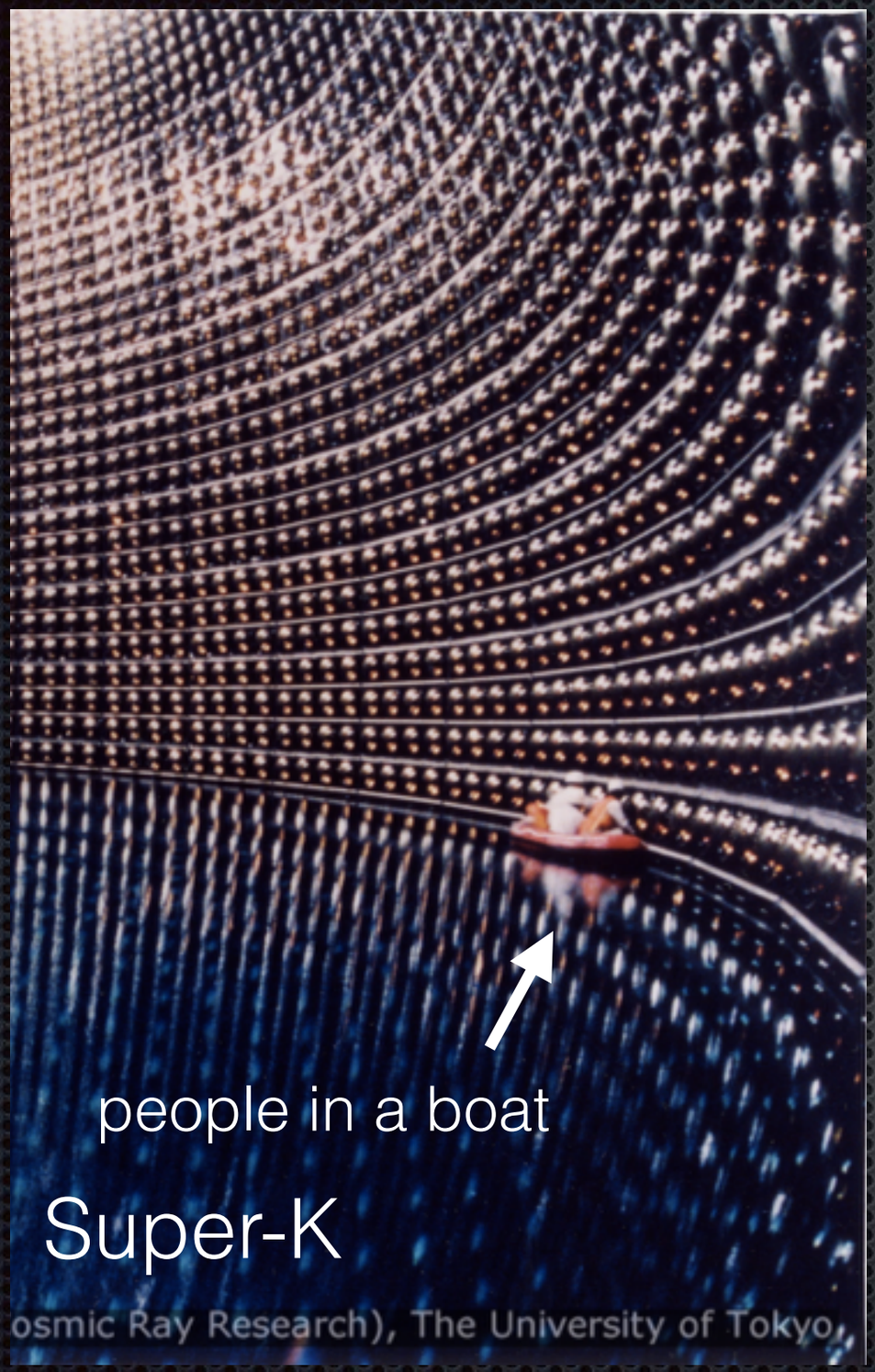
Neutrino sources

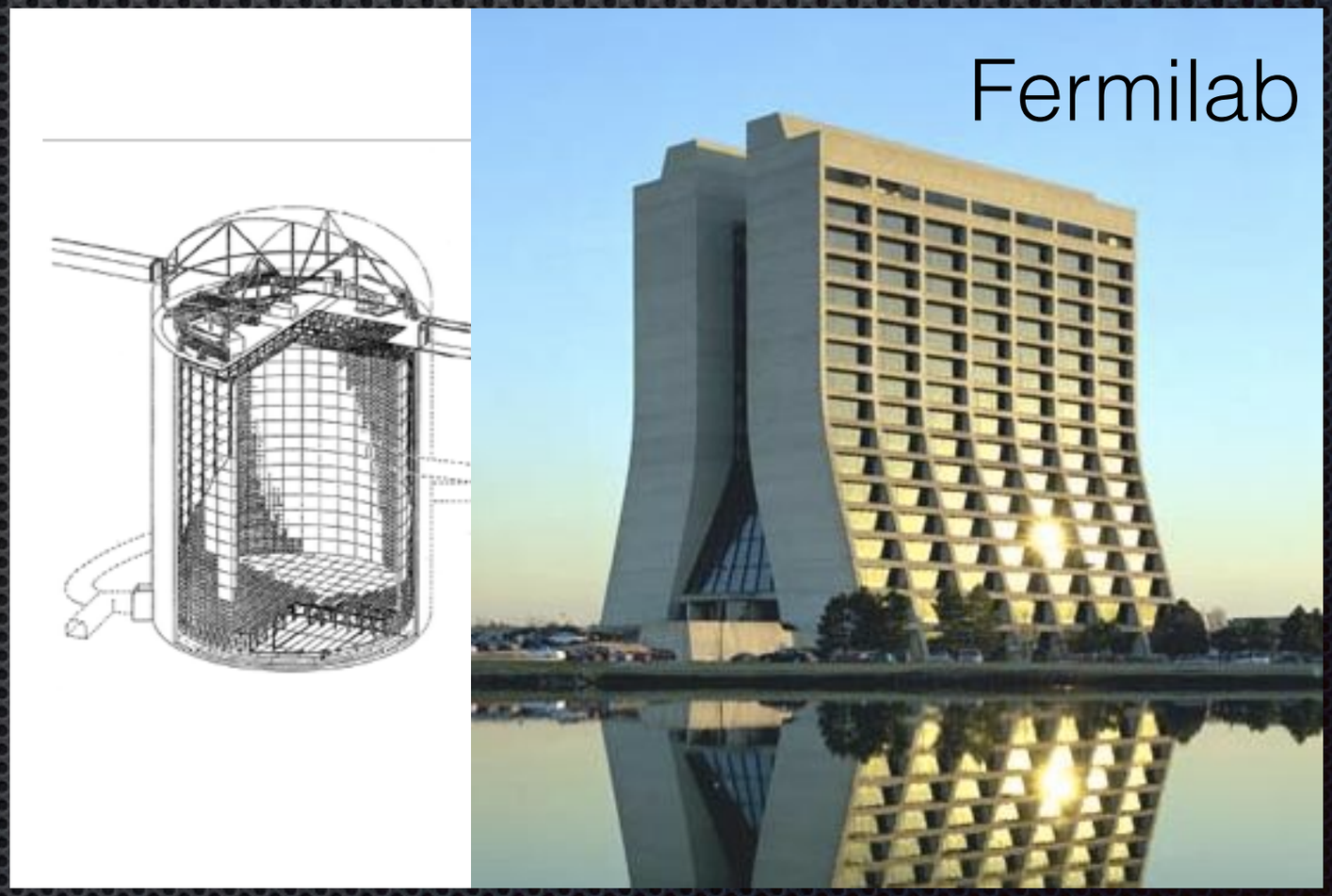


Neutrino sources



Scale of neutrino experiments

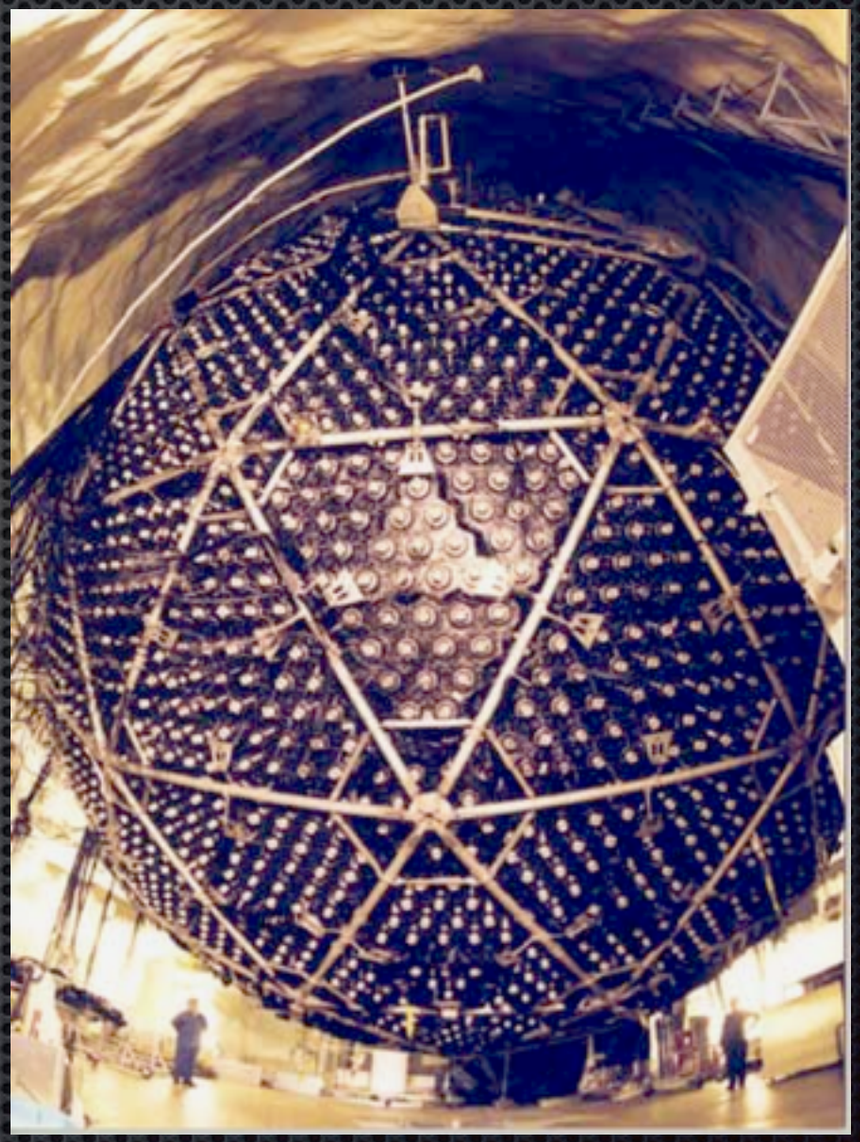




Solar

Scale of neutrino experiments

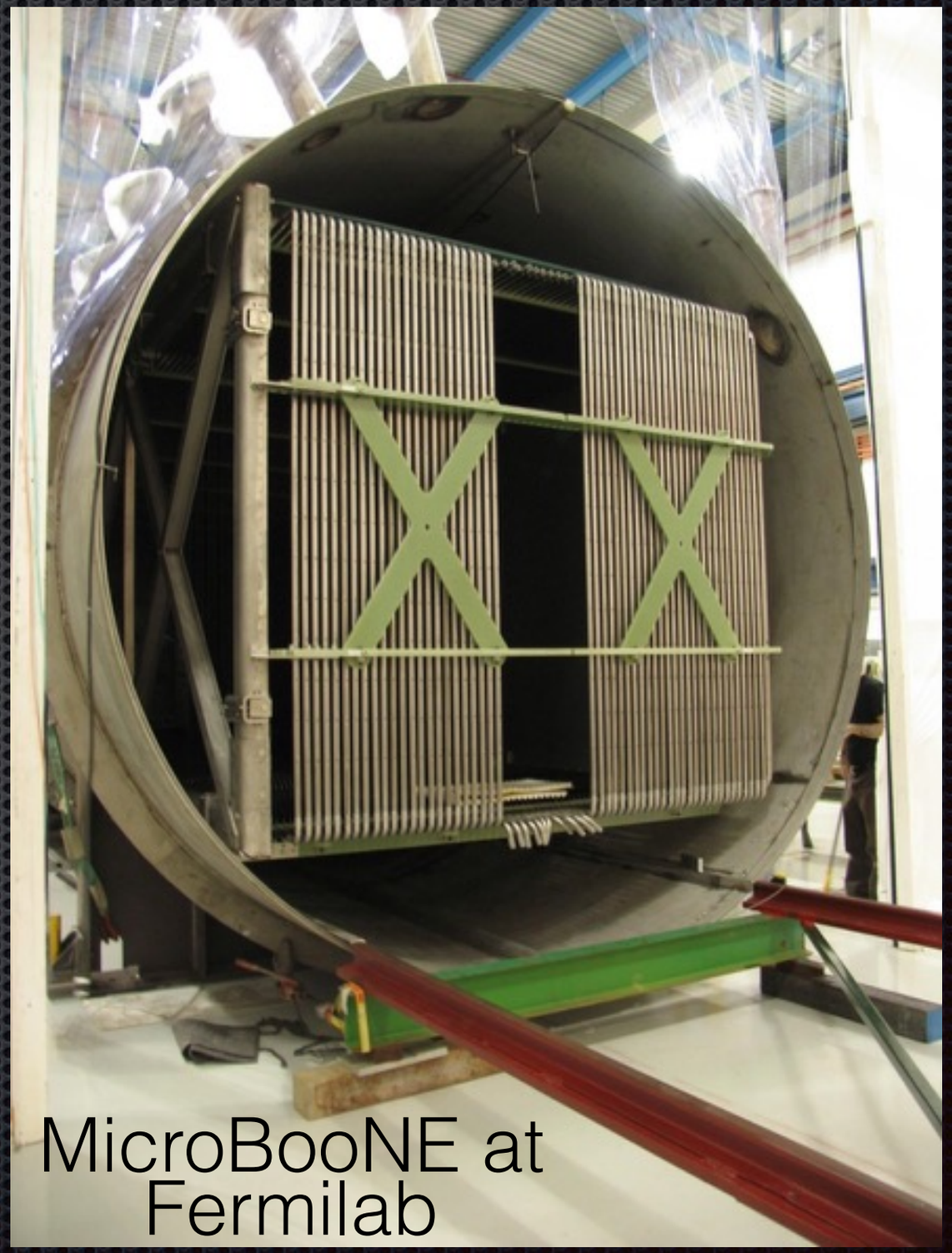
Aspen East



Sudbury Neutrino Experiment (SNO)

Solar

Scale of neutrino experiments



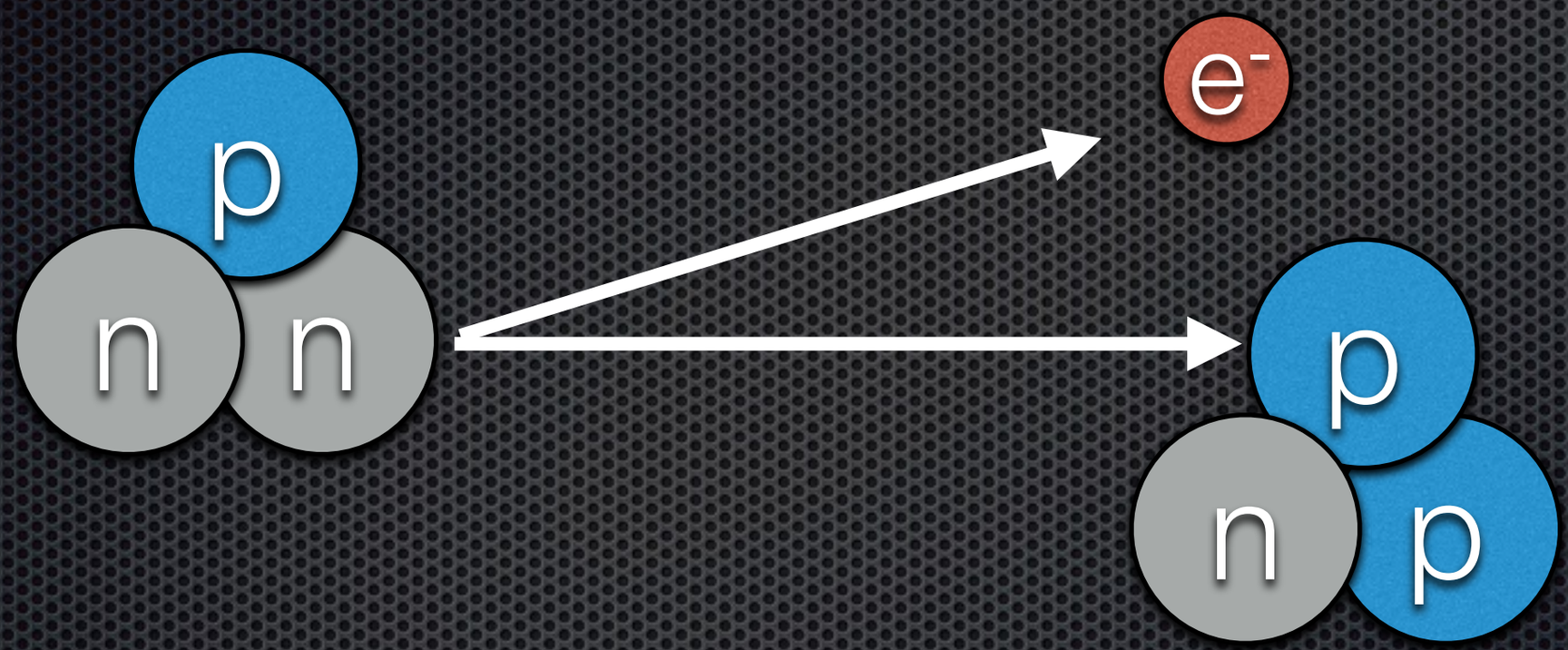
MicroBooNE at Fermilab

Beam

- What's particle physics? (translated into "Tia Speak")
- Where are we in neutrino physics? (Story time!)
  - FUN MATH for neutrino masses!
- Where are we going in neutrino physics? (A.K.A why Tia researches neutrinos)

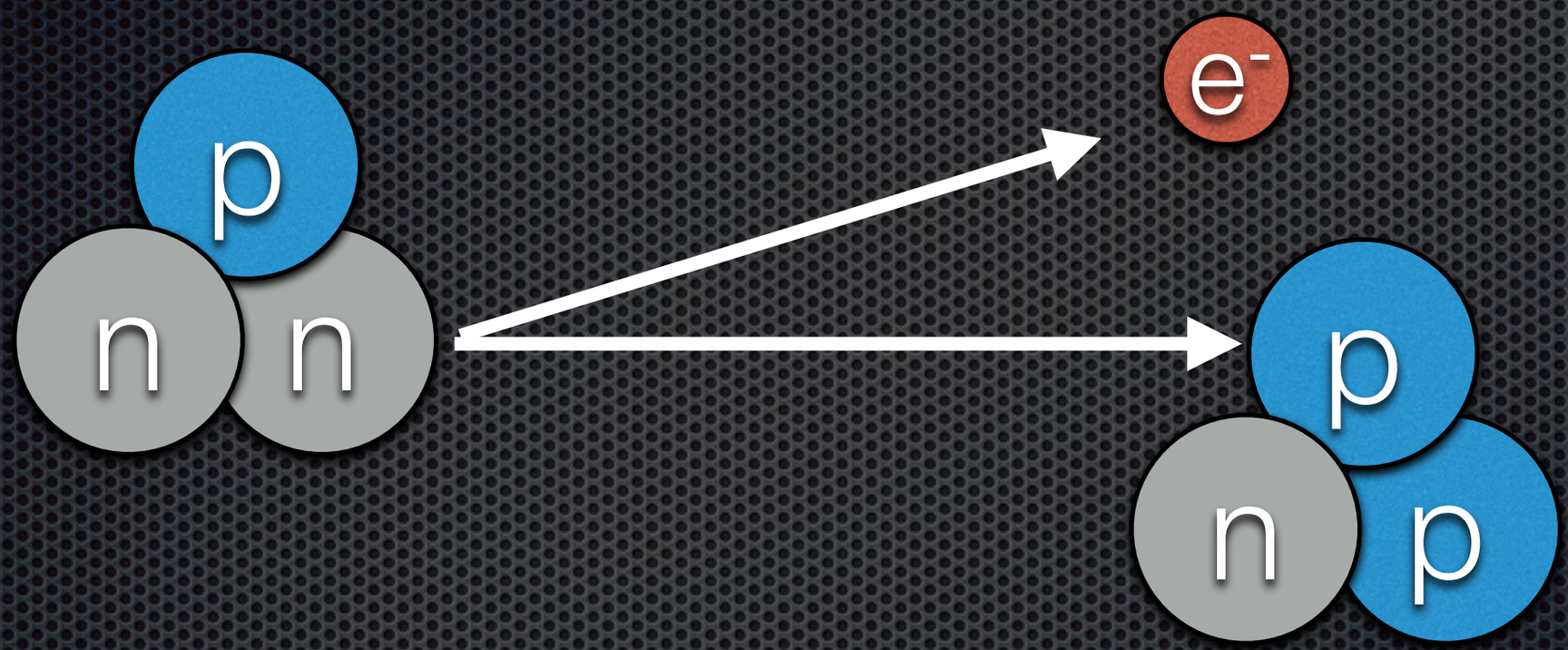
At the beginning, there was beta decay…

Ex. Tritium → Helium + e^-



At the beginning, there was beta decay…

Ex. Tritium → Helium + e^-



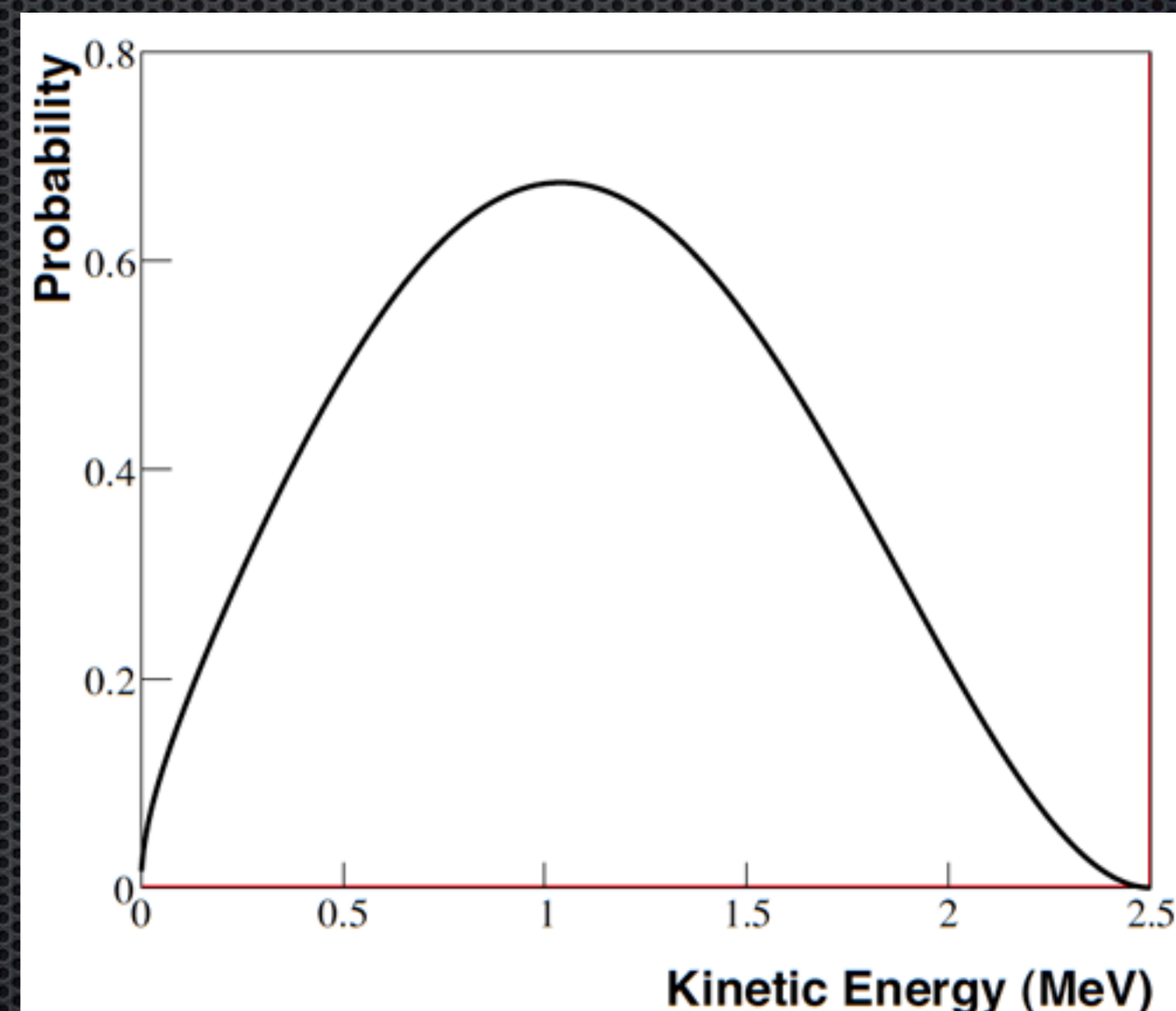
$E_{initial} > E_{final}$

products were missing energy

Observation of e^- energy spectrum

- Energy conservation was either violated, or a hidden particle carried away energy.

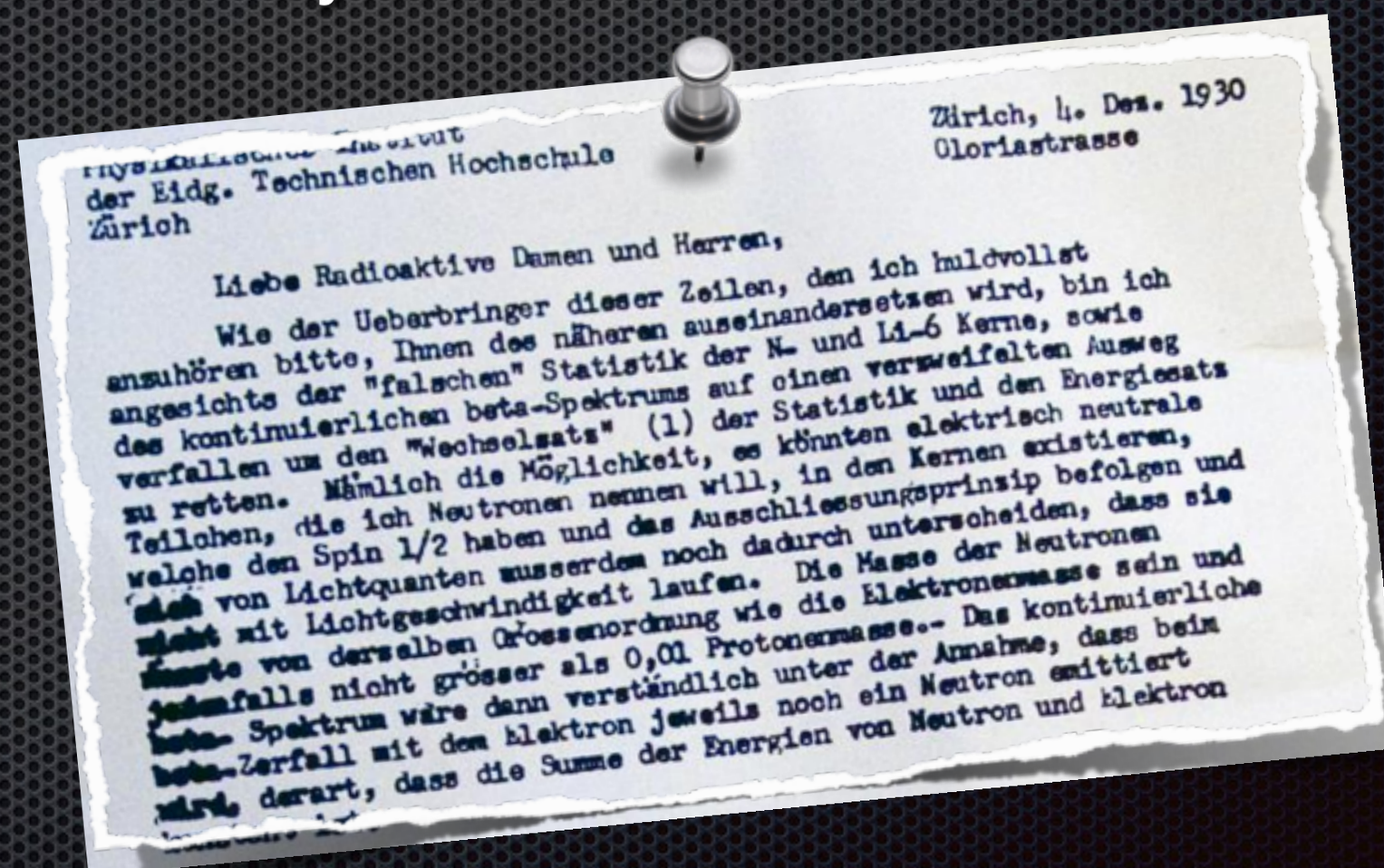
Sir James Chadwick 1914



A desperate remedy!

- Energy and spin 1/2 carried away by a light “invisible” particle.
  - Later named “neutrino” by Enrico Fermi

Wolfgang Pauli, 1930



Zürich, 4. Dez. 1930
Gloriastrasse

der Eidg. Technischen Hochschule
Zürich

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron

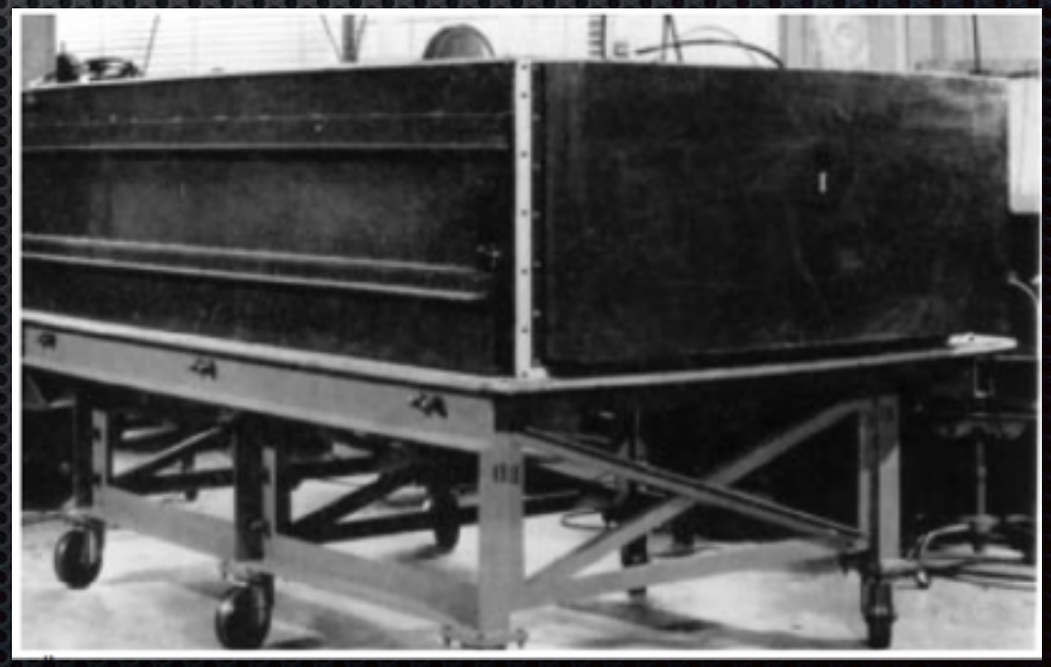
Letter to Zurich Conference

Neutrino Wanted!

- "Project Poltergeist"

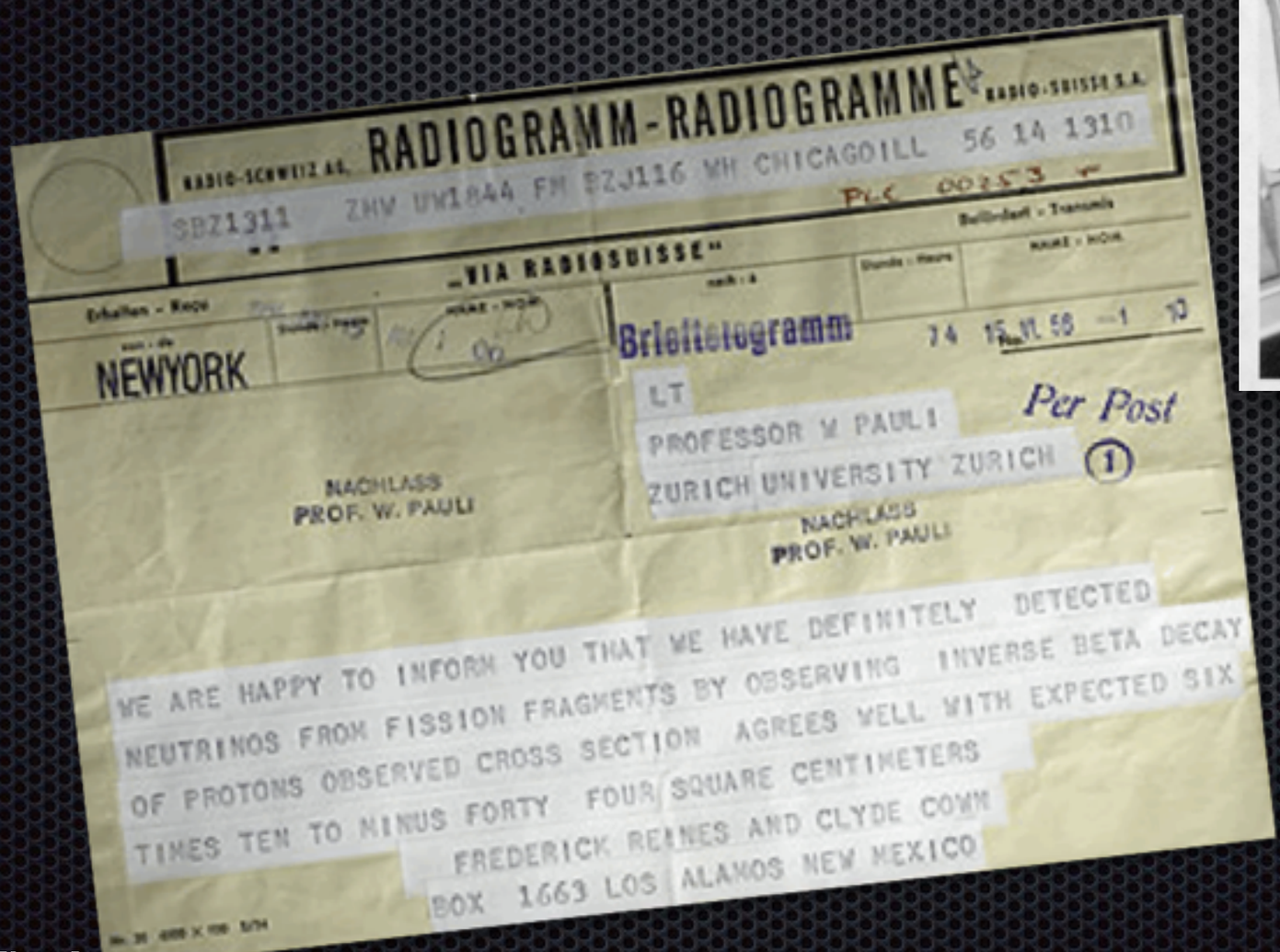
Savannah River
Nuclear Reactor,
South Carolina. 1955

Neutrino Detector:
tank of water and
liquid scintillator



Neutrino Caught!

- $\bar{\nu}_e$ discovered 14 June 1956


RADIO-SCHWEIZ AG. RADIOGRAMM-RADIOGRAMME RADIO-SUISSE S.A.

SBZ1311 ZHV UW1844 FM BZJ116 WH CHICAGOILL 56 14 1310

PLC 00253

„VIA RADIOSUISSE"

Empfangen - Reçu

NEWYORK

NACHLASS PROF. W. PAULI

Brieftelegramm

LT

PROFESSOR W PAULI

ZURICH UNIVERSITY ZURICH

Per Post

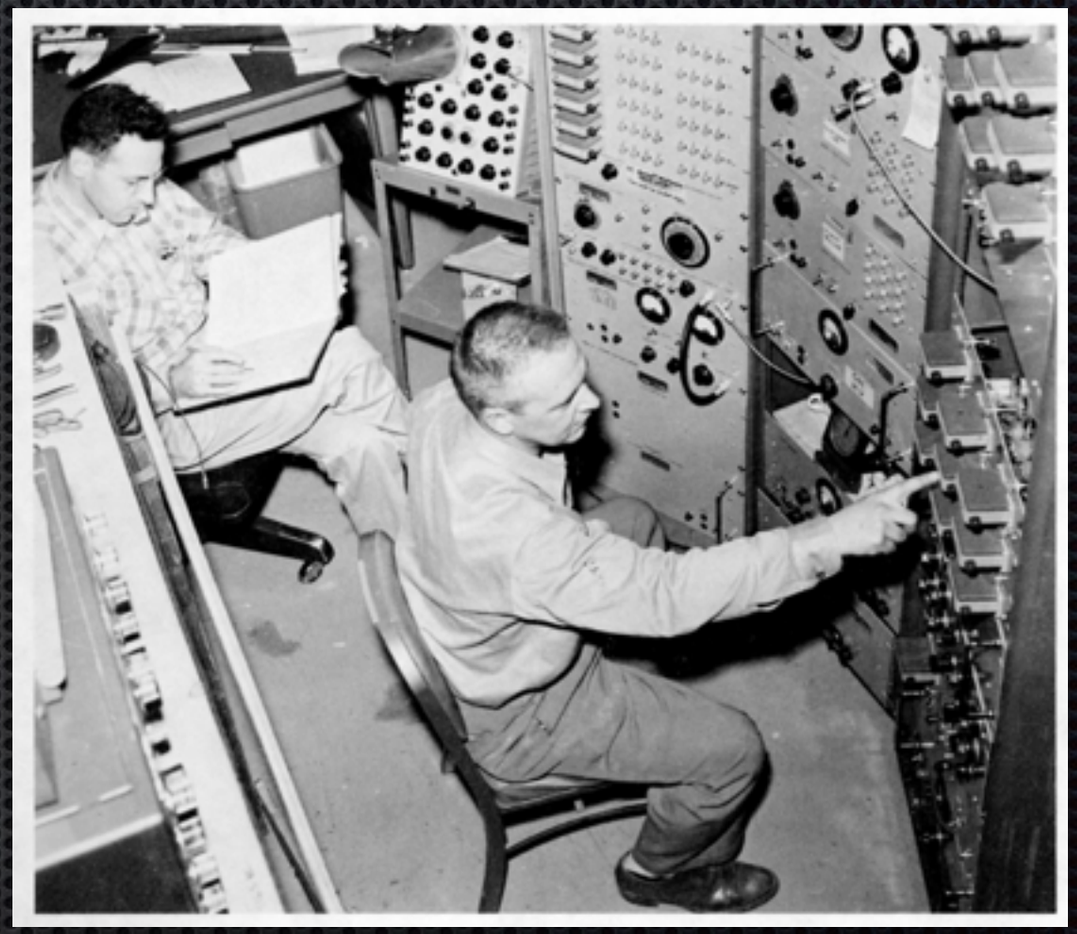
1

NACHLASS PROF. W. PAULI

WE ARE HAPPY TO INFORM YOU THAT WE HAVE DEFINITELY DETECTED NEUTRINOS FROM FISSION FRAGMENTS BY OBSERVING INVERSE BETA DECAY OF PROTONS OBSERVED CROSS SECTION AGREES WELL WITH EXPECTED SIX TIMES TEN TO MINUS FORTY FOUR SQUARE CENTIMETERS

FREDERICK REINES AND CLYDE COWN

BOX 1663 LOS ALAMOS NEW MEXICO



Clyde Cowan & Frederick Reines

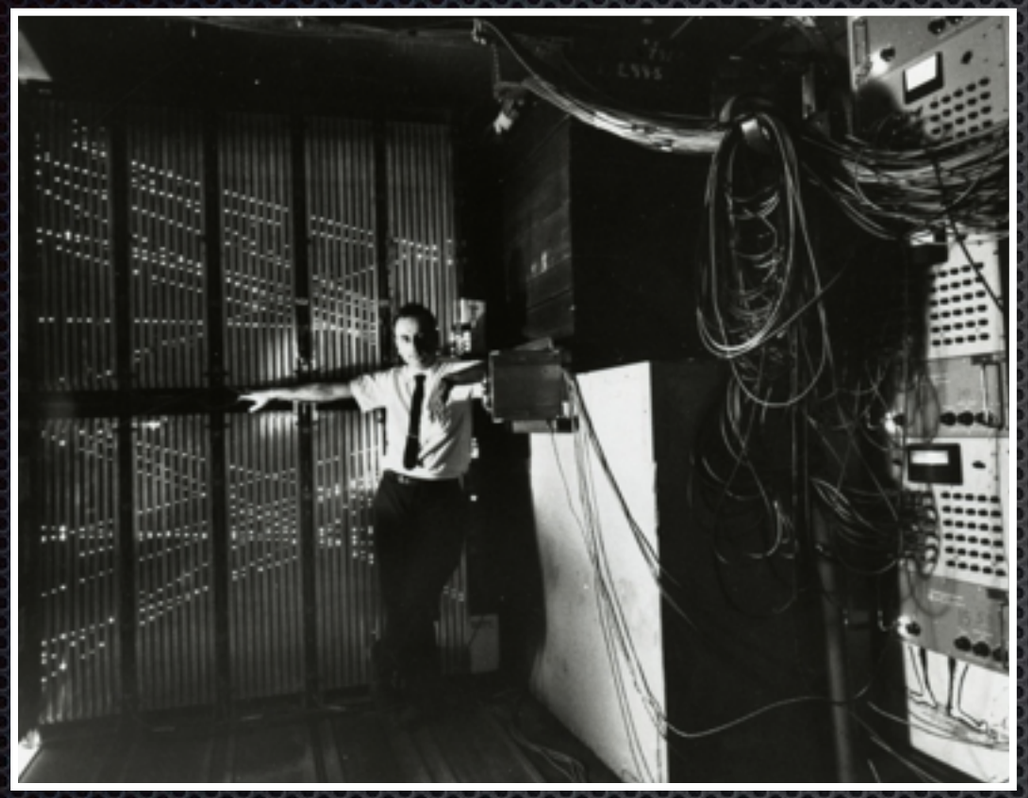
awarded 1995

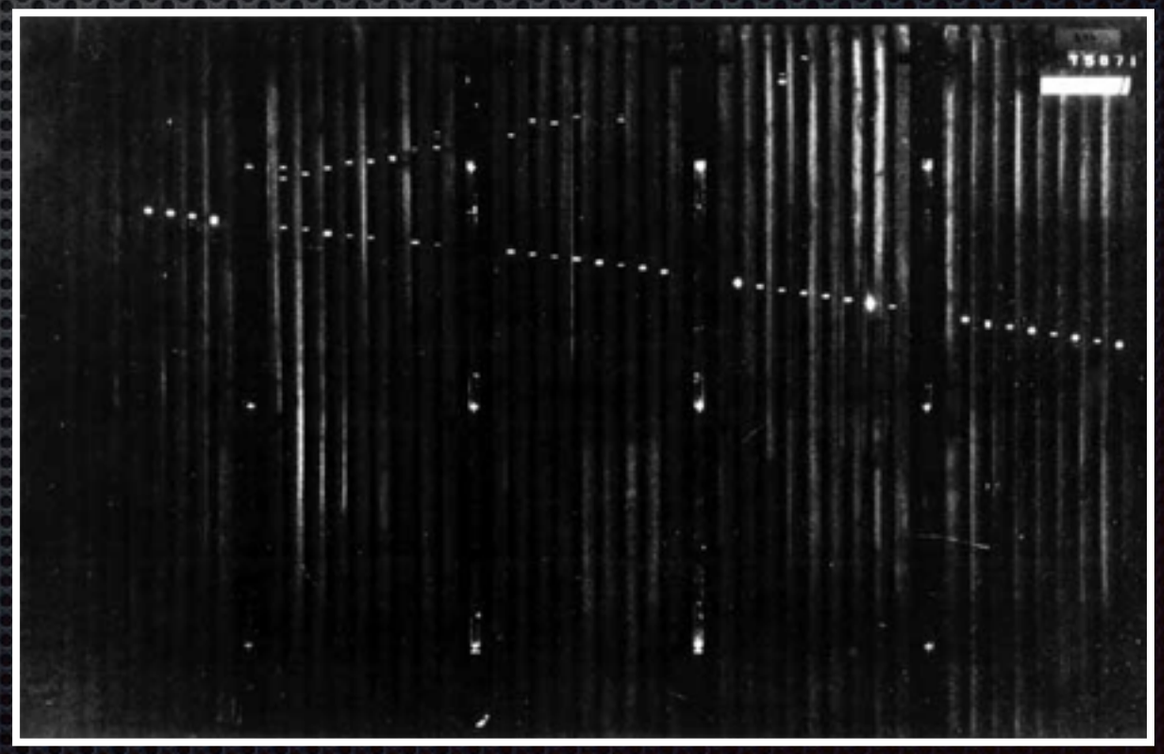
to Frederick for detection
of the neutrino

Spark chamber reveals ν_μ

- ν_μ suspected, then discovered in 1962

Leon M. Lederman, Melvin Schwartz and Jack Steinberger





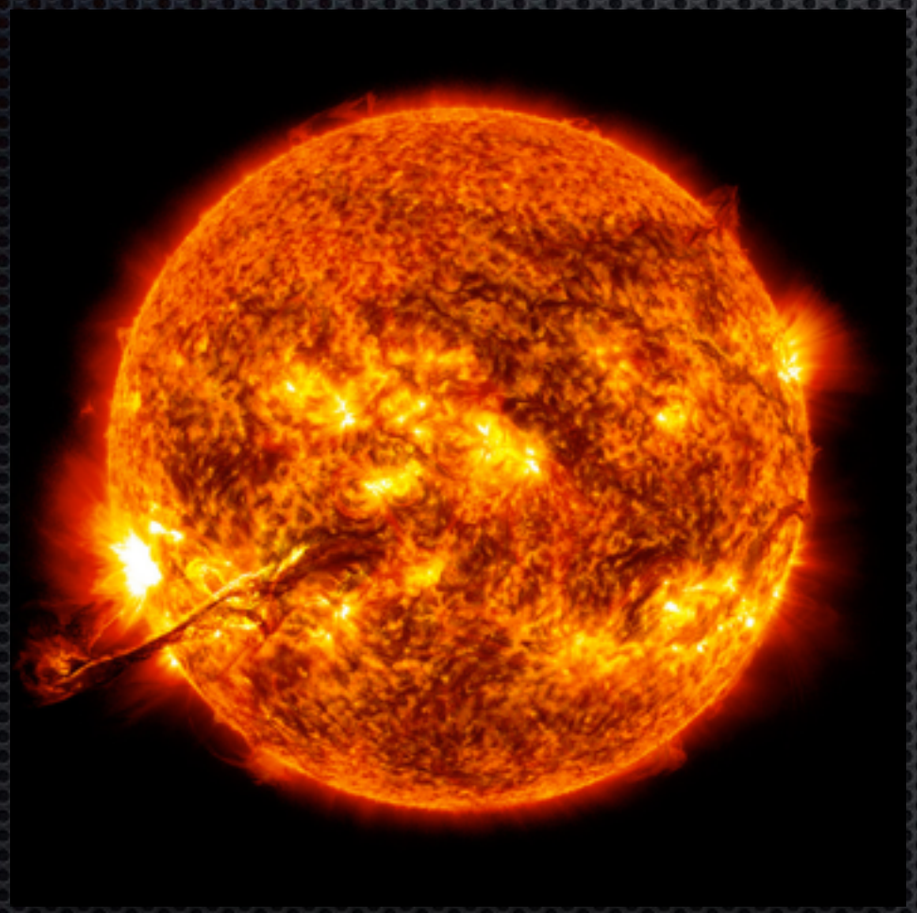
Muons created by a muon neutrino beam in a spark chamber

awarded 1998

to Lederman, Schwartz and Steinberger
for discovery of the ν_μ

Solar fusion should emit ν_e

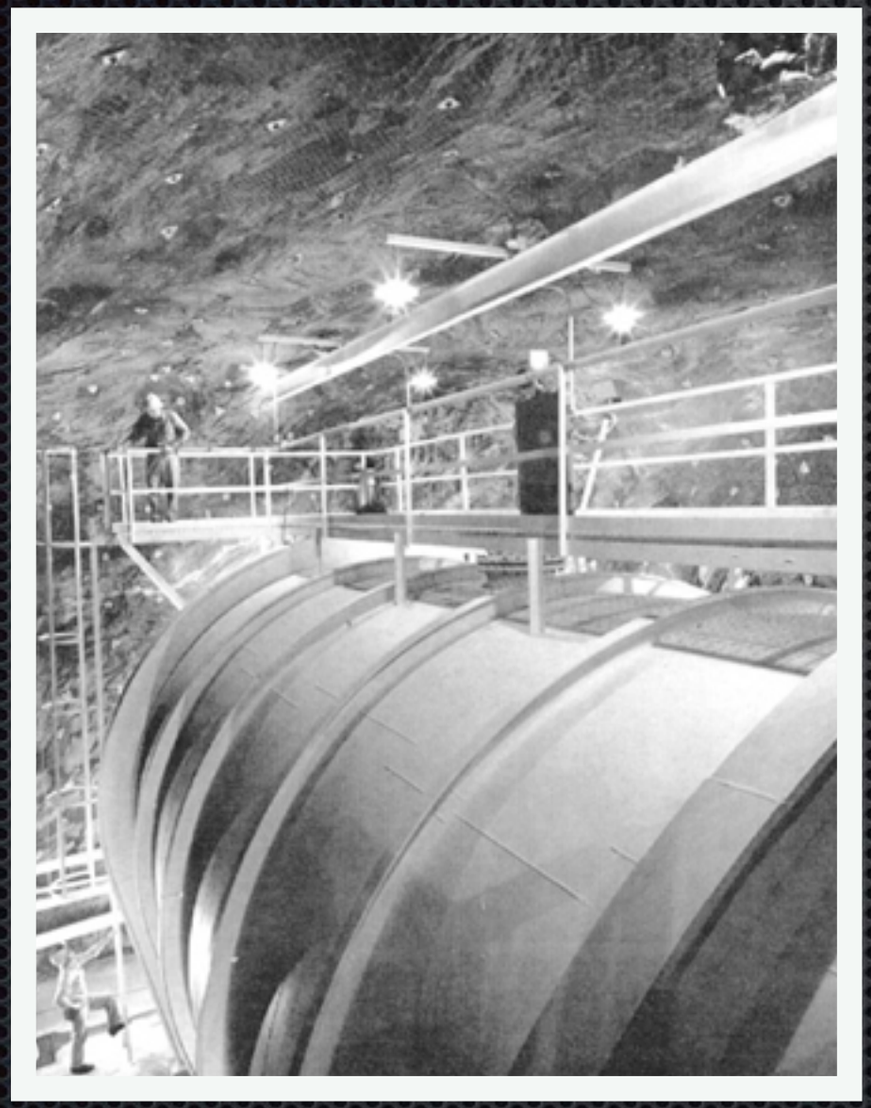
- Bahcall, under current solar models, calculated the number of electron flavor neutrinos coming from sun.



John Bahcall

Teaming up!

- 1968, South Dakota.
- Only saw ~1/3 of the expected rate!



Homestake Experiment, 1960s

Ray Davis and John Bahcall

Neutrino mixing postulated

- Just a few years earlier, such a deficit was predicted due to different types of neutrinos mixing. (PMNS matrix!)
  - Possibly the missing electron neutrinos oscillated to another flavor.

Bruno Pontecorvo

Ziro Maki

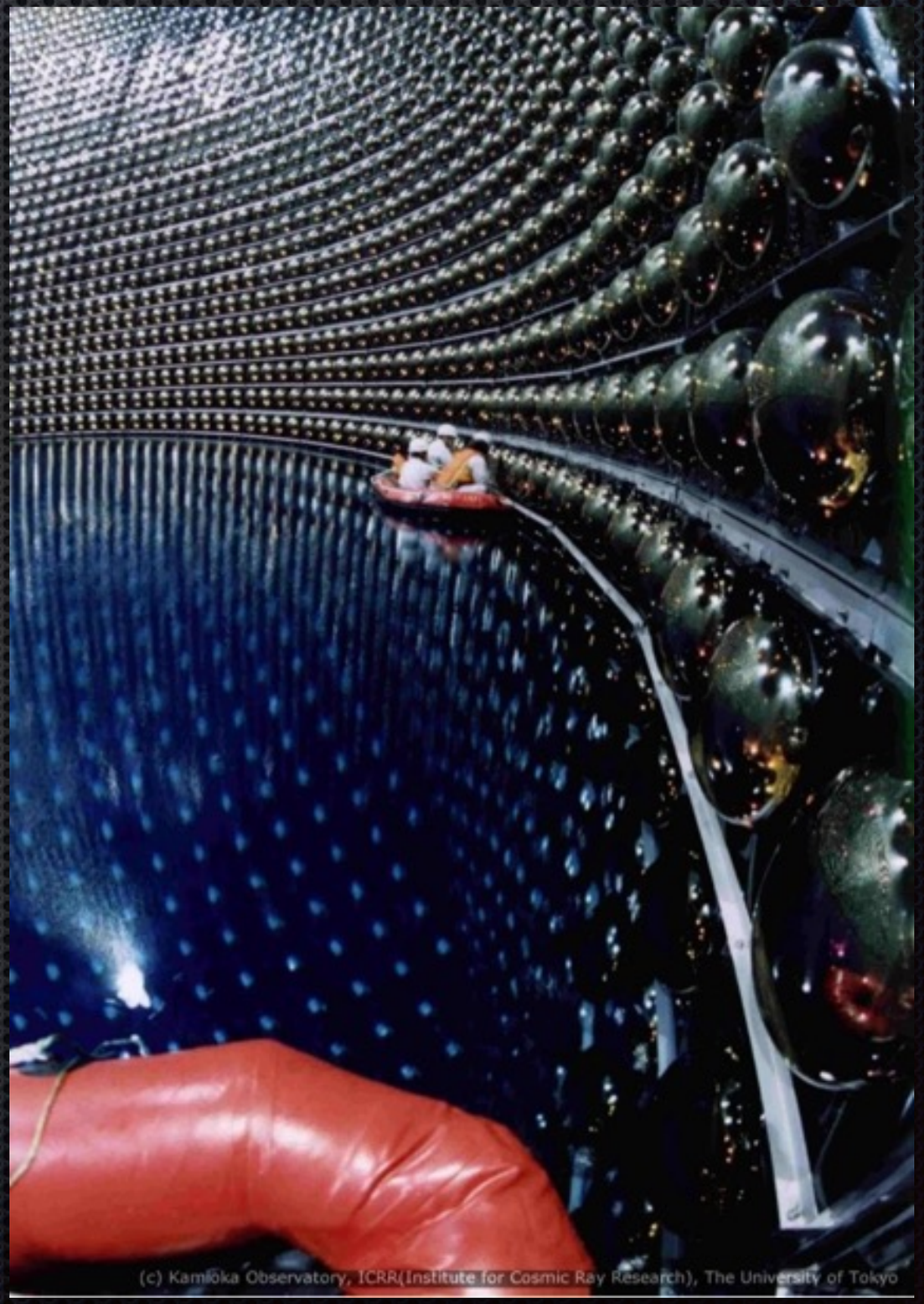
Masami Nakagawa

Shoichi Sakata

Conclusive evidence

- 3 July 1998, Japan's Super-K
  - Water chernkov detector, sensitive to electron and muon flavor neutrinos.

$$\nu_e \text{ sun} \rightarrow \nu_e,\ \nu_\mu \ldots$$

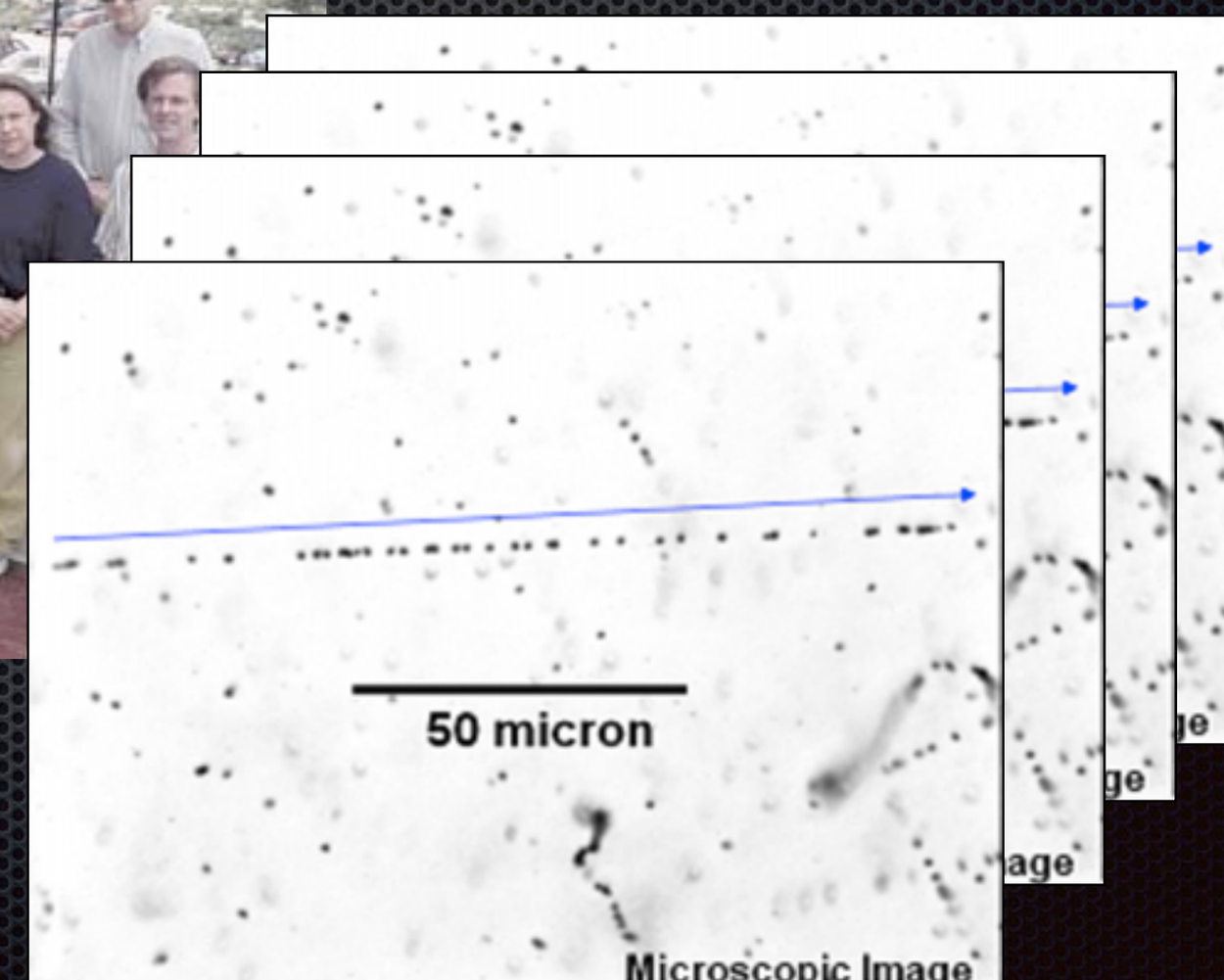


awarded 2002

to Davis and Koshiba
for “cosmic neutrinos” (solar, atmospheric, supernova)

Tau neutrino

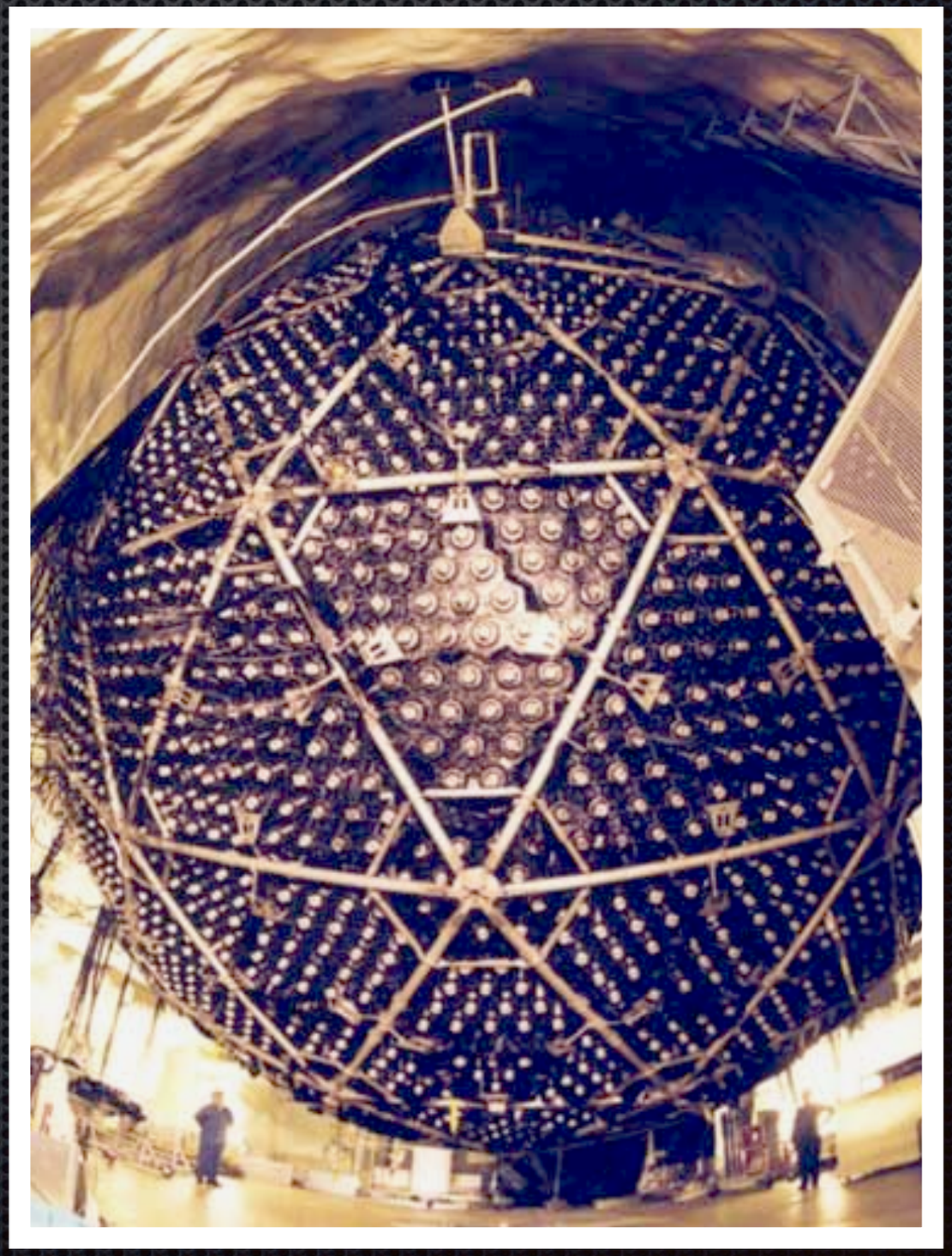
- 2000 Fermilab, DONUT experiment (Direct Observation of the Nu Tau)



Solar Oscillations Complete!

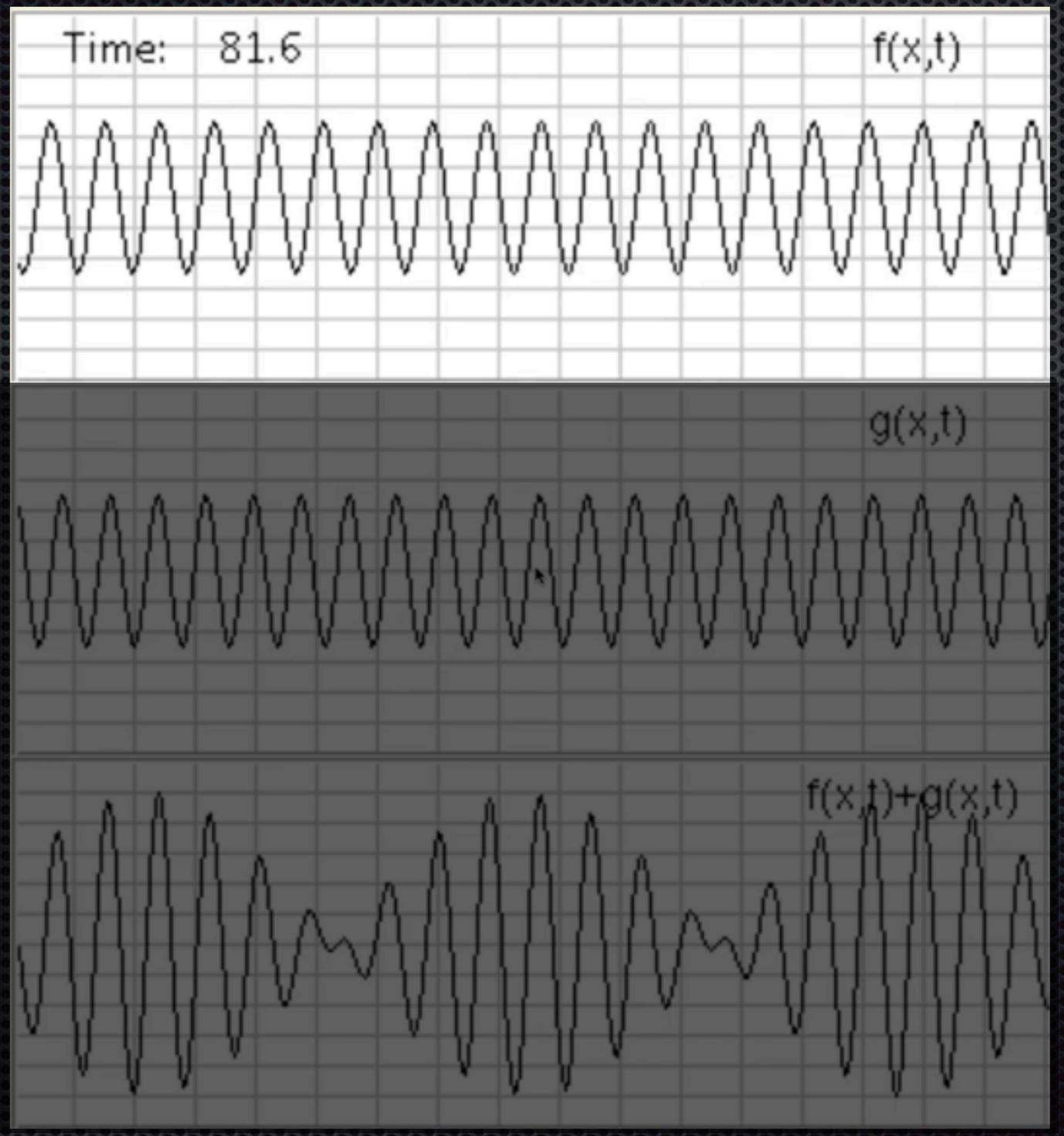
- 2001, SNO experiment (Sudbury Neutrino Observatory) in Canada.
  - Heavy water cherenkov detector, sensitive to all three neutrino flavors.

$$\nu_e \text{ sun} \rightarrow \nu_e,\ \nu_\mu,\ \nu_\tau$$



Neutrino oscillation probability

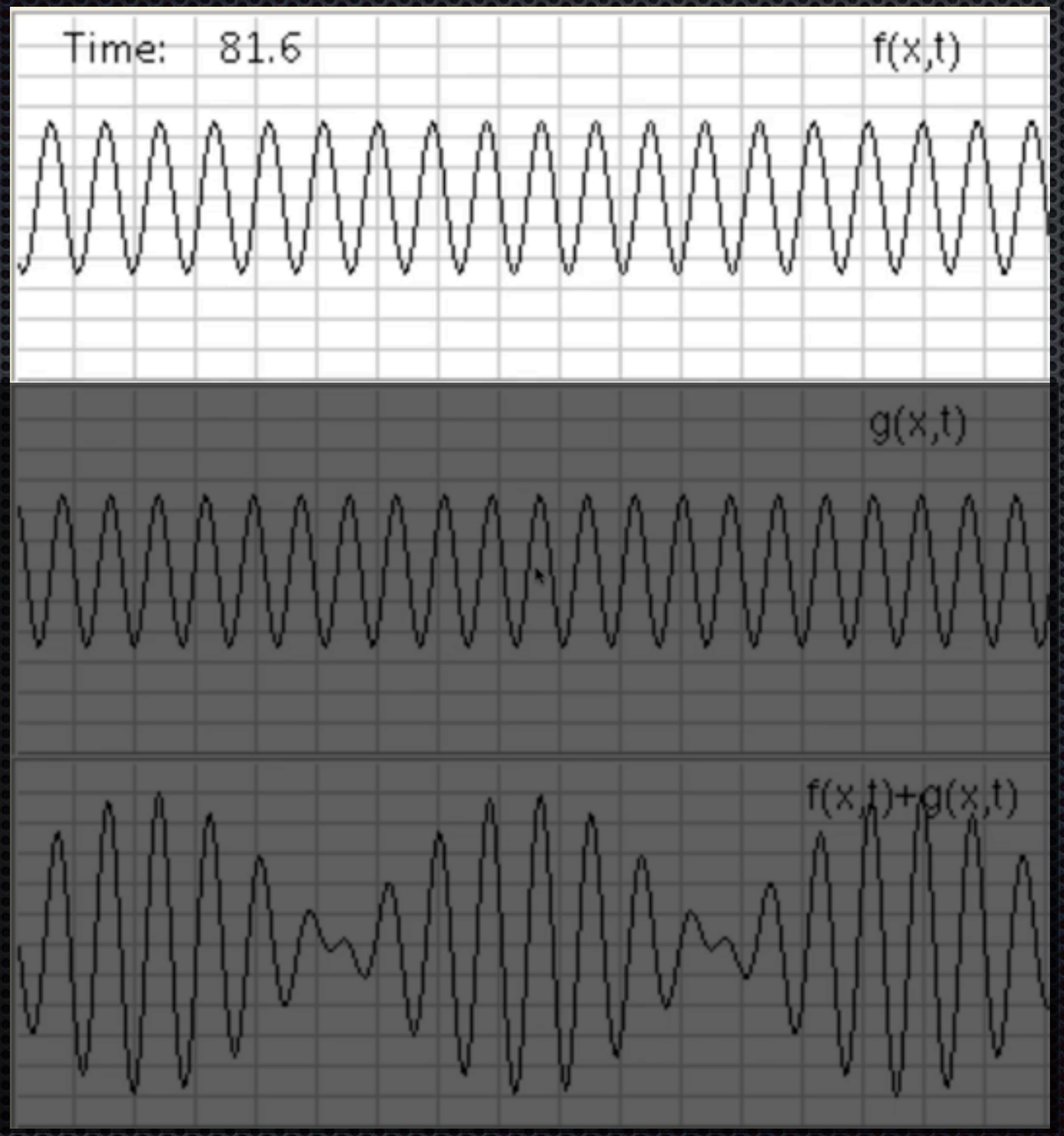
$\nu_e \rightarrow \nu_\mu$



440 Hz

Neutrino oscillation probability

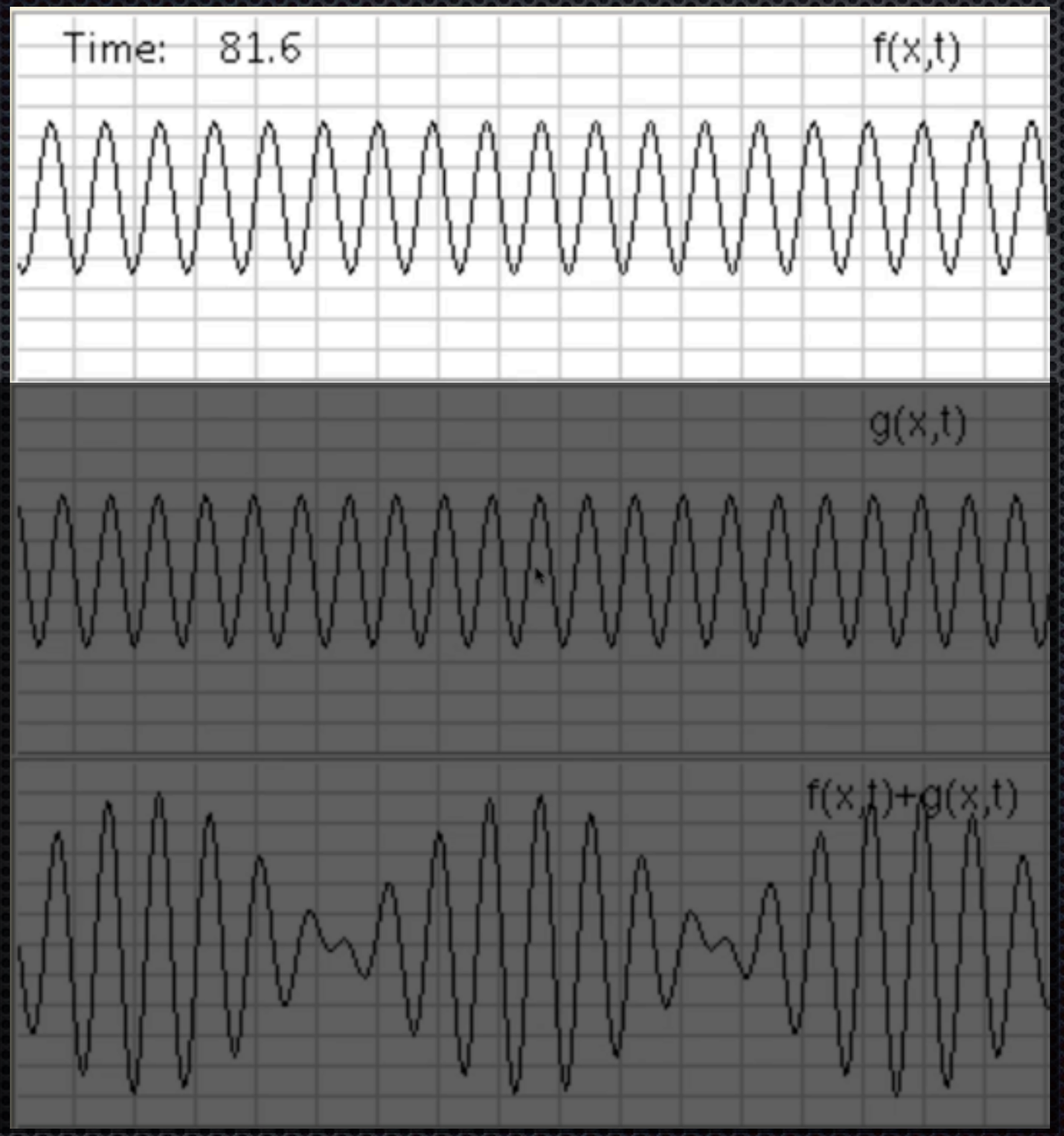
$\nu_e \rightarrow \nu_\mu$



440 Hz

Neutrino oscillation probability

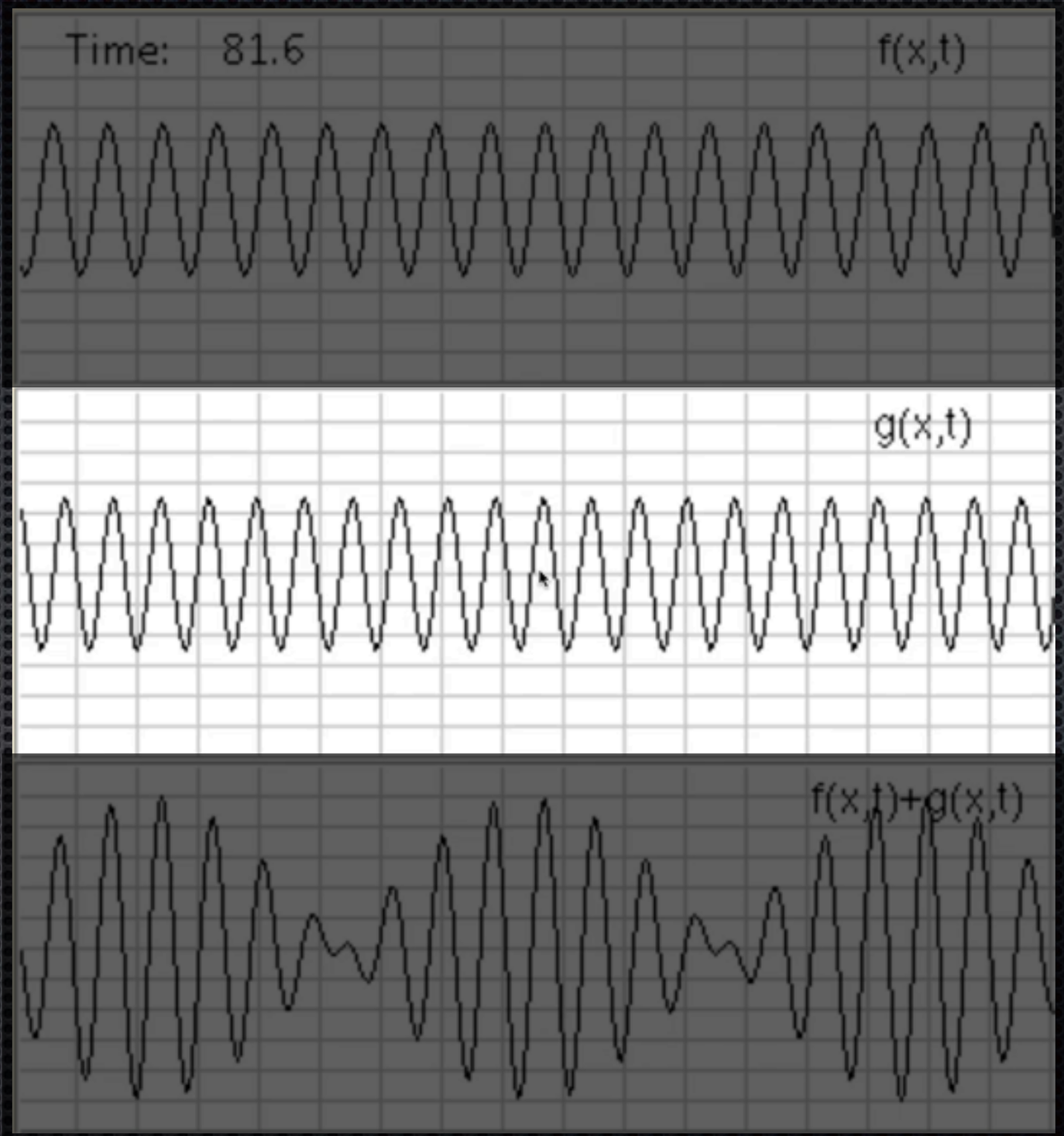
$\nu_e \rightarrow \nu_\mu$



440 Hz

Neutrino oscillation probability

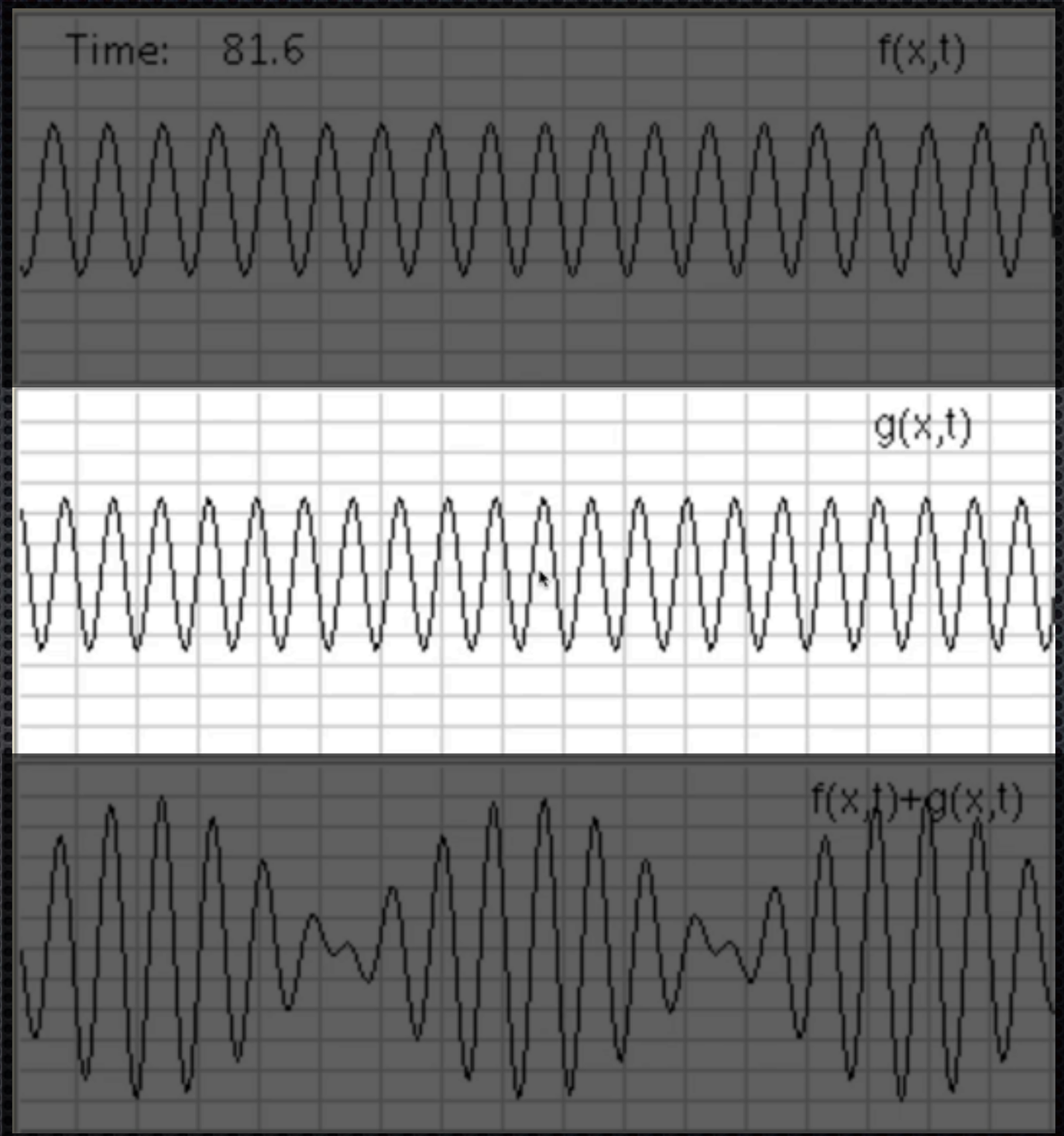
$\nu_e \rightarrow \nu_\tau$



441 Hz

Neutrino oscillation probability

$\nu_e \rightarrow \nu_\tau$



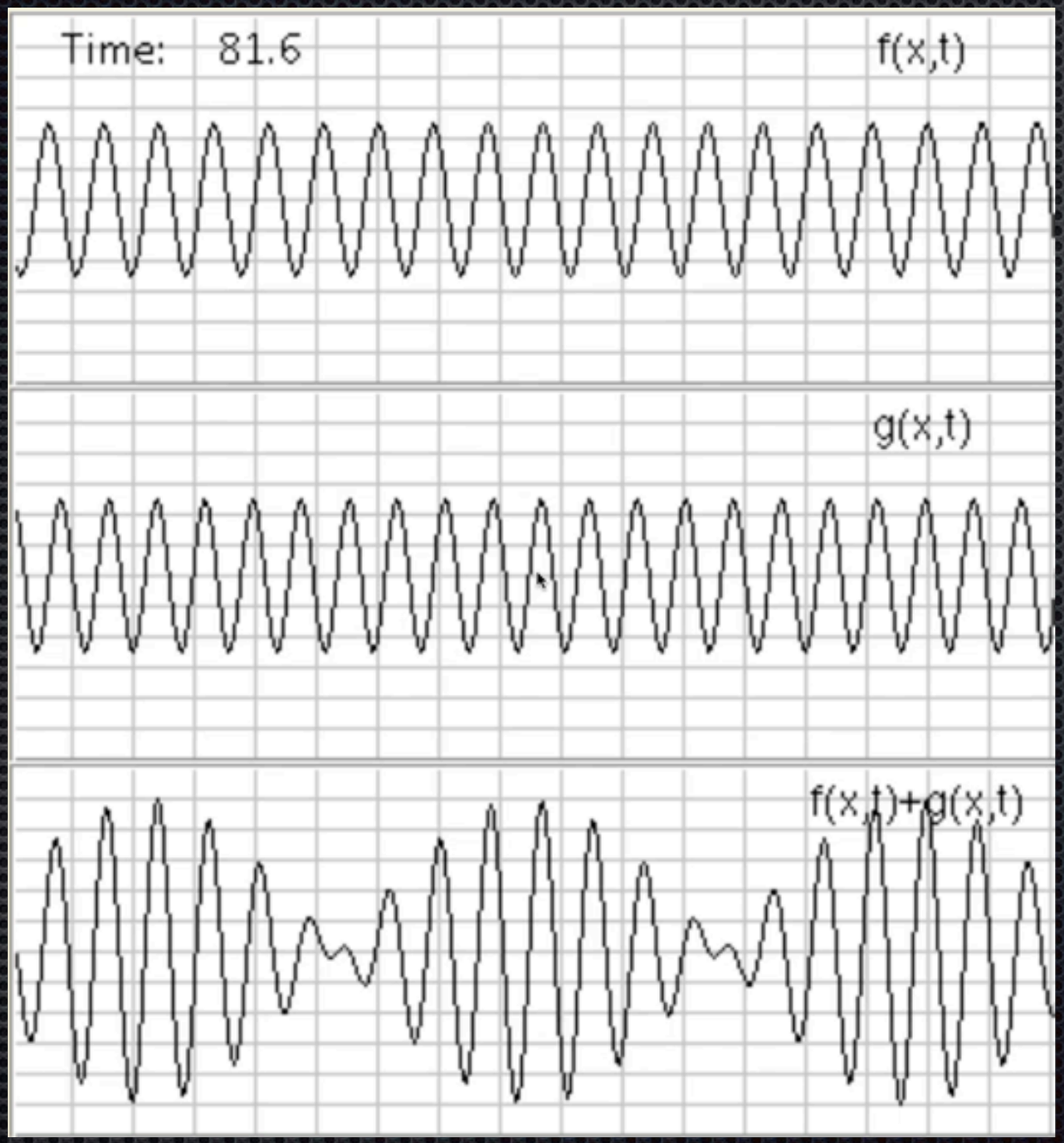
441 Hz

Neutrino oscillation probability

$\nu_e \rightarrow \nu_\mu$

$\nu_e \rightarrow \nu_\tau$

$\nu_e \not\rightarrow \nu_e$



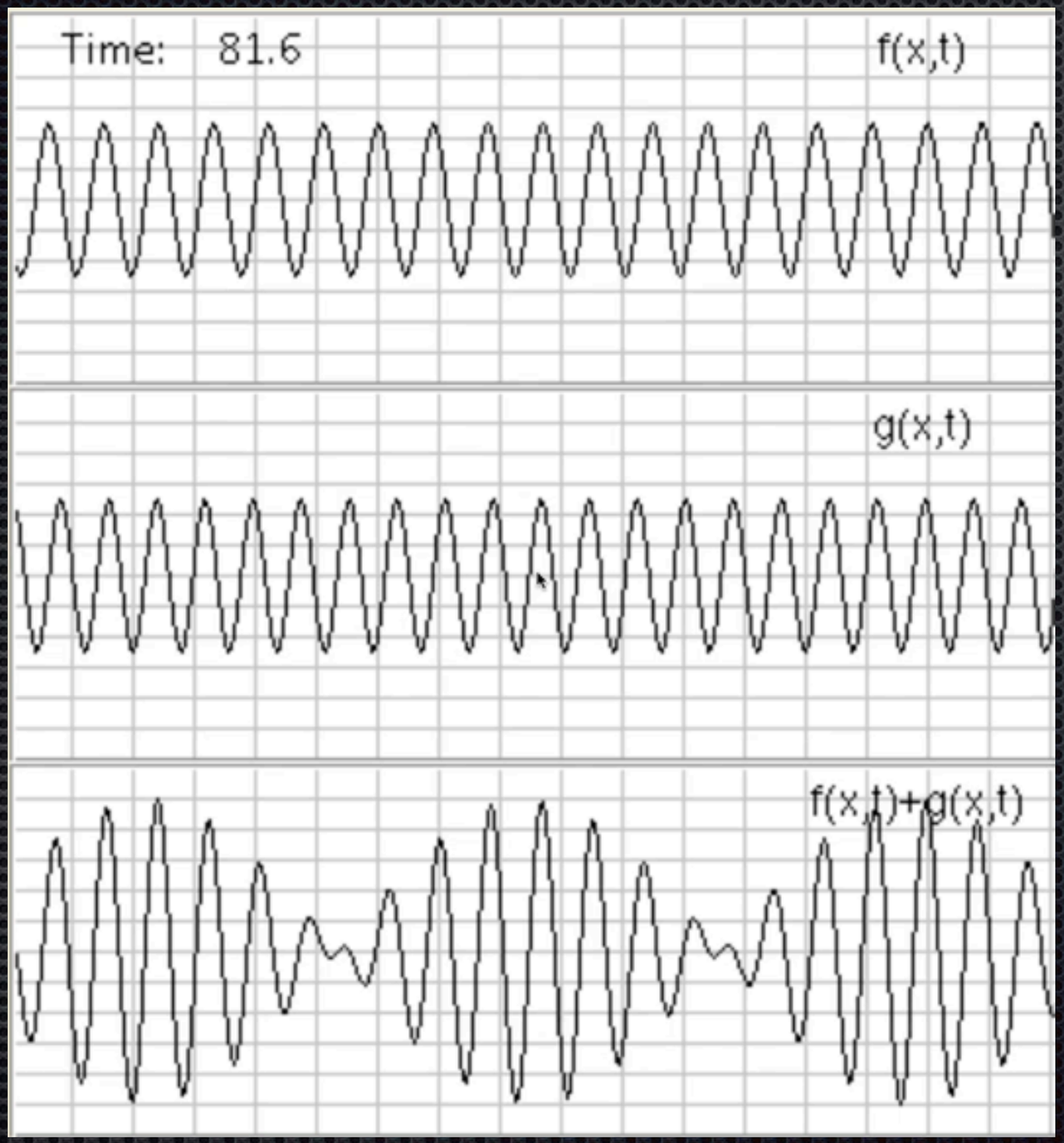
440 Hz + 441 Hz

Neutrino oscillation probability

$\nu_e \rightarrow \nu_\mu$

$\nu_e \rightarrow \nu_\tau$

$\nu_e \not\rightarrow \nu_e$



440 Hz + 441 Hz

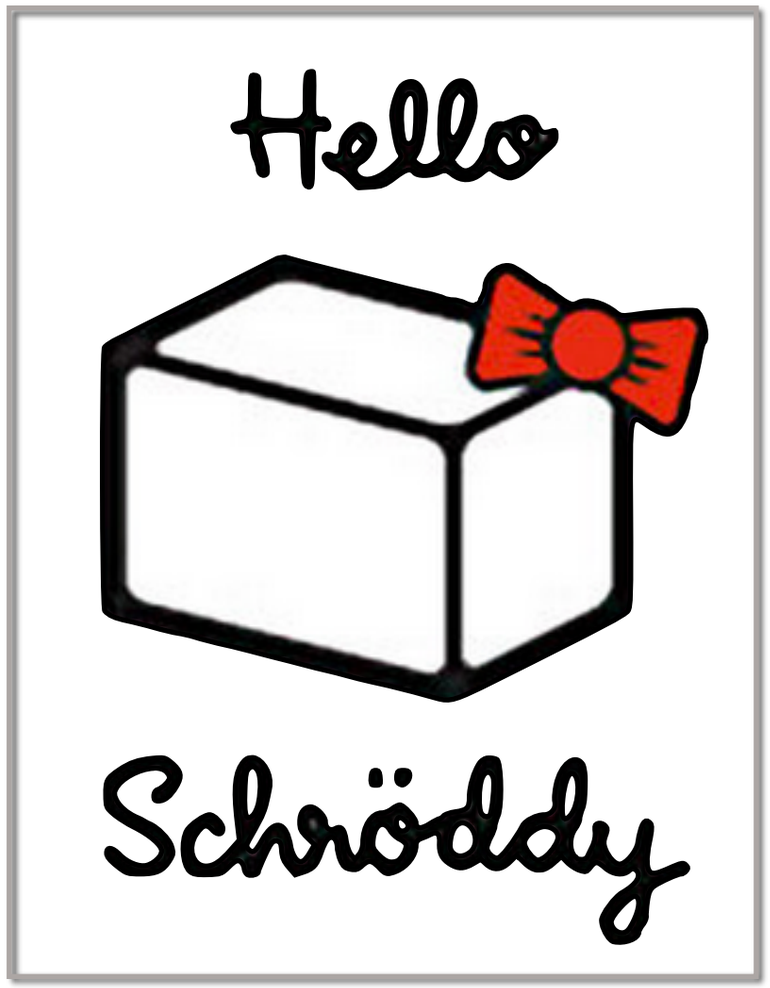
- What's particle physics? (translated into "Tia Speak")
- Where are we in neutrino physics? (Story time!)
  - FUN MATH for neutrino masses!
- Where are we going in neutrino physics? (A.K.A why Tia researches neutrinos)

Why do neutrinos oscillate?

- Review Schrödinger's Equation
- Free particle solution
- Assume some neutrino mixing
- See what happens later

Schrödinger's Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \hat{H}\Psi(\mathbf{r},t)$$



Schrödinger's Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \hat{H}\Psi(\mathbf{r},t)$$

Schrödinger's Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \hat{H}\Psi(\mathbf{r},t)$$

$$\hat{H} = \hat{T} + \hat{V}$$

Schrödinger's Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \hat{H}\Psi(\mathbf{r},t)$$

$$\hat{H} = \hat{T} + \hat{V}$$

$$\hat{T} = \frac{\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}}{2m} = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2$$

Schrödinger's Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \hat{H}\Psi(\mathbf{r},t)$$

$$\hat{H} = \hat{T} + \hat{V}$$

$$\hat{T} = \frac{\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}}{2m} = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2$$

$$\hat{V} = V(\mathbf{r},t)$$

Schrödinger's Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \hat{H}\Psi(\mathbf{r},t)$$

$$\hat{H} = \hat{T} + \hat{V}$$

$$\hat{T} = \frac{\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}}{2m} = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2$$

$$\hat{V} = V(\mathbf{r},t)$$

$$\boxed{i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r},t)\Psi(\mathbf{r},t)}$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r},t)\Psi(\mathbf{r},t)$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r},t)\Psi(\mathbf{r},t)$$

- For free particle, let potential be zero
  - V(r,t) = 0

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r},t)\Psi(\mathbf{r},t)$$

- For free particle, let potential be zero
  - V(r,t) = 0

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t)$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r},t)\Psi(\mathbf{r},t)$$

- For free particle, let potential be zero
  - V(r,t) = 0

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t)$$

- Simplify laplacian, by selecting function to propagate in x direction: r = $\hat{x}$.

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r},t)\Psi(\mathbf{r},t)$$

- For free particle, let potential be zero
  - V(r,t) = 0

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t)$$

- Simplify laplacian, by selecting function to propagate in x direction: r = $\hat{x}$.

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- For free particle, let it be in a definite energy state, E.

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- For free particle, let it be in a definite energy state, E.

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- For free particle, let it be in a definite energy state, E.

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- To solve this partial differential equation, note that the energy, E, is independent of x and independent of t: Break it up into two!

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- For free particle, let it be in a definite energy state, E.

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- To solve this partial differential equation, note that the energy, E, is independent of x and independent of t: Break it up into two!

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- For free particle, let it be in a definite energy state, E.

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- To solve this partial differential equation, note that the energy, E, is independent of x and independent of t: Break it up into two!

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi \qquad\qquad E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi$$

$$E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi \qquad E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- Let's try a solution of the form:

$$\Psi(x,t) = Ae^{ax}e^{bt}$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi \qquad E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- Let's try a solution of the form:

$$\Psi(x,t) = Ae^{ax}e^{bt}$$

  - computing the derivatives:

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi \qquad E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- Let's try a solution of the form:

$$\Psi(x,t) = Ae^{ax}e^{bt}$$

  - computing the derivatives:

$$\frac{\partial}{\partial t}\Psi(x,t) = bAe^{ax}e^{bt}$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi \qquad E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- Let's try a solution of the form:

$$\Psi(x,t) = Ae^{ax}e^{bt}$$

  - computing the derivatives:

$$\frac{\partial}{\partial t}\Psi(x,t) = bAe^{ax}e^{bt}$$

$$\frac{\partial}{\partial t}\Psi(x,t) = b\Psi(x,t)$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi \qquad E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- Let's try a solution of the form:

$$\Psi(x,t) = Ae^{ax}e^{bt}$$

  - computing the derivatives:

$$\frac{\partial}{\partial t}\Psi(x,t) = bAe^{ax}e^{bt} \qquad \frac{\partial}{\partial x}\Psi(x,t) = aAe^{ax}e^{bt}$$

$$\frac{\partial}{\partial t}\Psi(x,t) = b\Psi(x,t)$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi \qquad E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- Let's try a solution of the form:

$$\Psi(x,t) = Ae^{ax}e^{bt}$$

  - computing the derivatives:

$$\frac{\partial}{\partial t}\Psi(x,t) = bAe^{ax}e^{bt} \qquad \frac{\partial}{\partial x}\Psi(x,t) = aAe^{ax}e^{bt}$$

$$\frac{\partial}{\partial t}\Psi(x,t) = b\Psi(x,t) \qquad \frac{\partial^2}{\partial x^2}\Psi(x,t) = a^2Ae^{ax}e^{bt}$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi \qquad E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- Let's try a solution of the form:

$$\Psi(x,t) = Ae^{ax}e^{bt}$$

  - computing the derivatives:

$$\frac{\partial}{\partial t}\Psi(x,t) = bAe^{ax}e^{bt} \qquad \frac{\partial}{\partial x}\Psi(x,t) = aAe^{ax}e^{bt}$$

$$\frac{\partial}{\partial t}\Psi(x,t) = b\Psi(x,t) \qquad \frac{\partial^2}{\partial x^2}\Psi(x,t) = a^2Ae^{ax}e^{bt}$$

$$\frac{\partial^2}{\partial x^2}\Psi(x,t) = a^2\Psi(x,t)$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = E\Psi$$

$$E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x,t)$$

- Let's try a solution of the form:

$$\Psi(x,t) = Ae^{ax}e^{bt}$$

  - computing the derivatives:

$$\frac{\partial}{\partial t}\Psi(x,t) = bAe^{ax}e^{bt}$$

$$\frac{\partial}{\partial x}\Psi(x,t) = aAe^{ax}e^{bt}$$

$$\frac{\partial}{\partial t}\Psi(x,t) = b\Psi(x,t)$$

$$\frac{\partial^2}{\partial x^2}\Psi(x,t) = a^2Ae^{ax}e^{bt}$$

$$\frac{\partial^2}{\partial x^2}\Psi(x,t) = a^2\Psi(x,t)$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi = E\Psi$$

$$E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi = E\Psi$$

$$i\hbar b\Psi = E\Psi$$

$$E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi = E\Psi$$

$$i\hbar b\Psi = E\Psi$$

$$b = -\frac{iE}{\hbar}$$

$$E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi = E\Psi$$

$$i\hbar b\Psi = E\Psi$$

$$b = -\frac{iE}{\hbar}$$

$$E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi$$

$$E\Psi = -\frac{\hbar^2}{2m}a^2\Psi$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi = E\Psi$$

$$i\hbar b\Psi = E\Psi$$

$$b = -\frac{iE}{\hbar}$$

$$E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi$$

$$E\Psi = -\frac{\hbar^2}{2m}a^2\Psi$$

$$a = i\sqrt{\frac{2mE}{\hbar^2}}$$

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi = E\Psi$$

$$i\hbar b\Psi = E\Psi$$

$$b = -\frac{iE}{\hbar}$$

$$E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi$$

$$E\Psi = -\frac{\hbar^2}{2m}a^2\Psi$$

$$a = i\sqrt{\frac{2mE}{\hbar^2}}$$

- Use: $E = \frac{p^2}{2m}$ since we only have kinetic energy

Free Particle Solution

$$i\hbar\frac{\partial}{\partial t}\Psi = E\Psi$$

$$i\hbar b\Psi = E\Psi$$

$$b = -\frac{iE}{\hbar}$$

$$E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi$$

$$E\Psi = -\frac{\hbar^2}{2m}a^2\Psi$$

$$a = i\sqrt{\frac{2mE}{\hbar^2}}$$

- Use: $E = \frac{p^2}{2m}$ since we only have kinetic energy

$$a = i\frac{p}{\hbar}$$

Free Particle Solution

$$b = -\frac{iE}{\hbar} \qquad a = i\frac{p}{\hbar}$$

$$\Psi(x,t) = Ae^{ax}e^{bt}$$

Free Particle Solution

$$b = -\frac{iE}{\hbar} \qquad a = i\frac{p}{\hbar}$$

$$\Psi(x,t) = Ae^{ax}e^{bt}$$

$$\Psi(x,t) = Ae^{\frac{i}{\hbar}px}e^{-\frac{i}{\hbar}Et}$$

Free Particle Solution

$$b = -\frac{iE}{\hbar} \qquad a = i\frac{p}{\hbar}$$

$$\Psi(x,t) = Ae^{ax}e^{bt}$$

$$\Psi(x,t) = Ae^{\frac{i}{\hbar}px}e^{-\frac{i}{\hbar}Et}$$

$$\Psi(x,t) = Ae^{\frac{1}{\hbar}}e^{-i(Et-px)}$$

Free Particle Solution

$$b = -\frac{iE}{\hbar} \qquad a = i\frac{p}{\hbar}$$

$$\Psi(x,t) = Ae^{ax}e^{bt}$$

$$\Psi(x,t) = Ae^{\frac{i}{\hbar}px}e^{-\frac{i}{\hbar}Et}$$

$$\Psi(x,t) = Ae^{\frac{1}{\hbar}}e^{-i(Et-px)}$$

$$\boxed{\Psi(x,t) = Ae^{-i(Et-px)}}$$

Free Particle Solution

$b =$

$$\Psi(x,t) = Ae^{-i(Et-px)}$$

Free Neutrino

$$\Psi(x,t) = Ae^{-i(Et-px)}$$

Free Neutrino

$$\Psi(x,t) = Ae^{-i(Et-px)}$$

- In bra-ket notation: $\alpha = e, \mu, \tau$

$$|\nu_\alpha(x,t) >= \Psi(x,t)$$

Free Neutrino

$$\Psi(x,t) = Ae^{-i(Et-px)}$$

- In bra-ket notation: $\alpha = e, \mu, \tau$

$$|\nu_\alpha(x,t)>= \Psi(x,t)$$
$$|\nu_\alpha(x,t)>= Ae^{-i(E_\alpha t - p_\alpha x)}$$

Free Neutrino

$$\Psi(x,t) = Ae^{-i(Et-px)}$$

- In bra-ket notation: $\alpha = e, \mu, \tau$

$$|\nu_\alpha(x,t)>= \Psi(x,t)$$

$$|\nu_\alpha(x,t)>= Ae^{-i(E_\alpha t-p_\alpha x)}$$

$$|\nu_\alpha(0,0)>= A$$

Free Neutrino

$$\Psi(x,t) = Ae^{-i(Et-px)}$$

- In bra-ket notation: $\alpha = e, \mu, \tau$

$$|\nu_\alpha(x,t)>= \Psi(x,t)$$

$$|\nu_\alpha(x,t)>= Ae^{-i(E_\alpha t-p_\alpha x)}$$

$$|\nu_\alpha(0,0)>= A$$

$$|\nu_\alpha(x,t)>= e^{-i(E_\alpha t-p_\alpha x)}|\nu_\alpha(0,0)>$$

Free Neutrino

$$\Psi(x,t) = Ae^{-i(Et-px)}$$

- In bra-ket notation: $\alpha = e, \mu, \tau$

$$|\nu_\alpha(x,t)>= \Psi(x,t)$$

$$|\nu_\alpha(x,t)>= Ae^{-i(E_\alpha t-p_\alpha x)}$$

$$|\nu_\alpha(0,0)>= A$$

$$|\nu_\alpha(x,t)>= e^{-i(E_\alpha t-p_\alpha x)}|\nu_\alpha(0,0)>$$

$$|\nu_\alpha(x,t)>= e^{-i\phi_\alpha}|\nu_\alpha(0,0)> \qquad \phi_\alpha = (E_\alpha t - p_\alpha x)$$

Free Neutrino

$$\Psi(x,t) = Ae^{-i(Et-px)}$$

- In bra-ket notation: $\alpha = e, \mu, \tau$

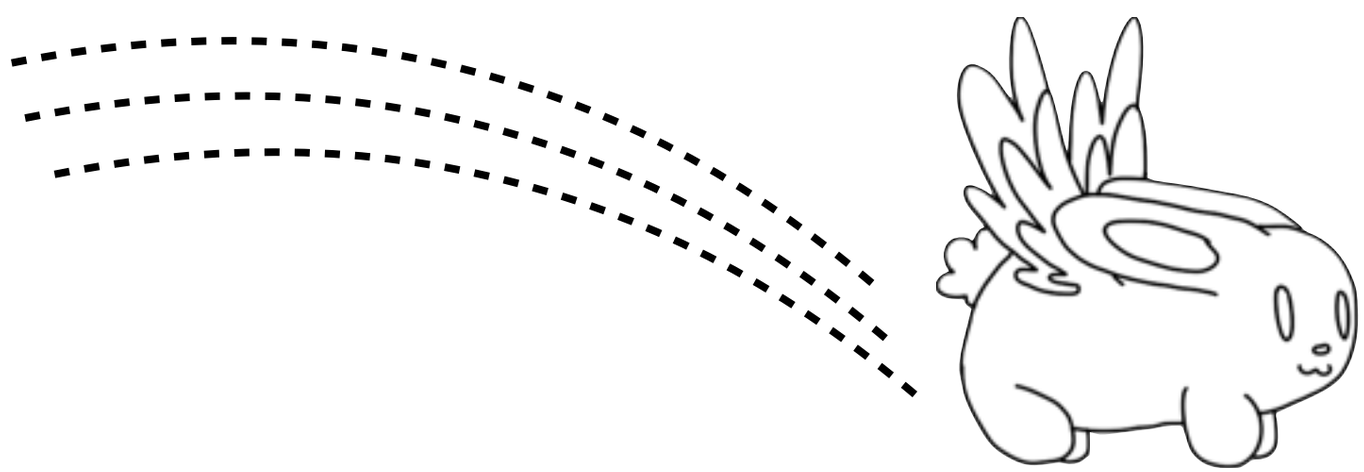
$$|\nu_\alpha(x,t)>= \Psi(x,t)$$

$$|\nu_\alpha(x,t)>= Ae^{-i(E_\alpha t - p_\alpha x)}$$

$$|\nu_\alpha(0,0)>= A$$

$$|\nu_\alpha(x,t)>= e^{-i(E_\alpha t - p_\alpha x)}|\nu_\alpha(0,0)>$$

$$|\nu_\alpha(x,t)>= e^{-i\phi_\alpha}|\nu_\alpha(0,0)> \qquad \phi_\alpha = (E_\alpha t - p_\alpha x)$$



Assume some mixing

- To simplify, let us consider the 2 neutrino mixing case. Often used in oscillation experiment papers!

Assume some mixing

- To simplify, let us consider the 2 neutrino mixing case. Often used in oscillation experiment papers!

$$|\nu_e> = +\cos\theta|\nu_1> + \sin\theta|\nu_2>$$
$$|\nu_\mu> = -\sin\theta|\nu_1> + \cos\theta|\nu_2>$$

Assume some mixing

- To simplify, let us consider the 2 neutrino mixing case. Often used in oscillation experiment papers!

$$|\nu_e> = +\cos\theta|\nu_1> + \sin\theta|\nu_2>$$
$$|\nu_\mu> = -\sin\theta|\nu_1> + \cos\theta|\nu_2>$$

Assume flavor eigenstates can be written as a linear combination of mass eigenstates.

θ is the “mixing angle”

Seeing what happens

- Let's compute probability of ν_e oscillating some distance later to ν_μ.

Seeing what happens

- Let's compute probability of ν_e oscillating some distance later to ν_μ.

$$P(\nu_e \to \nu_\mu) = |< \nu_\mu(x,t)|\nu_e(0,0) >|^2$$

Seeing what happens

- Let's compute probability of ν_e oscillating some distance later to ν_μ.

$$P(\nu_e \to \nu_\mu) = |< \nu_\mu(x,t)|\nu_e(0,0) >|^2$$

$$< \nu_\mu(x,t)|\nu_e(0,0) > = (-\sin\theta < \nu_1(x,t)| + \cos\theta < \nu_2(x,t)|) \cdot (\cos\theta|\nu_1(0,0) > + \sin\theta|\nu_2(0,0) >)$$

Seeing what happens

- Let's compute probability of ν_e oscillating some distance later to ν_μ.

$$P(\nu_e \to \nu_\mu) = |< \nu_\mu(x,t)|\nu_e(0,0) >|^2$$

$$< \nu_\mu(x,t)|\nu_e(0,0) > = (-\sin\theta < \nu_1(x,t)| + \cos\theta < \nu_2(x,t)|)$$
$$\cdot (\cos\theta|\nu_1(0,0) > + \sin\theta|\nu_2(0,0) >)$$

$$= (-\sin\theta e^{i\phi_1} < \nu_1(0,0)| + \cos\theta e^{i\phi_2} < \nu_2(0,0)|)$$
$$\cdot (\cos\theta|\nu_1(0,0) > + \sin\theta|\nu_2(0,0) >)$$

Seeing what happens

- Let's compute probability of ν_e oscillating some distance later to ν_μ.

$$P(\nu_e \to \nu_\mu) = |< \nu_\mu(x,t)|\nu_e(0,0) >|^2$$

$$< \nu_\mu(x,t)|\nu_e(0,0) > = (-\sin\theta < \nu_1(x,t)| + \cos\theta < \nu_2(x,t)|)$$
$$\cdot (\cos\theta|\nu_1(0,0) > + \sin\theta|\nu_2(0,0) >)$$

$$= (-\sin\theta e^{i\phi_1} < \nu_1(0,0)| + \cos\theta e^{i\phi_2} < \nu_2(0,0)|)$$
$$\cdot (\cos\theta|\nu_1(0,0) > + \sin\theta|\nu_2(0,0) >)$$

$$= -\sin\theta\cos\theta e^{i\phi_1} + \sin\theta\cos\theta e^{i\phi_2}$$

Seeing what happens

$$< \nu_\mu(x,t)|\nu_e(0,0) > \; = -\sin\theta\cos\theta e^{i\phi_1} + \sin\theta\cos\theta e^{i\phi_2}$$

Seeing what happens

$$< \nu_\mu(x,t)|\nu_e(0,0) > = -\sin\theta\cos\theta e^{i\phi_1} + \sin\theta\cos\theta e^{i\phi_2}$$

$$= -sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2}$$

Seeing what happens

$$< \nu_\mu(x,t)|\nu_e(0,0) > = -\sin\theta\cos\theta e^{i\phi_1} + \sin\theta\cos\theta e^{i\phi_2}$$
$$= -sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2}$$

- Square the amplitude to get the probability:

Seeing what happens

$$< \nu_\mu(x,t)|\nu_e(0,0) > = -\sin\theta\cos\theta e^{i\phi_1} + \sin\theta\cos\theta e^{i\phi_2}$$
$$= -sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2}$$

- Square the amplitude to get the probability:

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = |-sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2}|^2$$

Seeing what happens

$$< \nu_\mu(x,t)|\nu_e(0,0) > = -\sin\theta\cos\theta e^{i\phi_1} + \sin\theta\cos\theta e^{i\phi_2}$$
$$= -sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2}$$

- Square the amplitude to get the probability:

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = |-sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2}|^2$$
$$= (-sc\theta e^{-i\phi_1} + sc\theta e^{-i\phi_2})$$
$$\cdot(-sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2})$$

Seeing what happens

$$< \nu_\mu(x,t)|\nu_e(0,0) > = -\sin\theta\cos\theta e^{i\phi_1} + \sin\theta\cos\theta e^{i\phi_2}$$
$$= -sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2}$$

- Square the amplitude to get the probability:

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = |-sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2}|^2$$
$$= (-sc\theta e^{-i\phi_1} + sc\theta e^{-i\phi_2}) \cdot (-sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2})$$
$$= \sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta e^{-i(\phi_1-\phi_2)} + \sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta e^{i(\phi_1-\phi_2)}$$

Seeing what happens

$$< \nu_\mu(x,t)|\nu_e(0,0) > = -\sin\theta\cos\theta e^{i\phi_1} + \sin\theta\cos\theta e^{i\phi_2}$$
$$= -sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2}$$

- Square the amplitude to get the probability:

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = |-sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2}|^2$$
$$= (-sc\theta e^{-i\phi_1} + sc\theta e^{-i\phi_2}) \cdot (-sc\theta e^{i\phi_1} + sc\theta e^{i\phi_2})$$
$$= \sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta e^{-i(\phi_1-\phi_2)} + \sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta e^{i(\phi_1-\phi_2)}$$
$$= \sin^2\theta\cos^2\theta(2 - [e^{-i(\phi_1-\phi_2)} + e^{i(\phi_1-\phi_2)}])$$

Some tricks…

$$|< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = \sin^2\theta\cos^2\theta(2 - [e^{-i(\phi_1-\phi_2)} + e^{i(\phi_1-\phi_2)}])$$

Some tricks…

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = \sin^2\theta\cos^2\theta(2 - [e^{-i(\phi_1-\phi_2)} + e^{i(\phi_1-\phi_2)}])$$

- Trick 1: $2\cos a = e^{-ia} + e^{ia}$

Some tricks…

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = \sin^2\theta\cos^2\theta(2 - [e^{-i(\phi_1-\phi_2)} + e^{i(\phi_1-\phi_2)}])$$

- Trick 1: $2\cos a = e^{-ia} + e^{ia}$

$$= \sin^2\theta\cos^2\theta(2 - 2\cos(\phi_1 - \phi_2))$$

Some tricks…

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = \sin^2\theta\cos^2\theta(2 - [e^{-i(\phi_1-\phi_2)} + e^{i(\phi_1-\phi_2)}])$$

- Trick 1: $2\cos a = e^{-ia} + e^{ia}$

$$= \sin^2\theta\cos^2\theta(2 - 2\cos(\phi_1 - \phi_2))$$

$$= 2(\sin\theta\cos\theta)^2(1 - \cos(\phi_1 - \phi_2))$$

Some tricks…

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = \sin^2\theta\cos^2\theta(2 - [e^{-i(\phi_1-\phi_2)} + e^{i(\phi_1-\phi_2)}])$$

- Trick 1: $2\cos a = e^{-ia} + e^{ia}$

$$= \sin^2\theta\cos^2\theta(2 - 2\cos(\phi_1 - \phi_2))$$

$$= 2(\sin\theta\cos\theta)^2(1 - \cos(\phi_1 - \phi_2))$$

- Trick 2: $\sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$

Some tricks…

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = \sin^2\theta\cos^2\theta(2 - [e^{-i(\phi_1-\phi_2)} + e^{i(\phi_1-\phi_2)}])$$

- Trick 1: $2\cos a = e^{-ia} + e^{ia}$

$$= \sin^2\theta\cos^2\theta(2 - 2\cos(\phi_1 - \phi_2))$$

$$= 2(\sin\theta\cos\theta)^2(1 - \cos(\phi_1 - \phi_2))$$

- Trick 2: $\sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$

$$= 2\left(\frac{1}{2}sin(2\theta)\right)^2(1 - \cos(\phi_1 - \phi_2))$$

Some tricks…

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = \sin^2\theta\cos^2\theta(2 - [e^{-i(\phi_1-\phi_2)} + e^{i(\phi_1-\phi_2)}])$$

- Trick 1: $2\cos a = e^{-ia} + e^{ia}$

$$= \sin^2\theta\cos^2\theta(2 - 2\cos(\phi_1 - \phi_2))$$
$$= 2(\sin\theta\cos\theta)^2(1 - \cos(\phi_1 - \phi_2))$$

- Trick 2: $\sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$

$$= 2\left(\frac{1}{2}sin(2\theta)\right)^2(1 - \cos(\phi_1 - \phi_2))$$
$$= \frac{1}{2}sin^2(2\theta)(1 - \cos(\phi_1 - \phi_2))$$

Some tricks…

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = \sin^2\theta\cos^2\theta(2 - [e^{-i(\phi_1-\phi_2)} + e^{i(\phi_1-\phi_2)}])$$

- Trick 1: $2\cos a = e^{-ia} + e^{ia}$

$$= \sin^2\theta\cos^2\theta(2 - 2\cos(\phi_1 - \phi_2))$$
$$= 2(\sin\theta\cos\theta)^2(1 - \cos(\phi_1 - \phi_2))$$

- Trick 2: $\sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$

$$= 2\left(\frac{1}{2}sin(2\theta)\right)^2(1 - \cos(\phi_1 - \phi_2))$$
$$= \frac{1}{2}sin^2(2\theta)(1 - \cos(\phi_1 - \phi_2))$$

- Trick 3: $1 - \cos a = 2\sin^2\left(\frac{a}{2}\right)$

Some tricks…

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = \sin^2\theta\cos^2\theta(2 - [e^{-i(\phi_1-\phi_2)} + e^{i(\phi_1-\phi_2)}])$$

- Trick 1: $2\cos a = e^{-ia} + e^{ia}$

$$= \sin^2\theta\cos^2\theta(2 - 2\cos(\phi_1 - \phi_2))$$
$$= 2(\sin\theta\cos\theta)^2(1 - \cos(\phi_1 - \phi_2))$$

- Trick 2: $\sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$

$$= 2\left(\frac{1}{2}sin(2\theta)\right)^2(1 - \cos(\phi_1 - \phi_2))$$
$$= \frac{1}{2}sin^2(2\theta)(1 - \cos(\phi_1 - \phi_2))$$

- Trick 3: $1 - \cos a = 2\sin^2\left(\frac{a}{2}\right)$

$$= \frac{1}{2}sin^2(2\theta)\left(2\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)\right)$$

Some tricks…

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = \sin^2\theta\cos^2\theta(2 - [e^{-i(\phi_1-\phi_2)} + e^{i(\phi_1-\phi_2)}])$$

- Trick 1: $2\cos a = e^{-ia} + e^{ia}$

$$= \sin^2\theta\cos^2\theta(2 - 2\cos(\phi_1 - \phi_2))$$
$$= 2(\sin\theta\cos\theta)^2(1 - \cos(\phi_1 - \phi_2))$$

- Trick 2: $\sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$

$$= 2\left(\frac{1}{2}sin(2\theta)\right)^2(1 - \cos(\phi_1 - \phi_2))$$
$$= \frac{1}{2}sin^2(2\theta)(1 - \cos(\phi_1 - \phi_2))$$

- Trick 3: $1 - \cos a = 2\sin^2\left(\frac{a}{2}\right)$

$$= \frac{1}{2}sin^2(2\theta)\left(2\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)\right)$$
$$= sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

Some tricks…

$$< \nu_\mu(x,t)|\nu_e(0,0) >|^2 = \sin^2\theta\cos^2\theta(2 - [e^{-i(\phi_1-\phi_2)} + e^{i(\phi_1-\phi_2)}])$$

- Trick 1: $2\cos a = e^{-ia} + e^{ia}$

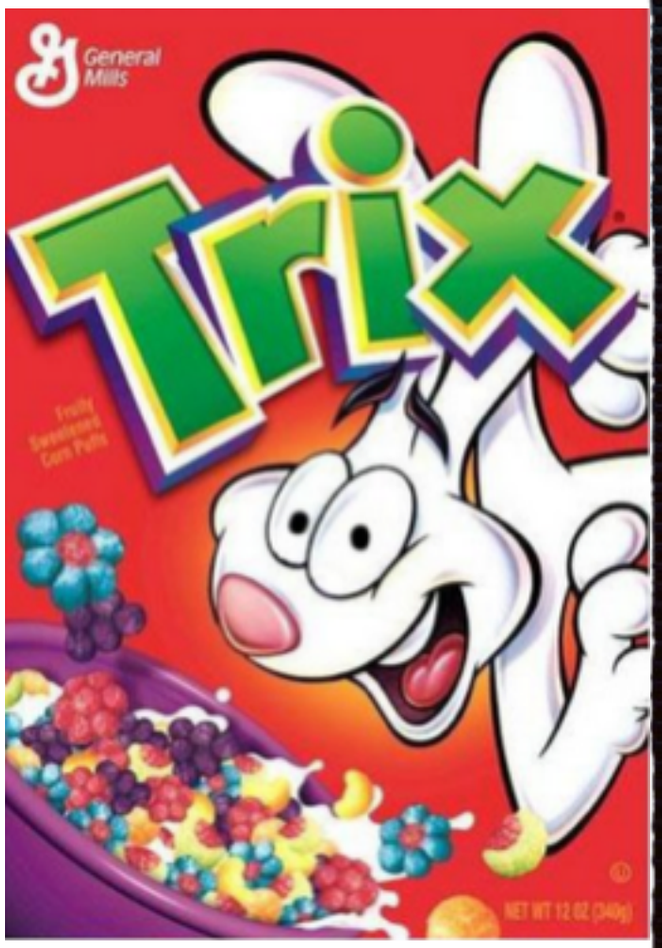
$$= \sin^2\theta\cos^2\theta(2 - 2\cos(\phi_1 - \phi_2))$$
$$= 2(\sin\theta\cos\theta)^2(1 - \cos(\phi_1 - \phi_2))$$

- Trick 2: $\sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$

$$= 2\left(\frac{1}{2}sin(2\theta)\right)^2(1 - \cos(\phi_1 - \phi_2))$$
$$= \frac{1}{2}sin^2(2\theta)(1 - \cos(\phi_1 - \phi_2))$$

- Trick 3: $1 - \cos a = 2\sin^2\left(\frac{a}{2}\right)$

$$= \frac{1}{2}sin^2(2\theta)\left(2\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)\right)$$
$$= sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$



Wait, what's ϕ again?

$$P(\nu_e \rightarrow \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

Wait, what's ϕ again?

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\phi_1 = E_1 t - p_1 x$$
$$\phi_2 = E_2 t - p_2 x$$

Wait, what's ϕ again?

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\phi_1 = E_1 t - p_1 x$$
$$\phi_2 = E_2 t - p_2 x$$

- Let's use high-energy units, $\hbar = c = 1$. Let t=x=L (L is the distance from a ν source to a detector).

Wait, what's ϕ again?

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\phi_1 = E_1 t - p_1 x$$
$$\phi_2 = E_2 t - p_2 x$$

- Let's use high-energy units, $\hbar = c = 1$. Let t=x=L (L is the distance from a ν source to a detector).

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

Wait, what's ϕ again?

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\phi_1 = E_1 t - p_1 x$$
$$\phi_2 = E_2 t - p_2 x$$

- Let's use high-energy units, $\hbar = c = 1$. Let t=x=L (L is the distance from a ν source to a detector).

  $$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Let's let $p_1 = p_2 = E$, reasonable if mass << E

Wait, what's ϕ again?

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\phi_1 = E_1 t - p_1 x$$
$$\phi_2 = E_2 t - p_2 x$$

- Let's use high-energy units, $\hbar = c = 1$. Let t=x=L (L is the distance from a ν source to a detector).

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Let's let $p_1 = p_2 = E$, reasonable if mass << E

$$E_1 = \sqrt{\mathbf{p_1}^2 + m_1^2}$$

Wait, what's ϕ again?

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\phi_1 = E_1 t - p_1 x$$
$$\phi_2 = E_2 t - p_2 x$$

- Let's use high-energy units, $\hbar = c = 1$. Let t=x=L (L is the distance from a ν source to a detector).

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Let's let $p_1 = p_2 = E$, reasonable if mass << E

$$E_1 = \sqrt{\mathbf{p_1}^2 + m_1^2} = \sqrt{E^2 + m_1^2}$$

Wait, what's ϕ again?

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\phi_1 = E_1 t - p_1 x$$
$$\phi_2 = E_2 t - p_2 x$$

- Let's use high-energy units, $\hbar = c = 1$. Let t=x=L (L is the distance from a ν source to a detector).

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Let's let $p_1 = p_2 = E$, reasonable if mass << E

$$E_1 = \sqrt{\mathbf{p_1}^2 + m_1^2} = \sqrt{E^2 + m_1^2} = E\sqrt{1 + \left(\frac{m_1}{E}\right)^2}$$

Wait, what's ϕ again?

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\phi_1 = E_1 t - p_1 x$$
$$\phi_2 = E_2 t - p_2 x$$

- Let's use high-energy units, $\hbar = c = 1$. Let t=x=L (L is the distance from a ν source to a detector).

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Let's let $p_1 = p_2 = E$, reasonable if mass << E

$$E_1 = \sqrt{\mathbf{p_1}^2 + m_1^2} = \sqrt{E^2 + m_1^2} = E\sqrt{1 + \left(\frac{m_1}{E}\right)^2}$$

- use this awesome taylor series approximation

$$\sqrt{1 + x^2} \approx 1 + \frac{x^2}{2}$$

Wait, what's ϕ again?

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\phi_1 = E_1 t - p_1 x$$
$$\phi_2 = E_2 t - p_2 x$$

- Let's use high-energy units, $\hbar = c = 1$. Let t=x=L (L is the distance from a ν source to a detector).

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Let's let $p_1 = p_2 = E$, reasonable if mass << E

$$E_1 = \sqrt{\mathbf{p_1}^2 + m_1^2} = \sqrt{E^2 + m_1^2} = E\sqrt{1 + \left(\frac{m_1}{E}\right)^2} = E\left(1 + \frac{m_1^2}{2E^2}\right)$$

- use this awesome taylor series approximation

$$\sqrt{1 + x^2} \approx 1 + \frac{x^2}{2}$$

Wait, what's ϕ again?

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\phi_1 = E_1 t - p_1 x$$
$$\phi_2 = E_2 t - p_2 x$$

- Let's use high-energy units, $\hbar = c = 1$. Let t=x=L (L is the distance from a ν source to a detector).

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Let's let $p_1 = p_2 = E$, reasonable if mass << E

$$E_1 = \sqrt{\mathbf{p_1}^2 + m_1^2} = \sqrt{E^2 + m_1^2} = E\sqrt{1 + \left(\frac{m_1}{E}\right)^2} = E\left(1 + \frac{m_1^2}{2E^2}\right) = E + \frac{m_1^2}{2E}$$

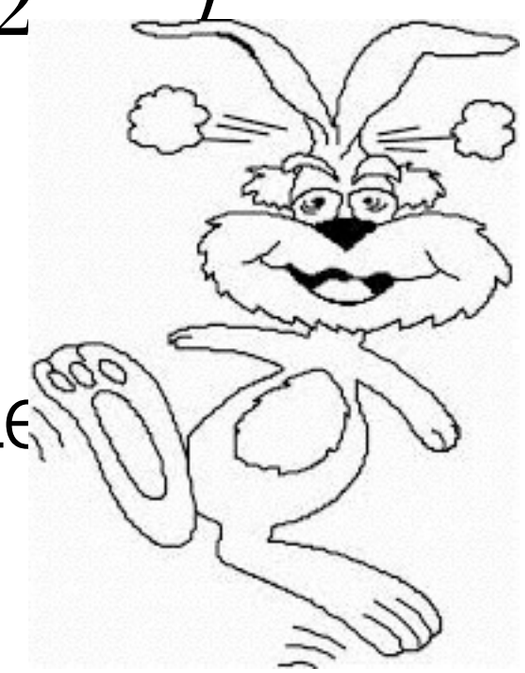
- use this awesome taylor series approximation

$$\sqrt{1 + x^2} \approx 1 + \frac{x^2}{2}$$

Wait, what's ϕ again?

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$\phi_1 = E_1 t - p_1 x$$
$$\phi_2 = E_2 t - p_2 x$$



- Let's use high-energy units, $\hbar = c = 1$. Le the distance from a ν source to a detector).

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Let's let $p_1 = p_2 = E$, reasonable if mass << E

$$E_1 = \sqrt{\mathbf{p_1}^2 + m_1^2} = \sqrt{E^2 + m_1^2} = E\sqrt{1 + \left(\frac{m_1}{E}\right)^2} = E\left(1 + \frac{m_1^2}{2E^2}\right) = E + \frac{m_1^2}{2E}$$

- use this awesome taylor series approximation

$$\sqrt{1 + x^2} \approx 1 + \frac{x^2}{2}$$

Wait, what's ϕ again?

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

Wait, what's ϕ again?

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Plug in $p_1 = p_2 = E$ and $E_1 = E + \frac{m_1^2}{2E}$ from last page

Wait, what's ϕ again?

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Plug in $p_1 = p_2 = E$ and $E_1 = E + \frac{m_1^2}{2E}$ from last page

$$= L(E + \frac{m_1^2}{2E} - E - E - \frac{m_2^2}{2E} + E)$$

Wait, what's ϕ again?

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Plug in $p_1 = p_2 = E$ and $E_1 = E + \frac{m_1^2}{2E}$ from last page

$$= L(E + \frac{m_1^2}{2E} - E - E - \frac{m_2^2}{2E} + E)$$

$$= L\frac{m_1^2 - m_2^2}{2E}$$

Wait, what's ϕ again?

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Plug in $p_1 = p_2 = E$ and $E_1 = E + \frac{m_1^2}{2E}$ from last page

$$= L(E + \frac{m_1^2}{2E} - E - E - \frac{m_2^2}{2E} + E)$$

$$= L\frac{m_1^2 - m_2^2}{2E}$$

$$\phi_1 - \phi_2 = L\frac{\Delta m_{12}^2}{2E}$$

Wait, what's ϕ again?

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Plug in $p_1 = p_2 = E$ and $E_1 = E + \frac{m_1^2}{2E}$ from last page

$$= L(E + \frac{m_1^2}{2E} - E - E - \frac{m_2^2}{2E} + E)$$

$$= L\frac{m_1^2 - m_2^2}{2E}$$

$$\phi_1 - \phi_2 = L\frac{\Delta m_{12}^2}{2E}$$

- And back to the probability for oscillation:

Wait, what's ϕ again?

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Plug in $p_1 = p_2 = E$ and $E_1 = E + \frac{m_1^2}{2E}$ from last page

$$= L(E + \frac{m_1^2}{2E} - E - E - \frac{m_2^2}{2E} + E)$$

$$= L\frac{m_1^2 - m_2^2}{2E}$$

$$\phi_1 - \phi_2 = L\frac{\Delta m_{12}^2}{2E}$$

- And back to the probability for oscillation:

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

Wait, what's ϕ again?

$$\phi_1 - \phi_2 = L(E_1 - p_1 - E_2 + p_2)$$

- Plug in $p_1 = p_2 = E$ and $E_1 = E + \frac{m_1^2}{2E}$ from last page

$$= L(E + \frac{m_1^2}{2E} - E - \frac{m_2^2}{2E})$$

$$= L\frac{m_1^2 - m_2^2}{2E}$$

$$\phi_1 - \phi_2 = L\frac{\Delta m_{12}^2}{2E}$$

- And back to the probability for oscil

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

Oscillation Probability

$$P(\nu_e \rightarrow \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

Oscillation Probability

$$P(\nu_e \rightarrow \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

$$= sin^2(2\theta)\sin^2\left(\frac{L\Delta m_{12}^2}{4E}\right)$$

Oscillation Probability

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$
$$= sin^2(2\theta)\sin^2\left(\frac{L\Delta m_{12}^2}{4E}\right)$$

- Taking care of a $1/\hbar c$

Oscillation Probability

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$
$$= sin^2(2\theta)\sin^2\left(\frac{L\Delta m_{12}^2}{4E}\right)$$

- Taking care of a $1/\hbar c$

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(1.27\Delta m_{12}^2\frac{L}{E}\right)$$

Oscillation Probability

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$
$$= sin^2(2\theta)\sin^2\left(\frac{L\Delta m_{12}^2}{4E}\right)$$

- Taking care of a $1/\hbar c$

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)\sin^2\left(1.27\Delta m_{12}^2\frac{L}{E}\right)$$

where Δm_{12}^2 is in units of eV^2, L is km, E is GeV

So what!?

$$P(\nu_e \rightarrow \nu_\mu) = sin^2(2\theta) sin^2\left(1.27 \Delta m^2_{12} \frac{L}{E}\right)$$

So what!?

$$P(\nu_e \rightarrow \nu_\mu) = sin^2(2\theta) sin^2\left(1.27 \Delta m^2_{12} \frac{L}{E}\right)$$

We observe this happening in experiments

So what!?

$$P(\nu_e \rightarrow \nu_\mu) = sin^2(2\theta) sin^2\left(1.27 \Delta m^2_{12} \frac{L}{E}\right)$$

We observe this happening in experiments

deduce ν mass > 0

So what!?

$$P(\nu_e \rightarrow \nu_\mu) = sin^2(2\theta) sin^2\left(1.27 \Delta m^2_{12} \frac{L}{E}\right)$$

We observe this happening in experiments

deduce ν mass > 0

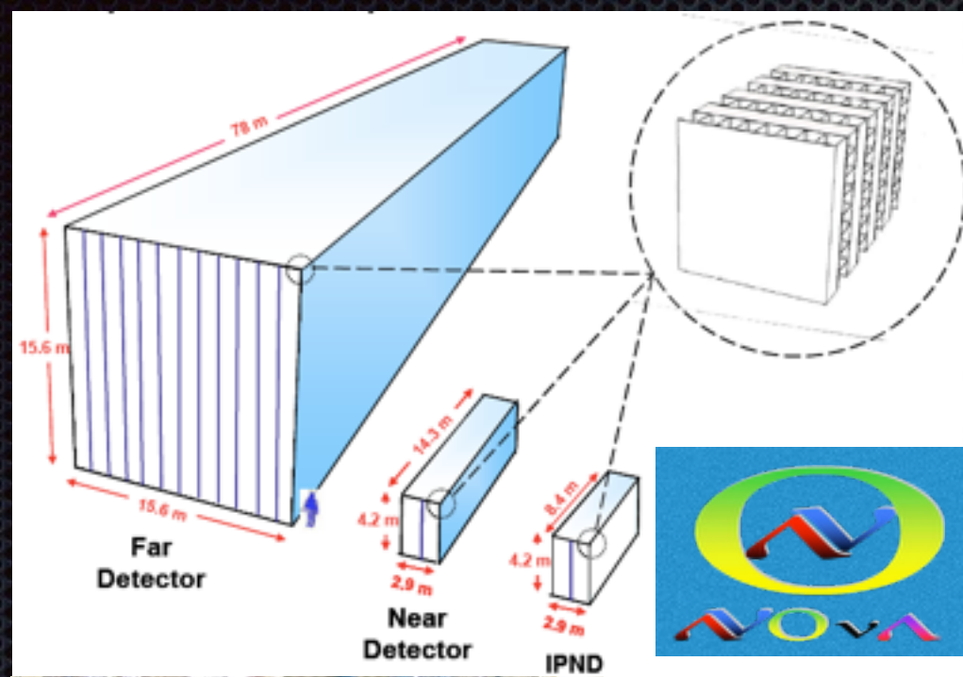
Wow! We just showed that oscillations imply that neutrinos have mass!

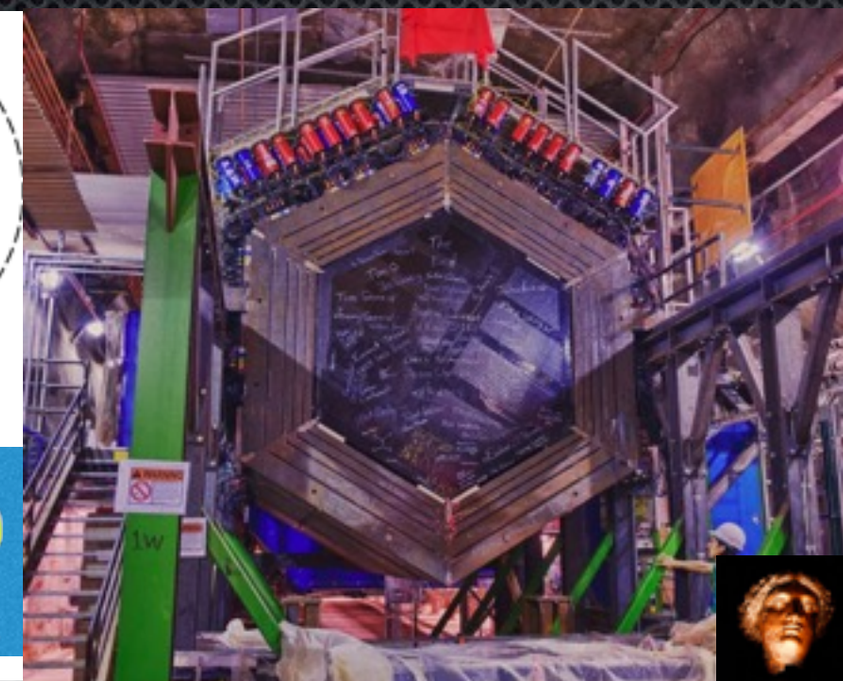
Still thirsty?

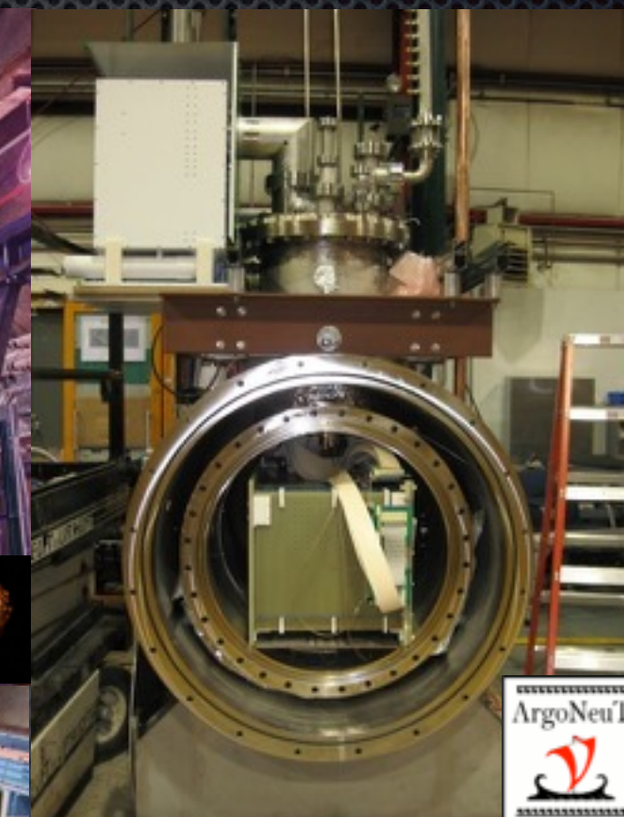
- It is left as an exercise to the reader to expand the 2 neutrino mixing case to the 3 neutrino case.

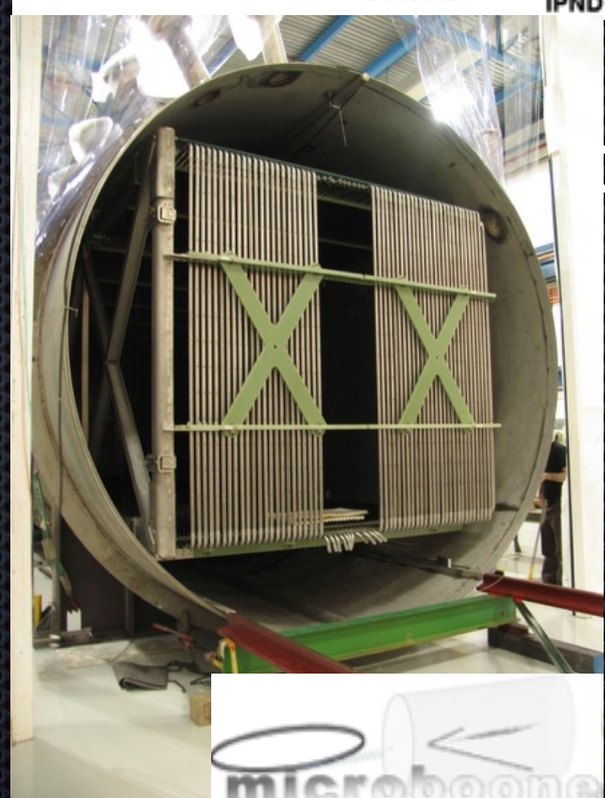
- What's particle physics? (translated into "Tia Speak")
- Where are we in neutrino physics? (Story time!)
  - FUN MATH for neutrino masses!
- Where are we going in neutrino physics? (A.K.A why Tia researches neutrinos)

At Fermilab


78 m
15.6 m
15.6 m
Far
Detector
14.3 m
4.2 m
2.9 m
Near
Detector
8.4 m
4.2 m
2.9 m
IPND



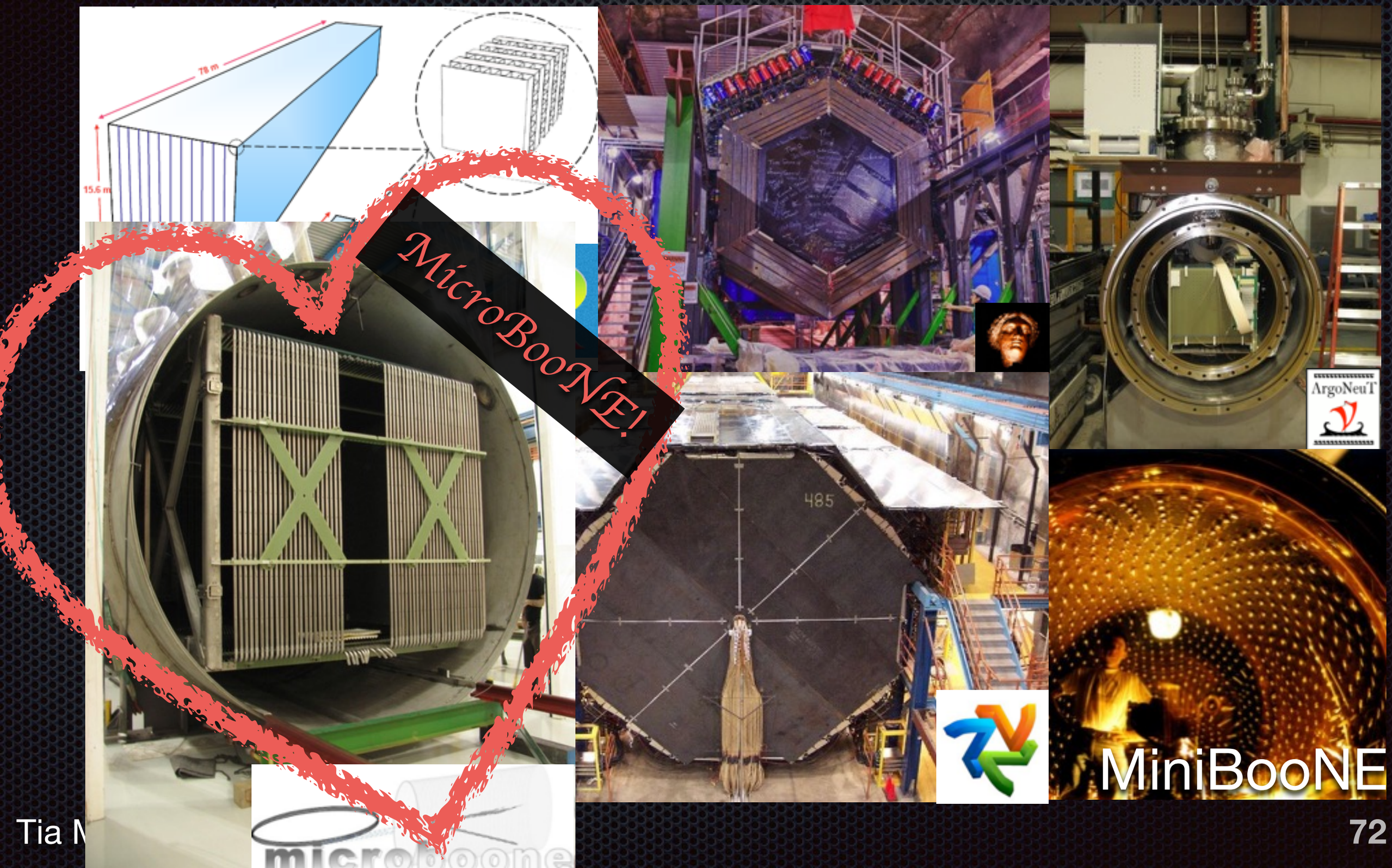

ArgoNeuT


microboone

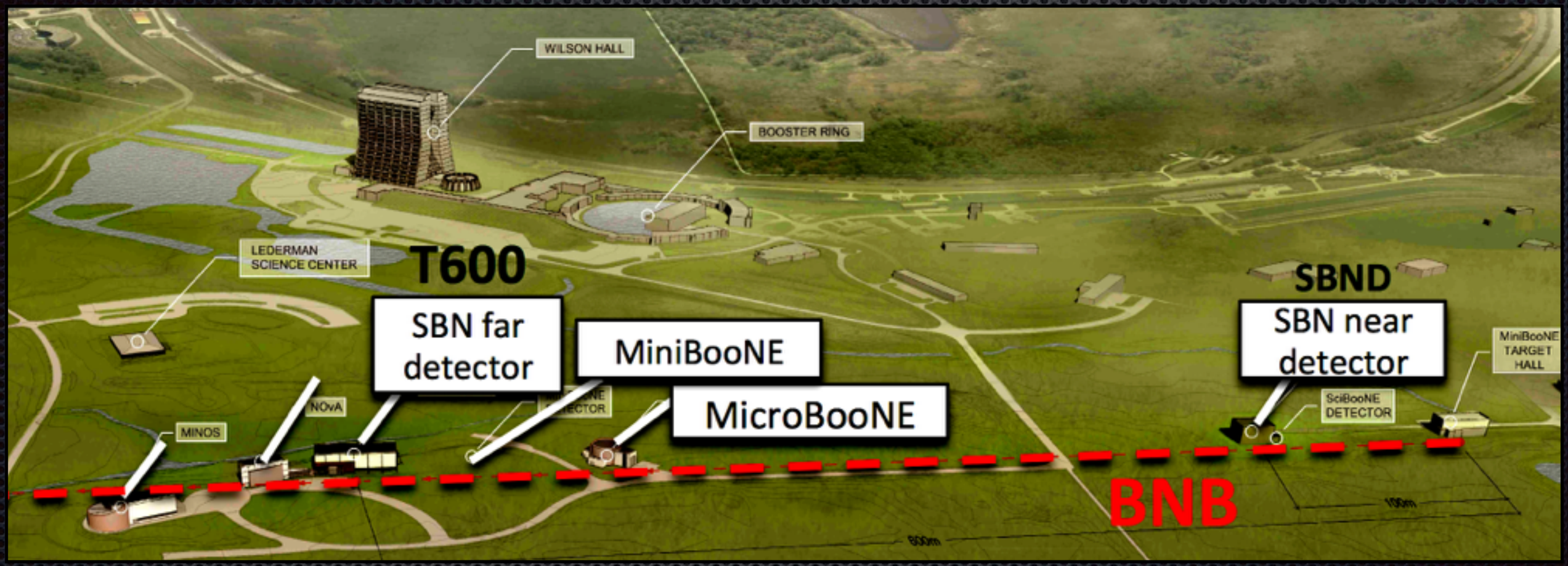
485

MiniBooNE

At Fermilab

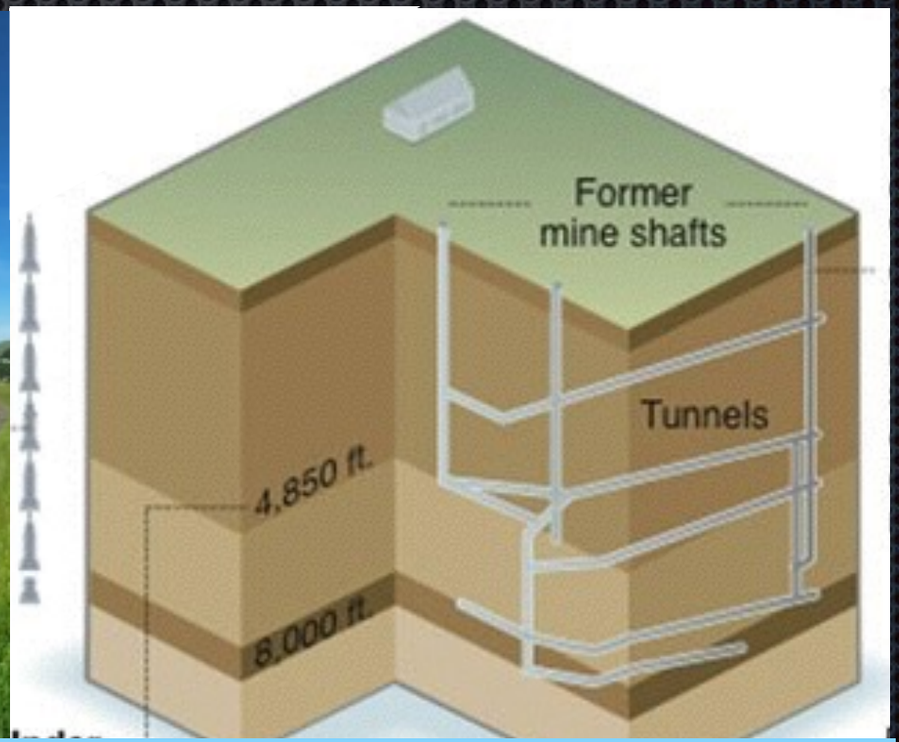


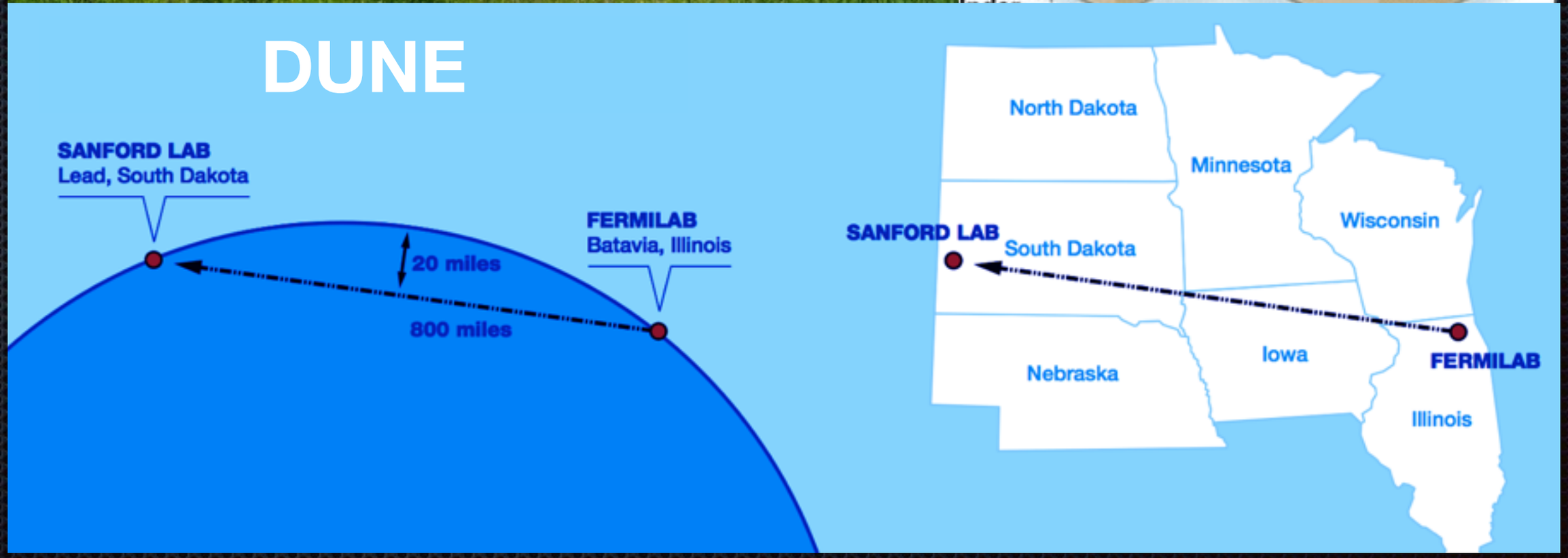
What the future holds for Fermilab



The Short Baseline Program

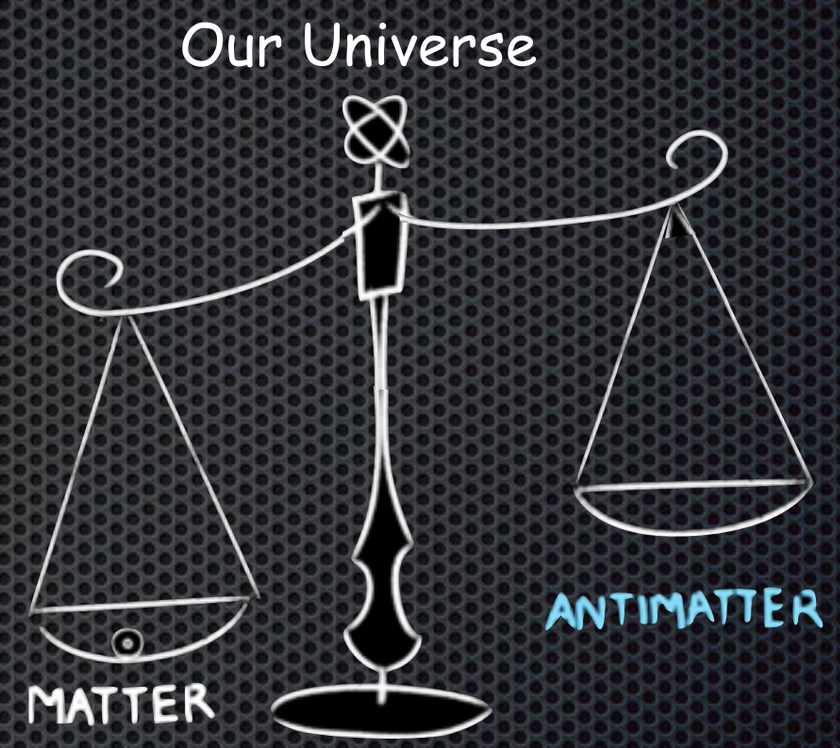
What the future holds for Fermilab





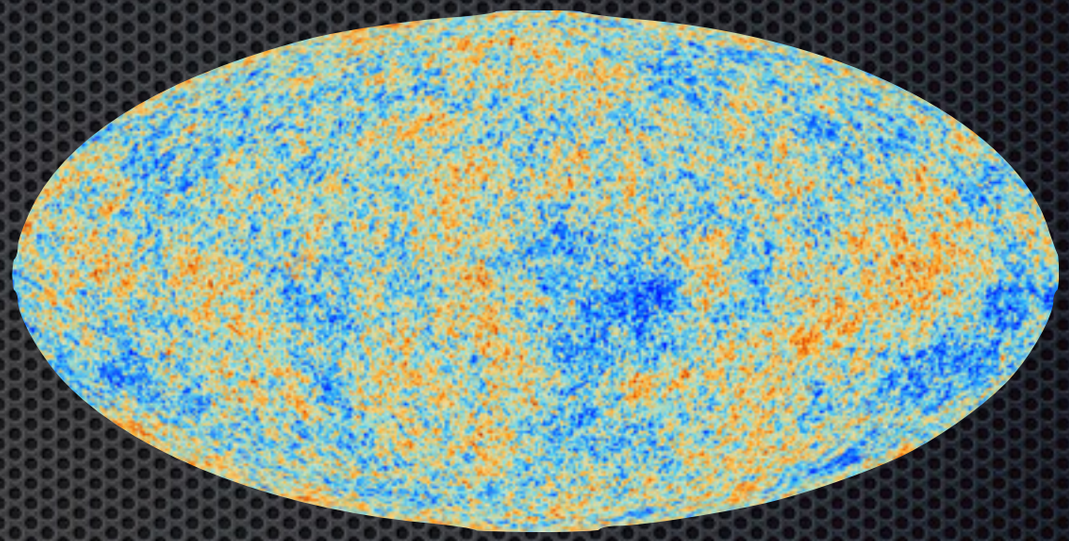
Why Tia researches neutrinos (1/3)

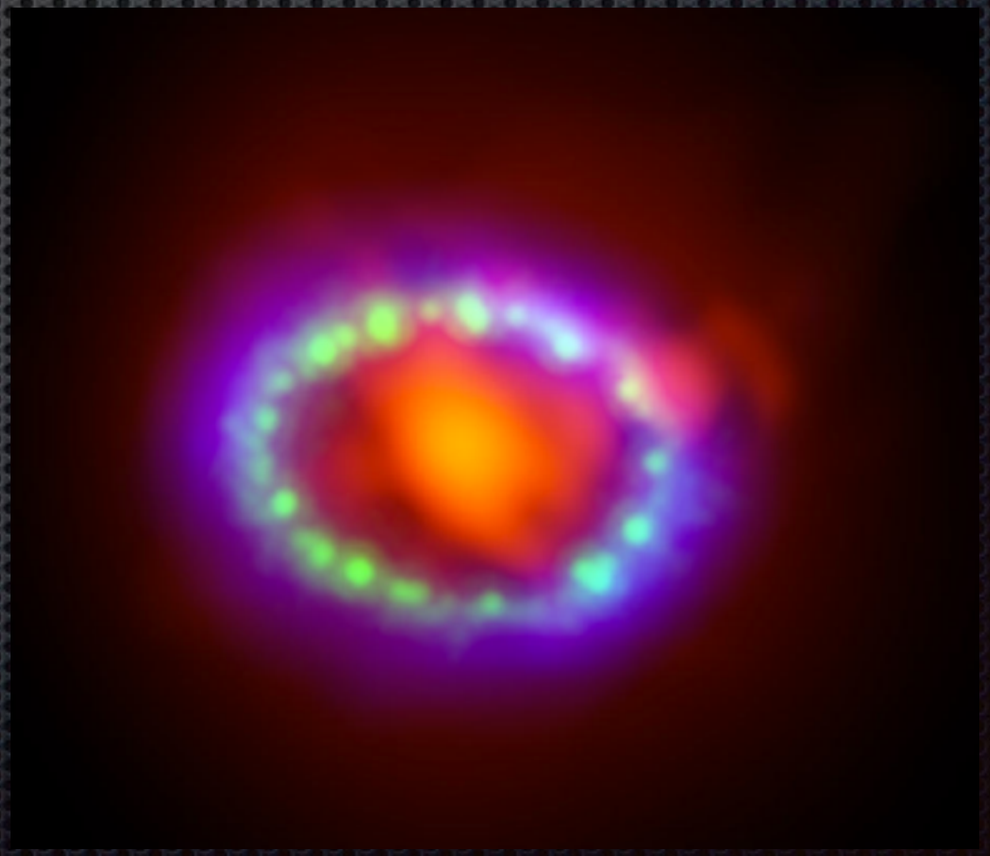
- Basic unknown properties!
  - Mass?
  - How many types?
  - Do neutrinos behave the same as anti-neutrinos?
  - neutrino = anti-neutrino? “Majorana”



Why Tia researches neutrinos (2/3)

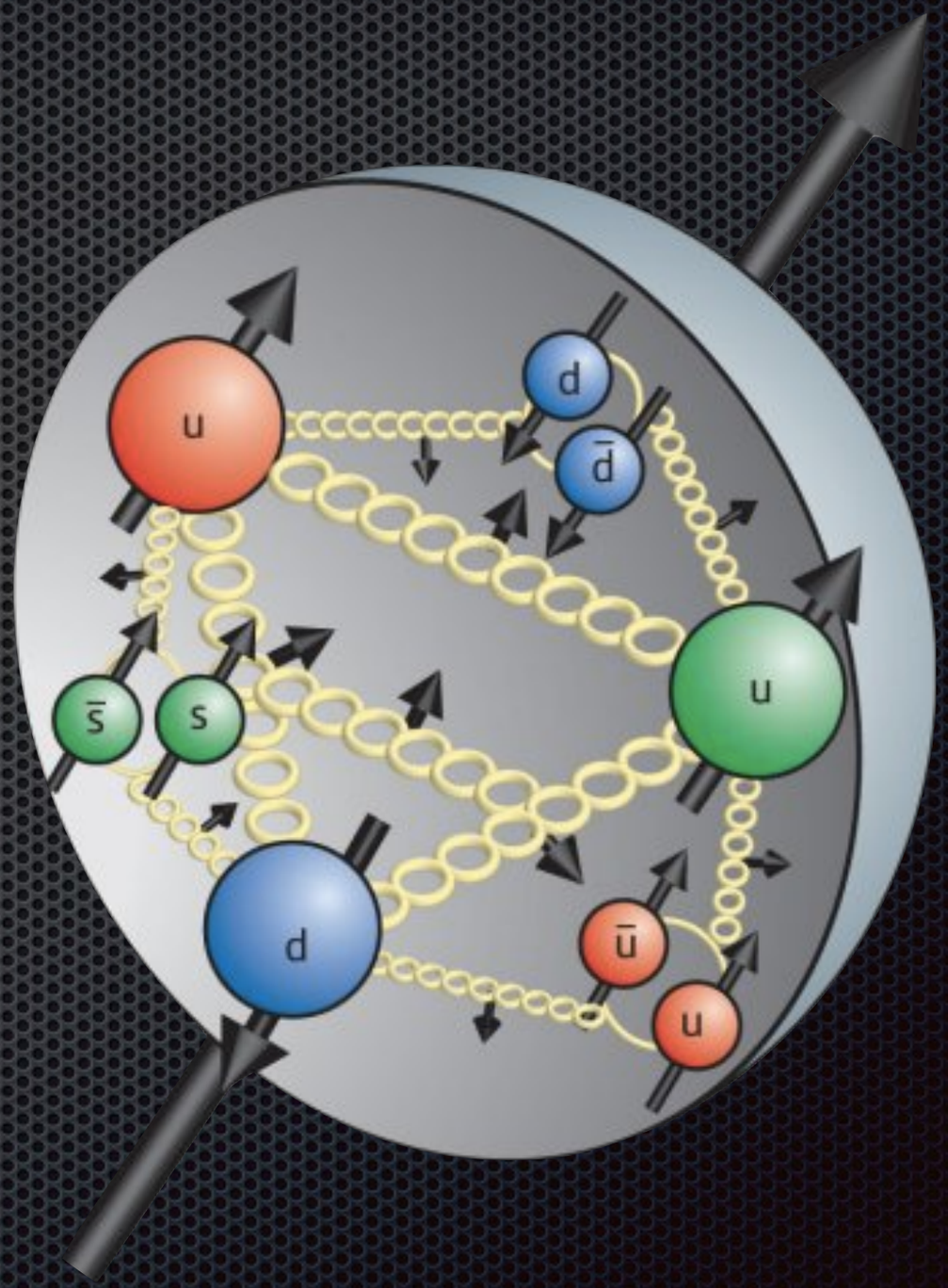
- Explore the universe
  - see where light can't! (Big bang relic neutrinos!?)
  - see inside stars! (solar, supernovae neutrinos)



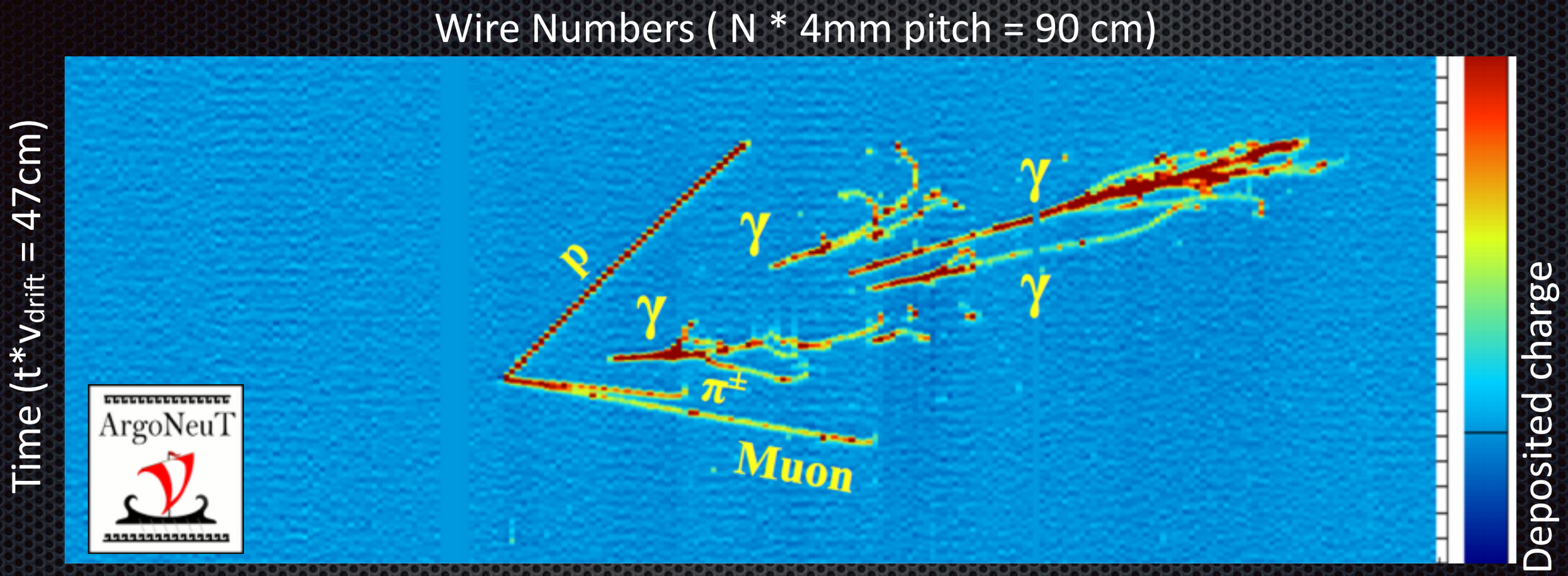


Why Tia researches neutrinos (3/3)

- Explore atoms
  - Neutrino scattering probabilities (cross-sections), used to refine experiments.
  - Spin composition of protons and neutrons.♡

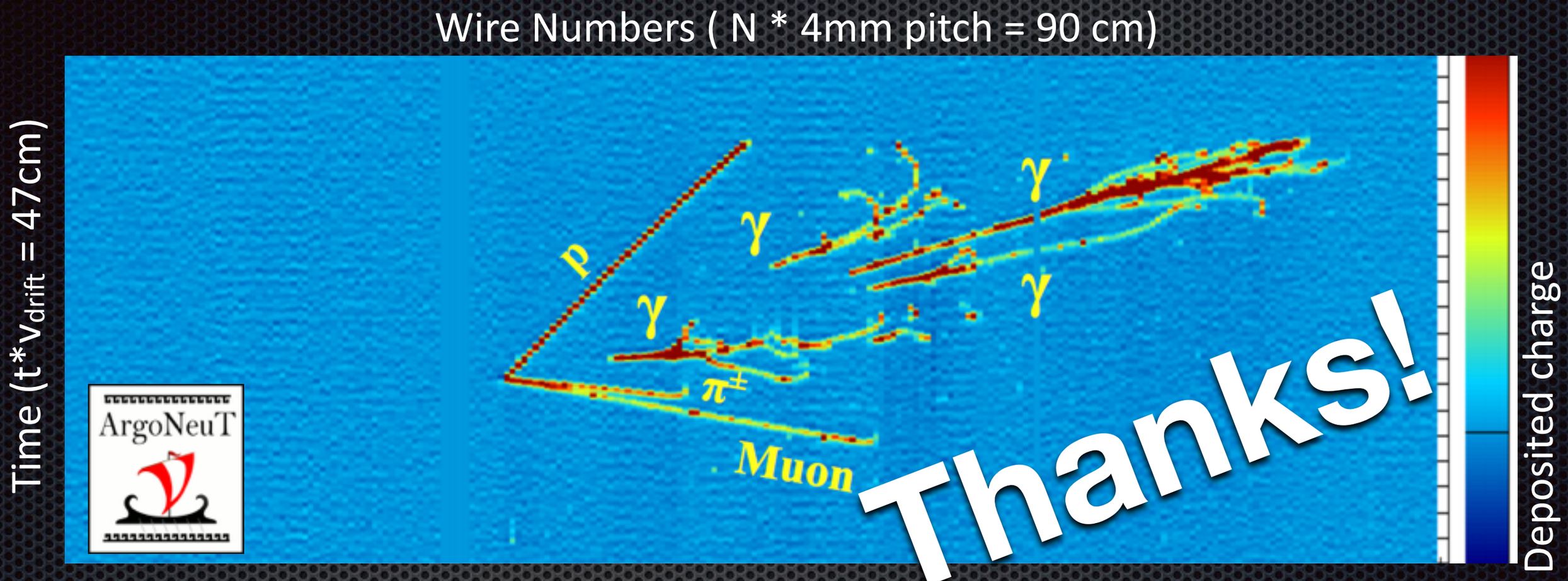


Amazing world of neutrinos!



What events will look like in MicroBooNE!
Commissioning now!
Everyone is anxious for data!

Amazing world of neutrinos!



What events will look like in MicroBooNE!
Commissioning now!
Everyone is anxious for data!