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"On the Applicability of Phase Space Tomography to Reconstruct the Phase Space of a Space-Charge-Dominated Electron Beam"

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Basic Underlying Theory (BUT)

- With knowledge of what the electron bunch looks like at some time (t) and the environmental changes endured by the bunch since the initial time (t_0) – which in this case include drift lengths, quad strength, and quad length – it is possible to obtain a very good representation of the bunch at time t_0 .

Experimental Setup

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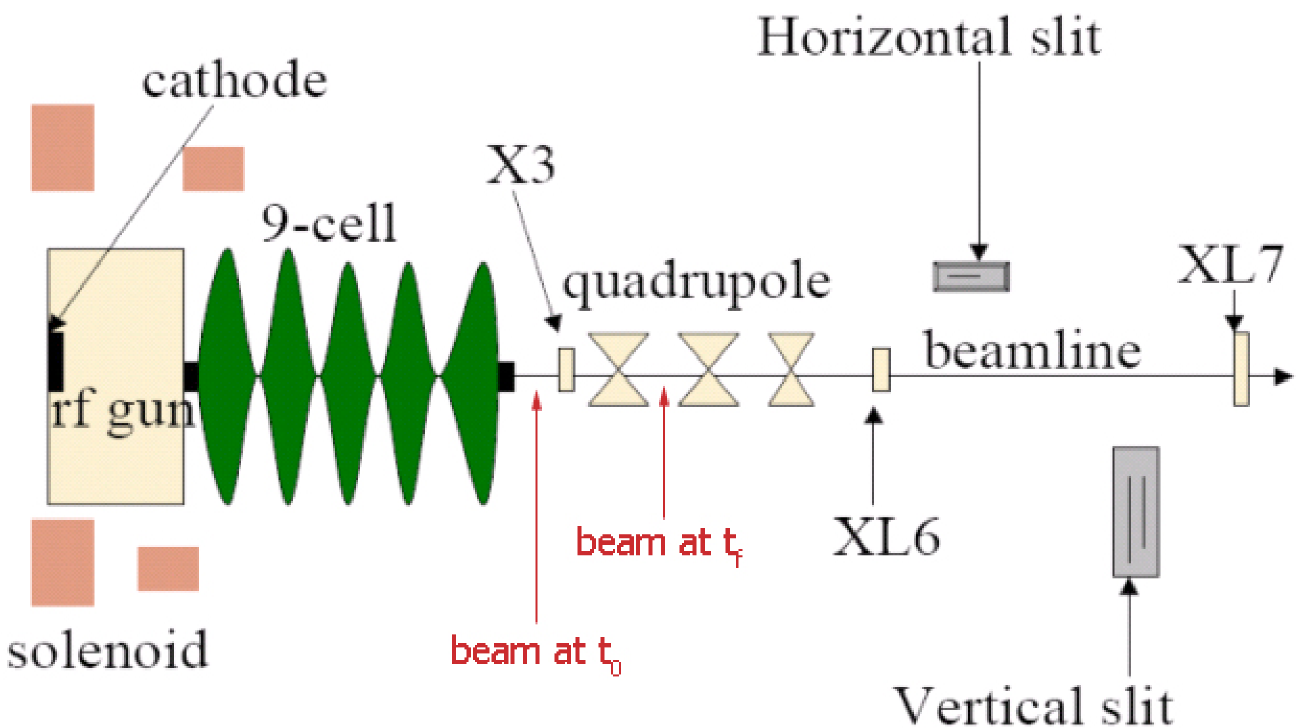
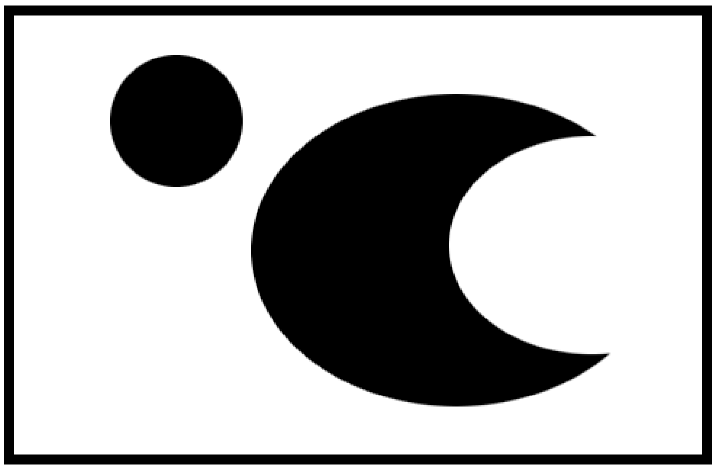


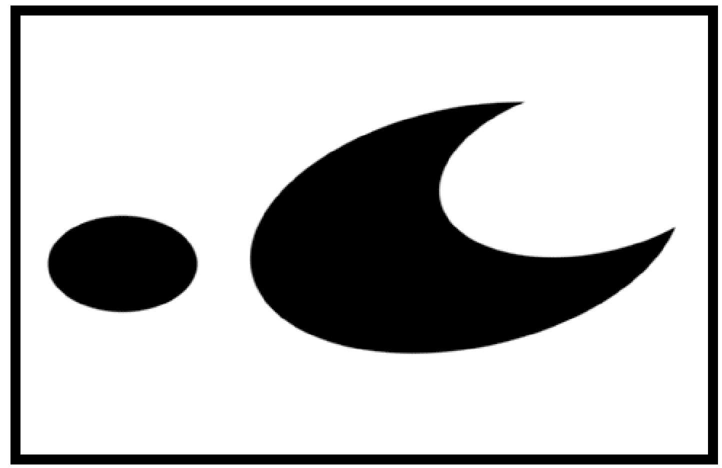
Transfer Matrices

Drifts: The drift is an extremely simple beam line element to model. In the drift, we do not have to take into account any forces on the particles. The only things that we must take into account are x', y' and the length (*L*) of the drift.

Quadrupoles: Quads are used to focus the beam on one plane and defocus it on the orthogonal plane. Because of this, the transformation is more difficult than the drift matrix, and is dependant on the angle (which is dependant on the quad strength, *k*)

Why The Matrices?





- As the bunch travels through the quadrupoles and the drift, the beam distribution becomes rotated and skewed.

Enter The Matrix

In our program, the drifts are given the following matrix:

$$\mathbf{M}_{\mathrm{D}} = \begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{Q}_{\mathrm{F}} = \begin{pmatrix} \cos(\Omega) & \frac{1}{\sqrt{k}} \sin(\Omega) \\ -\sqrt{k} \sin(\Omega) & \cos(\Omega) \end{pmatrix}$$

$$\mathbf{Q}_{\mathrm{D}} = \begin{pmatrix} \cosh(\Omega) & \frac{1}{\sqrt{k}} \sinh(\Omega) \\ \sqrt{k} \sinh(\Omega) & \cosh(\Omega) \end{pmatrix}$$

While quadrupole transfer matrices are solved via the matrix to the left.

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Two Programs

(Both Alike in Dignity)

<u>The Transfer Matrix Program</u>: A program developed by me at Fermilab. The purpose of the program is to solve the transfer matrices of the beam as it progresses through space. It then applies the final transfer matrix to a 1x4 vector of x, x’, y, and y’. The output of this is later applied to the Max Entropy program.

<u>The Max Entropy Program</u>: The brilliant part of the code was done at DESY in Germany. My contribution to the code was getting it execute its algorithms on external (real) data as opposed to the internal test which it came with. This program will take the histograms and their associated angles as input, and attempt to recreate the original bunch distribution.

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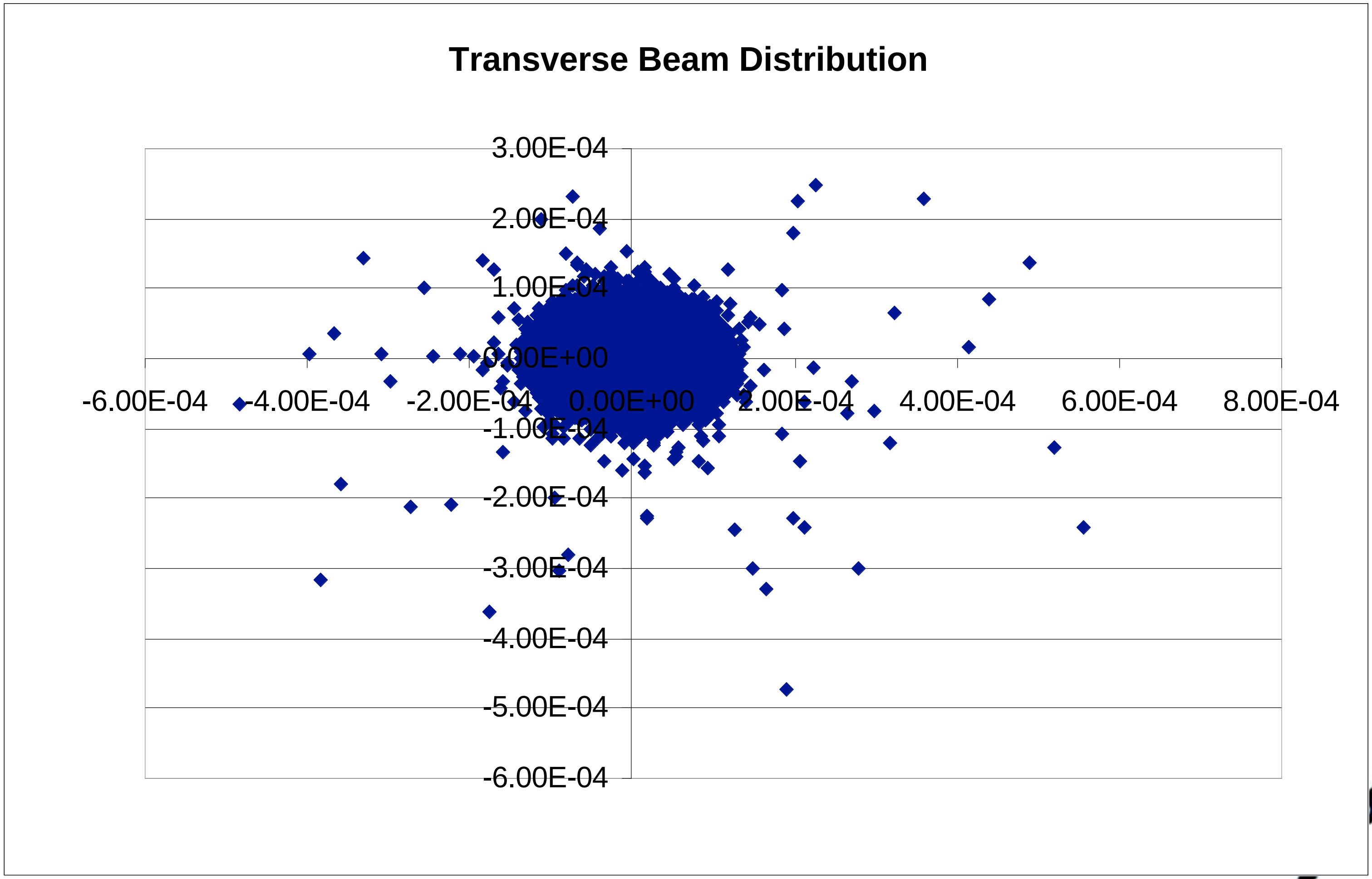
Transfer Matrix Program

```
[auriemma@a0pi-spitefire2 beam]$ ./a.out -q 1 -min -.003 -max .003 -b 500
Enter length of drift 1 in meters: 5
Enter strength of quadrupole 1: 20
Enter length of quadrupole 1: .1

Enter length of final drift in meters: .4

Angle:  1.45466
```

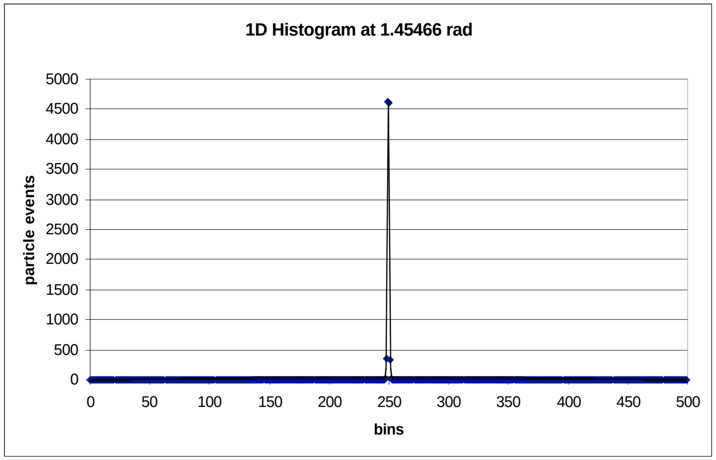
- The Transfer Matrix Program asks how many quads the beam went through, the strength of the quads, their lengths, and the length of the drifts. It will then calculate all of the transfer matrices and apply the final matrix to the data set. From this final matrix, we can then calculate the angle of polarization:

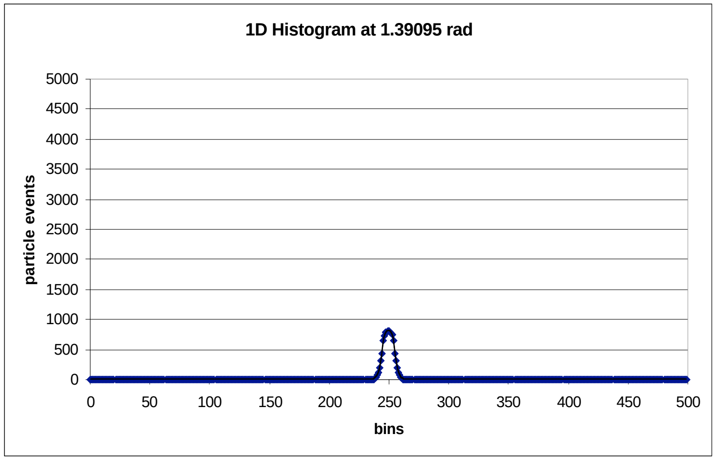
$$\_ = \arctan(x_{21}/x_{11})$$

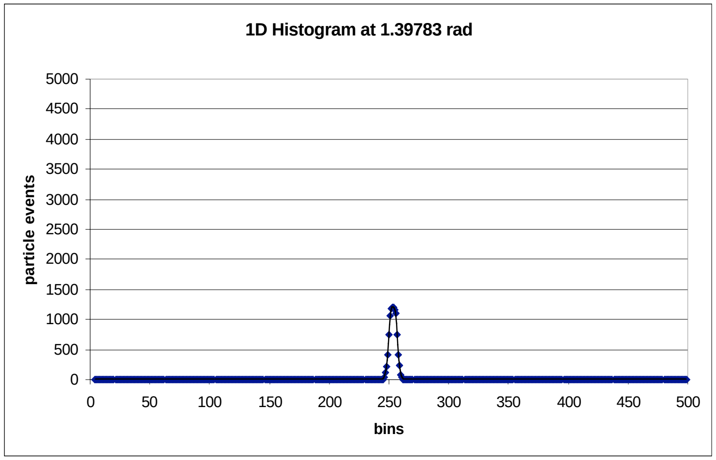
Sample TMP Input

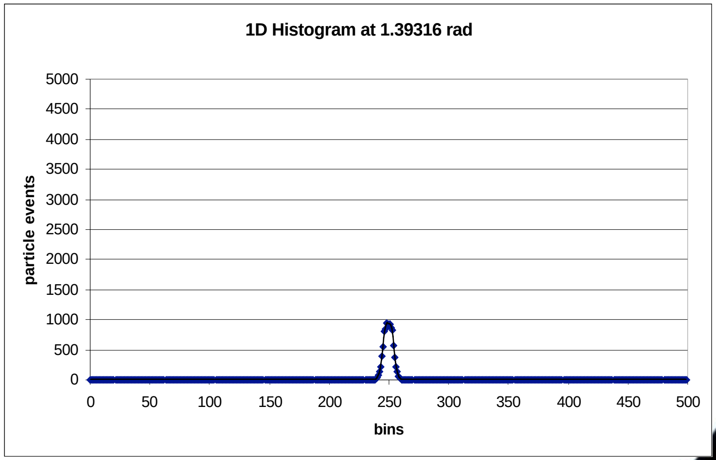


Sample TMP Output

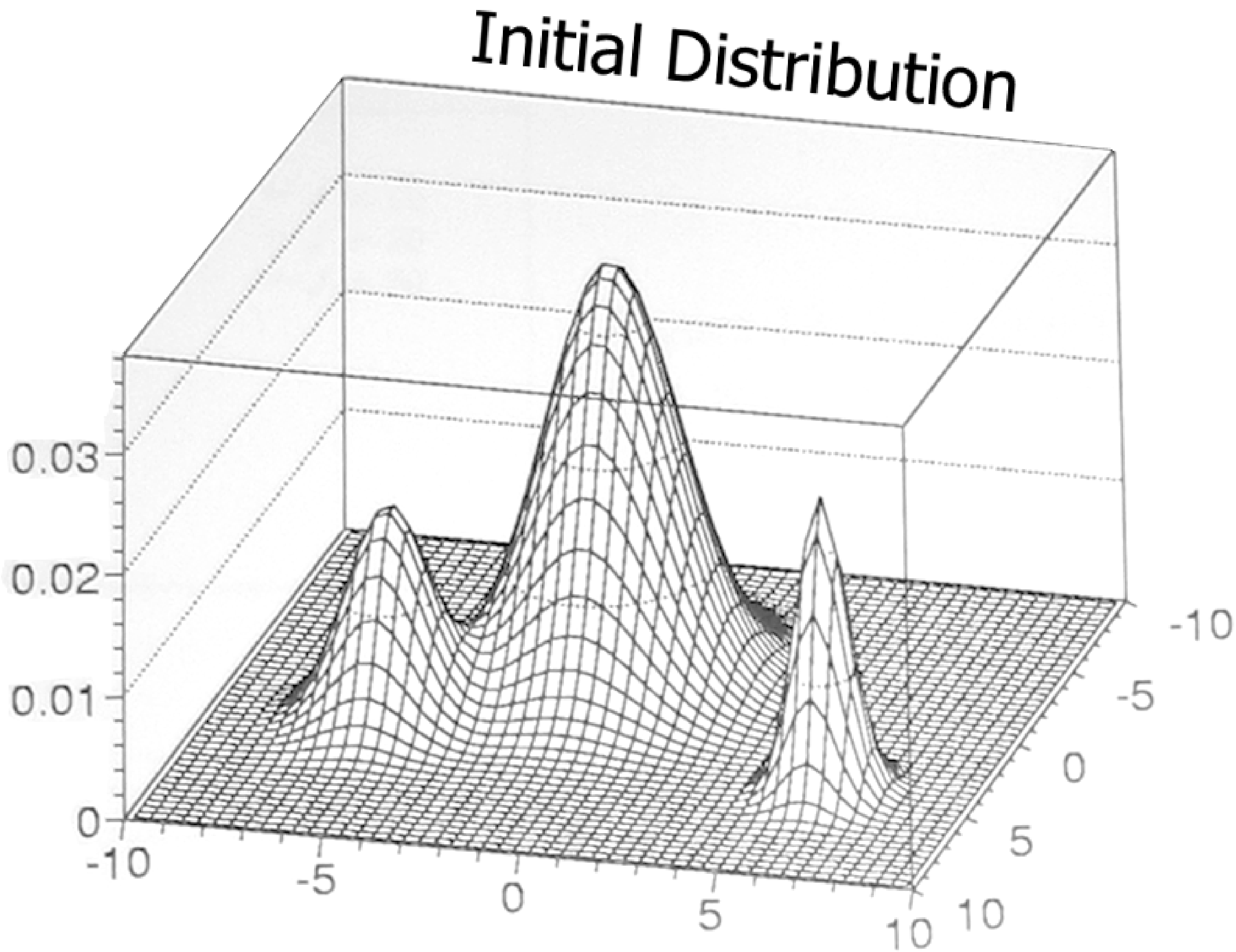




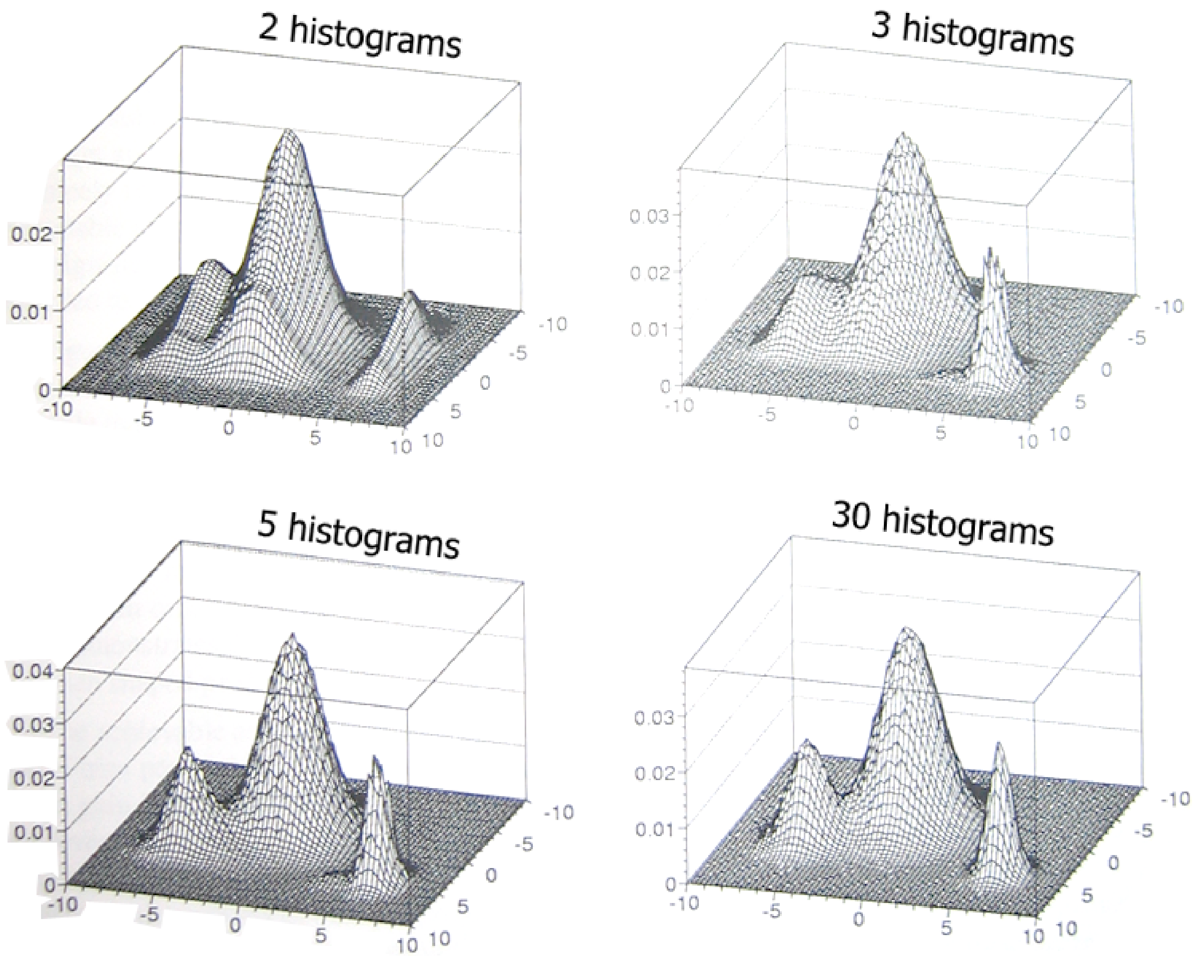




Max Entropy Program



Recreated Distributions



Conclusions

- Even Gaussian distributions can be very accurately recreated by using the maximum entropy algorithm.

- It is likely that an optimum number of histograms needed to accurately recreate the bunch will be obtained with further research.

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Contact, Thanks, Etc.

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Thanks to Roger, Erik, and Max for paying us to learn physics at the best particle accelerator in the world.

And most importantly, thanks to the interns whose interesting mesh of personalities and corny physics humor will leave me with fond memories for a long time to come.