Rotations: The Moon, MRI, g-2, etc.

Wu, Jinyuan
Fermilab
July. 2014

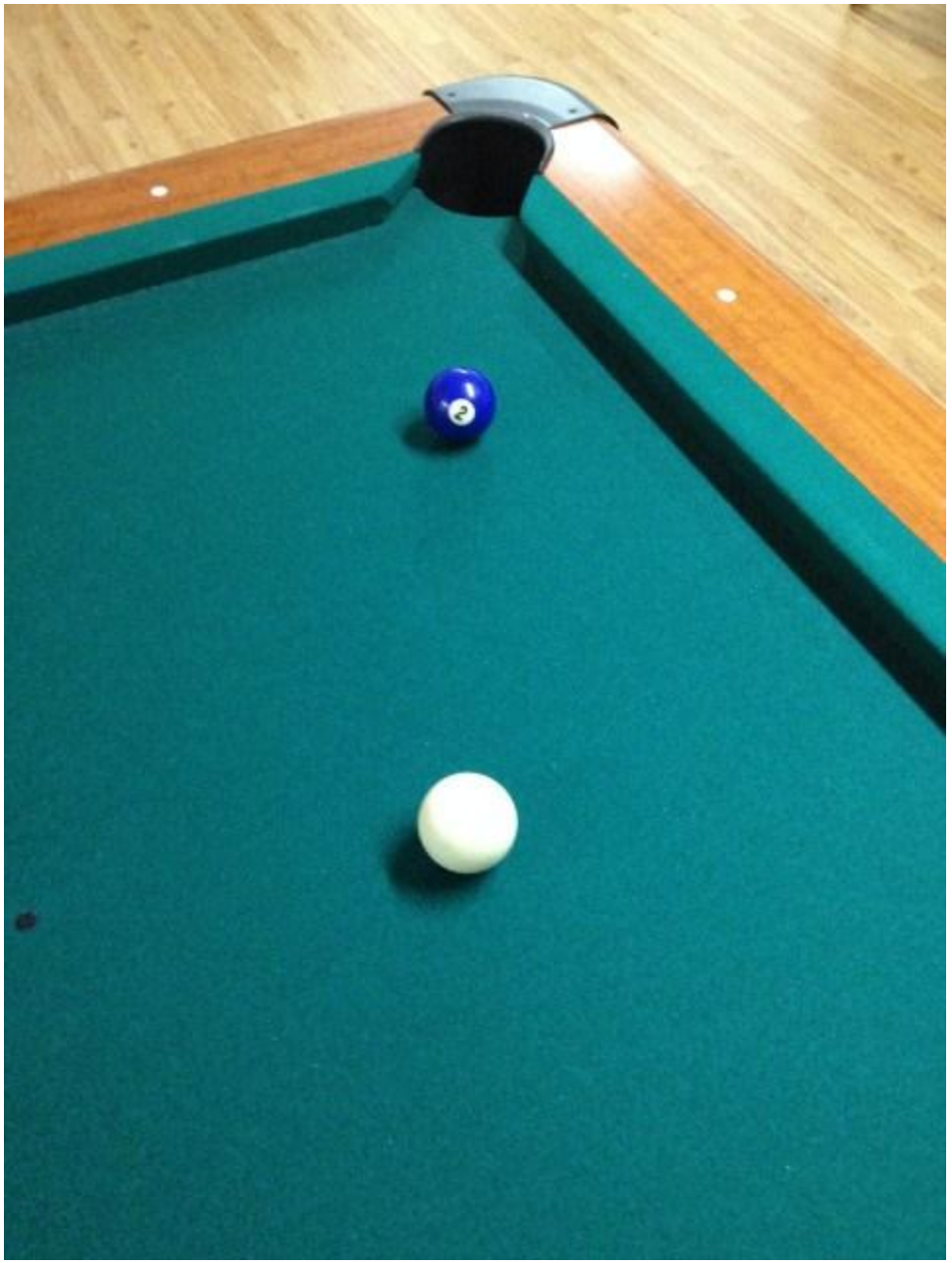
Introduction

- We will talk about classical physics only.
- We will try to show phenomena not commonly seen in text book.
- We will try to make connections between PHYS 101 to the research topics in research institutes such as national laboratories.

Many pictures in this file are taken via www.google.com
They are used for review and comments only.

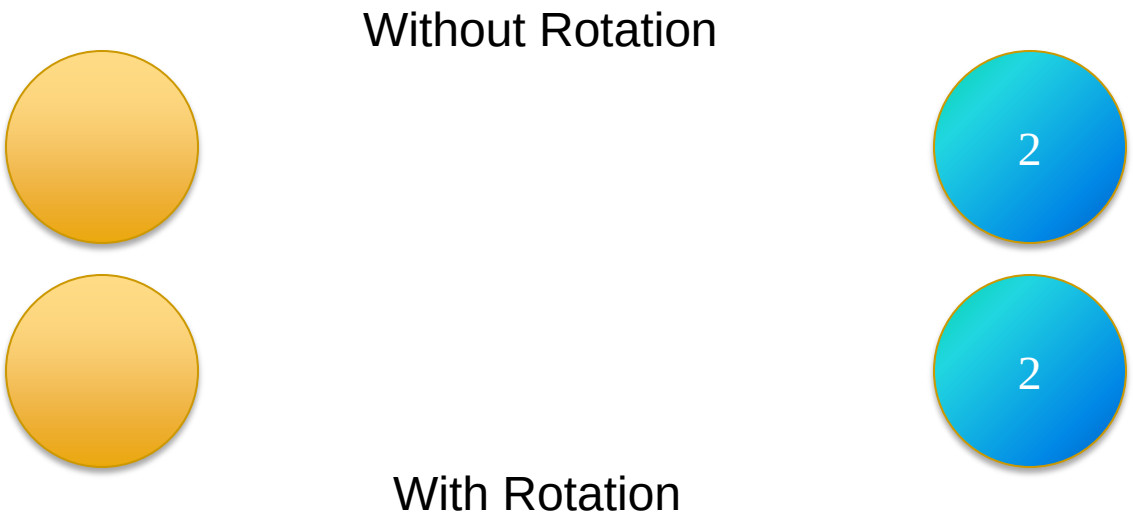
Intuitions vs. Phenomena with Rotations

Billiard Balls



- The goal is to pocket the numbered ball, but not to sink the cue ball.
- Should we hit the cue ball harder or lighter?

Elastic Collisions with and without Rotations

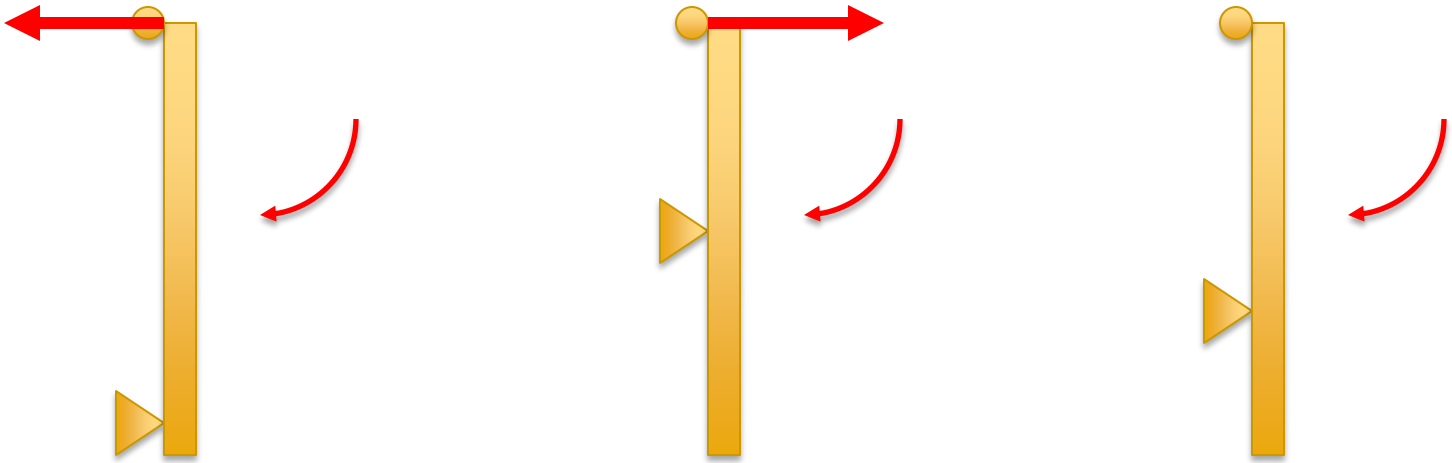


- When the cue ball moves without rotation and collide center-to-center to the ball 2, the energy and momentum exchange completely:
  - The cue ball stops.
  - The ball 2 moves at the same speed.
- If the cue ball moves with rotation, not all kinetic energy will be given to the ball 2:
  - The ball 2 moves.
  - The cue ball follows at slower speed.

Door Hinge and Stopper

- It is hard to image forces perpendicular to the door are applied to door hinges.
- It is even harder to image the door stopper may cause large force on the hinges.

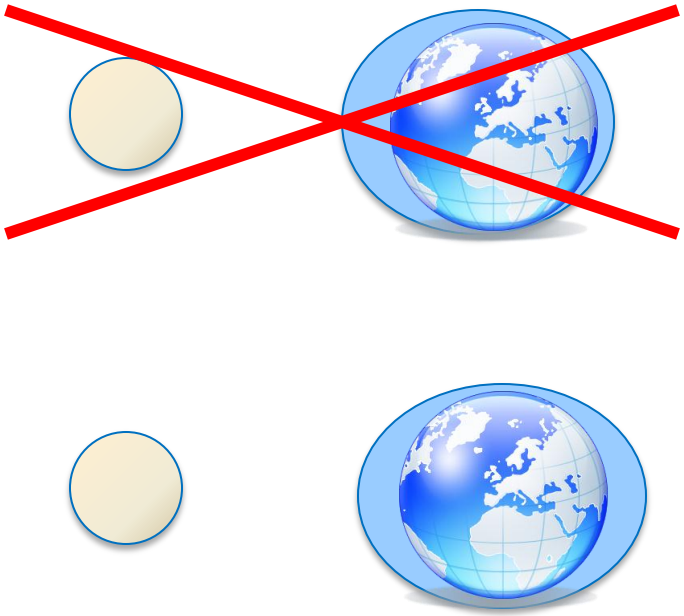
Positions of the Stopper



- When the door stopper is too far to too close to the hinge, a large force is applied to the hinge when the door hits the stopper.
- When the stopper is at a “magic” position, the force on hinge vanishes.

The Story of Tide

Tide: Twice a Day, Not Once a Day



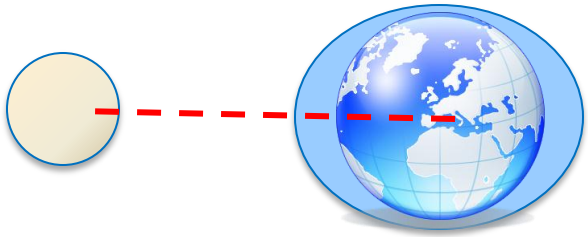
http://en.wikipedia.org/wiki/Tide

- The tide is caused primarily due to gravity of the moon.
- It seems that the high tide should only happen once per day.
- The reality is that the high tide happens twice a day (semi-diurnal) in most places.

Death Spiral

- Death spiral is a required element of pair skating.
- It resembles motions of the Earth and the Moon very well.
- The pair rotate around their center of mass.

High Tide on Both Sides of the Earth



- The sea water facing the Moon is “dragged” high due to gravity of the Moon.
- The sea water on the other side of the Earth is “thrown” away due to the rotation of the Earth-Moon system.

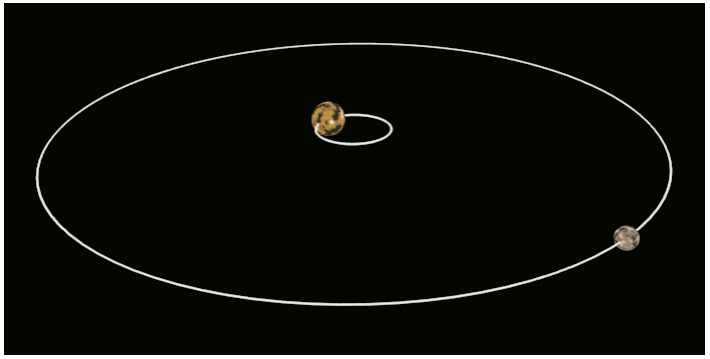
Solid Earth Tide

- The tidal force causes displacement of not only water but also the solid Earth's surface.
- The vertical amplitude can be as large as 30-40 cm.
- Horizontal displacement of the Earth's surface can also be seen.

Tidal Friction (Tidal Acceleration)

- The tidal force slows down the rotation of the Earth.
- The length of the Earth day decreases by 1.6 ms/100 years.

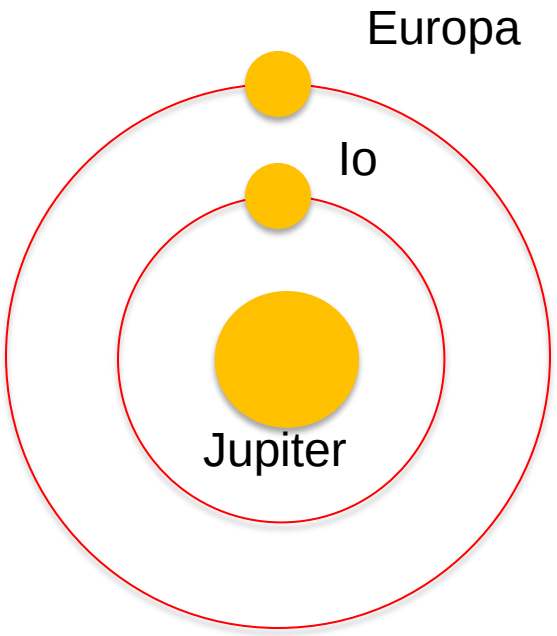
Tidal Locking



http://en.wikipedia.org/wiki/Charon_(moon)

- Tidal friction caused the Moon's self rotation to slow down.
- Now our Moon is tidal locked, i.e., the spin-orbit ratio is 1:1.
- Many natural moons in solar system are tidal locked.
  - Earth: Moon
  - Mars: Phobos, Deimos
  - Jupiter: Metis, Adrastea, Amalthea, Thebe, Io, Europa, Ganymede, Callisto
  - Saturn: 15, Uranus: 5, Neptune: 2
- Pluto & Charon are locked to each other.

Tidal Heating



- The tidal friction generates heat.
- It is believed the tidal head causes volcanos on Io and liquid water on Europa.
- Europa and Io are both tidal locked to Jupiter.
- The gravity between them caused their orbit heights change.
- The solid tide amplitude on Io is >100m.

Effect of Tidal Forces on the Accelerator

- The tidal force causes the solid Earth surface to stretch.
- The deformation of the ground causes accelerator parameters to change.
- The accelerator shown is the LEP of CERN.

Effect of Tidal Forces on the Accelerator

- Tidal force causes variations of the beam energy in LEP.
- The study was in 1992.

Effects of Tidal Forces on the Beam Energy in LEP

L. Arnaudon, F. Bordry, W. Coosemans, B. Dehning, K. Henrichsen, A. Hofmann, R. Jacobsen, J.P. Koutchouk, L. Lawson-Chroco, M. Mayoud, J. Miles, R. Olsen, M. Placidi, G. Ramseier, R. Schmidt, J. Wenninger, (CERN, Geneva, CH), A. Blondel, (Ecole Polytechnique, Paris), R. Aßmann, (Max-Planck-Institut, Munich, DE), G.E. Fischer, C. Pan, (SLAC, Stanford, U.S.A.) and R. Olivier, (IGL, Lausanne University, CH).

Abstract

The e^+e^- collider LEP is used to investigate the Z particle and to measure its energy and width. This requires energy calibrations with ~20 ppm precision achieved by measuring the frequency of a resonance which destroys the transverse beam polarization established by synchrotron radiation. To make this calibration valid over a longer period all effects causing an energy change have to be corrected for. Among those are the terrestrial tides due to the Moon and Sun. They move the Earth surface up and down by as much as ~0.25 m which represents a relative local change of the Earth radius of 0.04 ppm. This motion has also lateral components resulting in a change of the LEP circumference (C_c=26.7 km) by a similar relative amount. Since the length of the beam orbit is fixed by the constant RF-frequency the change of the machine circumference will force the beam to go off-center through the quadrupoles and receive an extra deflection leading to an energy change given by $\Delta C_c/C_c \sim -\alpha_c \Delta E/E$. With the momentum compaction $\alpha_c = 1.85 \cdot 10^{-4}$ for the present LEP optics this gives tide-driven p.t.p. energy excursion up to about 220 ppm, corresponding to ~18.5 MeV for the Z energy. A beam energy measurement carried out over a 24 hour period perfectly confirmed the effects expected from a more detailed calculation of the tides. A corresponding correction can be applied to energy calibrations.

I. Introduction

A significant improvement in the precision of the measurements providing the absolute energy calibration of the LEP beam was registered during the 1991 physics run with the availability of transverse beam polarization [1] and the implementation of the resonant depolarization method for the precise determination of the mean beam energy [2]. In particular a systematic uncertainty of ~ 6.3 MeV in the Z mass and ~ 4.9 Mev in the Z width were quoted [3] for the LEP results. The Z mass error is dominated by the knowledge of the absolute energy scale, while the Z width error stems from uncertainties in the differences between the various center-of-mass energies. The analysis of the 1991 beam energy data suggested that effects other than temperature changes in the dipoles contribute to the overall LEP energy reproducibility and the hypothesis that tidal forces might be responsible for it was anticipated in [4]. Fluctuations in the Energy Calibration data [2] were correlated to gravity variations in the Geneva area related to tidal forces. A rather strong correlation was found (Fig.1) despite the measurements having been taken over two months, [5]. Furthermore a tide related variation of the horizontal beam position was observed later [6], [7]. On the basis of this evidence the LEP Energy Working Group recommended the *TidExperiment* to be performed during the 1992 LEP run.

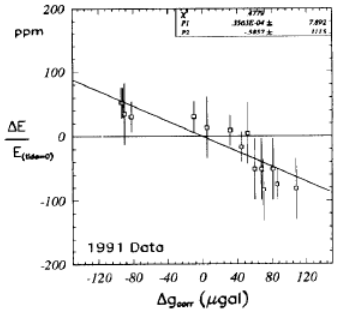


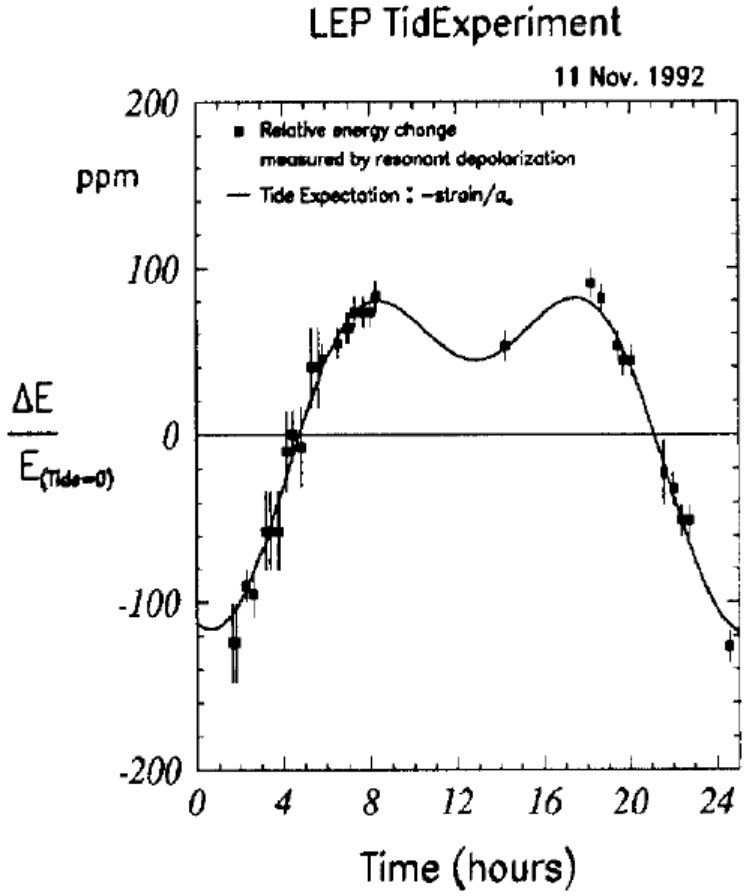
Figure 1: Correlation between relative beam energy deviations measured over the last two months of the 1991 LEP run and the corrections to the local gravity in the Geneva area ($1\,\mu\text{gal} = 10^{-6}$ cm s^{-2}).

II. Ground Motion and Accelerators

Alignment tolerances in modern and future accelerators have become more and more critical with the introduction of strong focusing magnetic elements to contain beam phase space and with increasing beam currents implying precise positional requirements to reduce interactions of wake fields with the beam environment [8],[9],[10]. Besides occasional motion from seasonal variations of water content in the soil, natural microseismic disturbances and other effects, Earth tides are the major example of *periodic ground motion*.

0-7803-1203-1/93$03.00 © 1993 IEEE

Energy Variations in 24 Hours



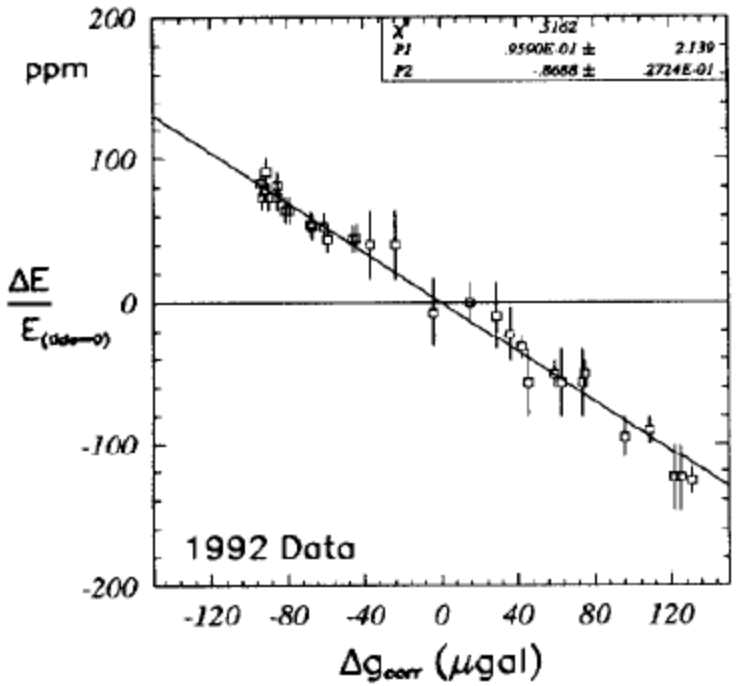


Figure 2: TidExperiment - Correlation between relative beam energy variations over the 24 hours measuring time and corrections to tide-induced local gravity changes. Fit: -0.869 ppm/μgal, expected: -0.882 ppm/μgal.

- November 11, 1992 was a full moon day.
- Two maxima and two minima are seen.

Precession

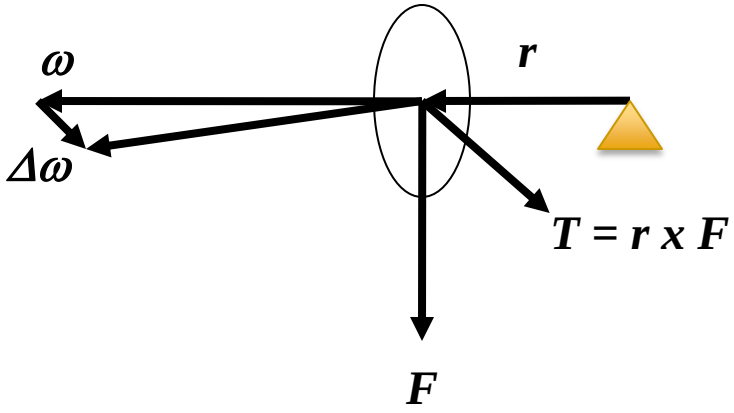
An Experiment

- Every physics student should do this experiment at least once in his life time.

Torque Causing Angular Momentum Direction Change

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

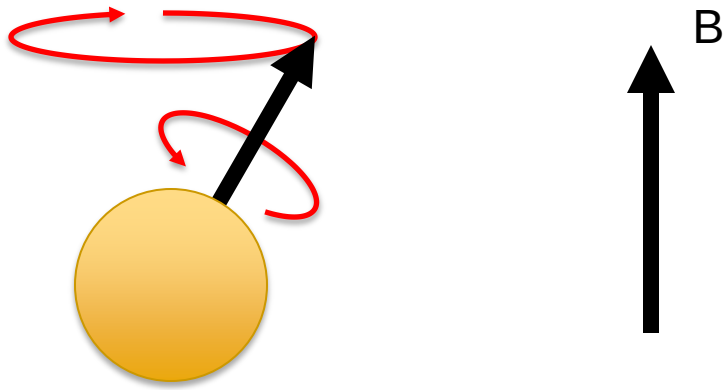
$$\vec{r} \times \vec{F} = I\frac{d\vec{\omega}}{dt}$$



- A torque causes angular momentum change.
- The angular momentum may only change its direction.

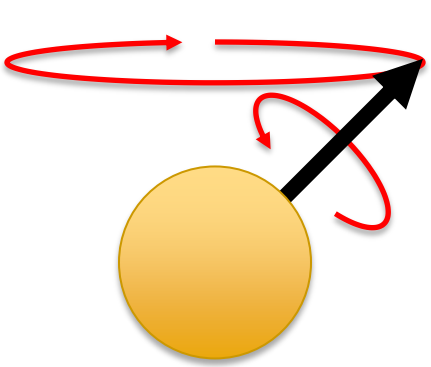
Larmor Precession

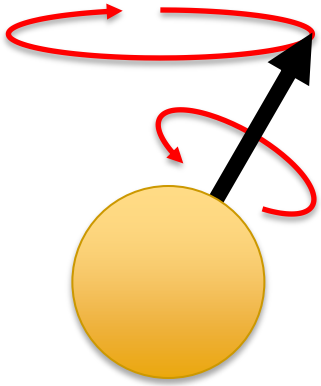
$$\vec{\mu} \times \vec{B} = \frac{d\vec{J}}{dt}$$

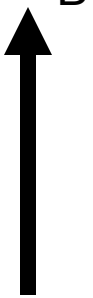


- A particle or a nucleus with a magnetic dipole moment μ receives a torque in magnetic field.
- The torque caused the angular momentum J of the particle to rotate around the B direction.

Larmor Frequency







$$\omega = \frac{egB}{2m}$$

Larger inclined angle
Larger torque
Larger ΔJ

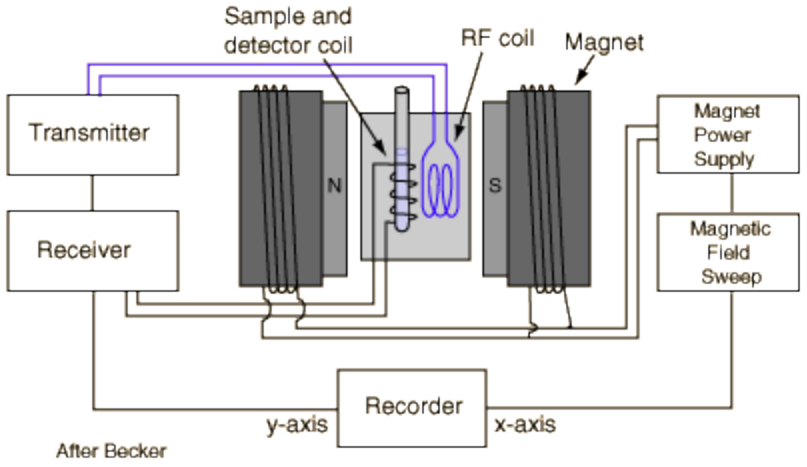
Same
Precession
Frequency

Smaller inclined angle
Smaller torque
Smaller ΔJ

- Same particles in a same magnetic field have an identical precession frequency.
- The g-factor g is a constant which is a ratio between the magnetic moment and the angular momentum.

Nuclear Magnetic Resonance Spectroscopy

Particle	Spin	ω_{Larmor}/B $s^{-1}T^{-1}$	ν/B
Electron	1/2	1.7608×10^{11}	28.025 GHz/T
Proton	1/2	2.6753×10^{8}	42.5781 MHz/T
Deuteron	1	0.4107×10^{8}	6.5357 MHz/T
Neutron	1/2	1.8326×10^{8}	29.1667 MHz/T
^{23}Na	3/2	0.7076×10^{8}	11.2618 MHz/T
^{31}P	1/2	1.0829×10^{8}	17.2349 MHz/T
^{14}N	1	0.1935×10^{8}	3.08 MHz/T
^{13}C	1/2	0.6729×10^{8}	10.71 MHz/T
^{19}F	1/2	2.518×10^{8}	40.08 MHz/T



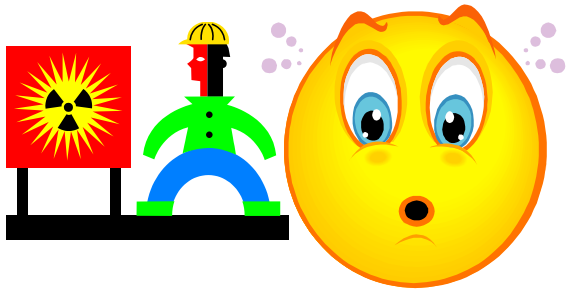
http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/nmr.html

- Put sample in magnetic field.
- Send RF to excite the particle.
- Receive the spin relaxation signal.
- Different particles have different signal frequencies.

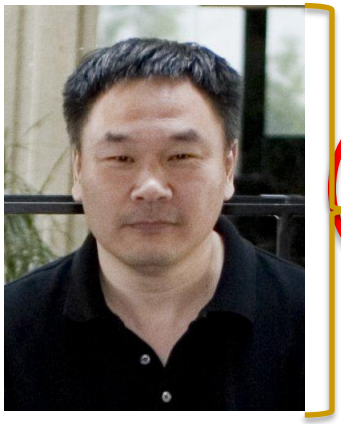
MRI

From NMR to MRI

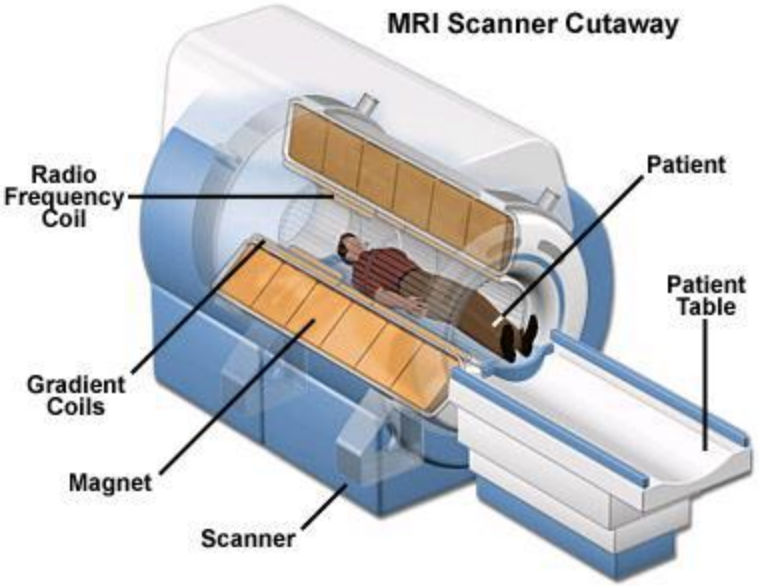
NMR => NMRI



=> MRI



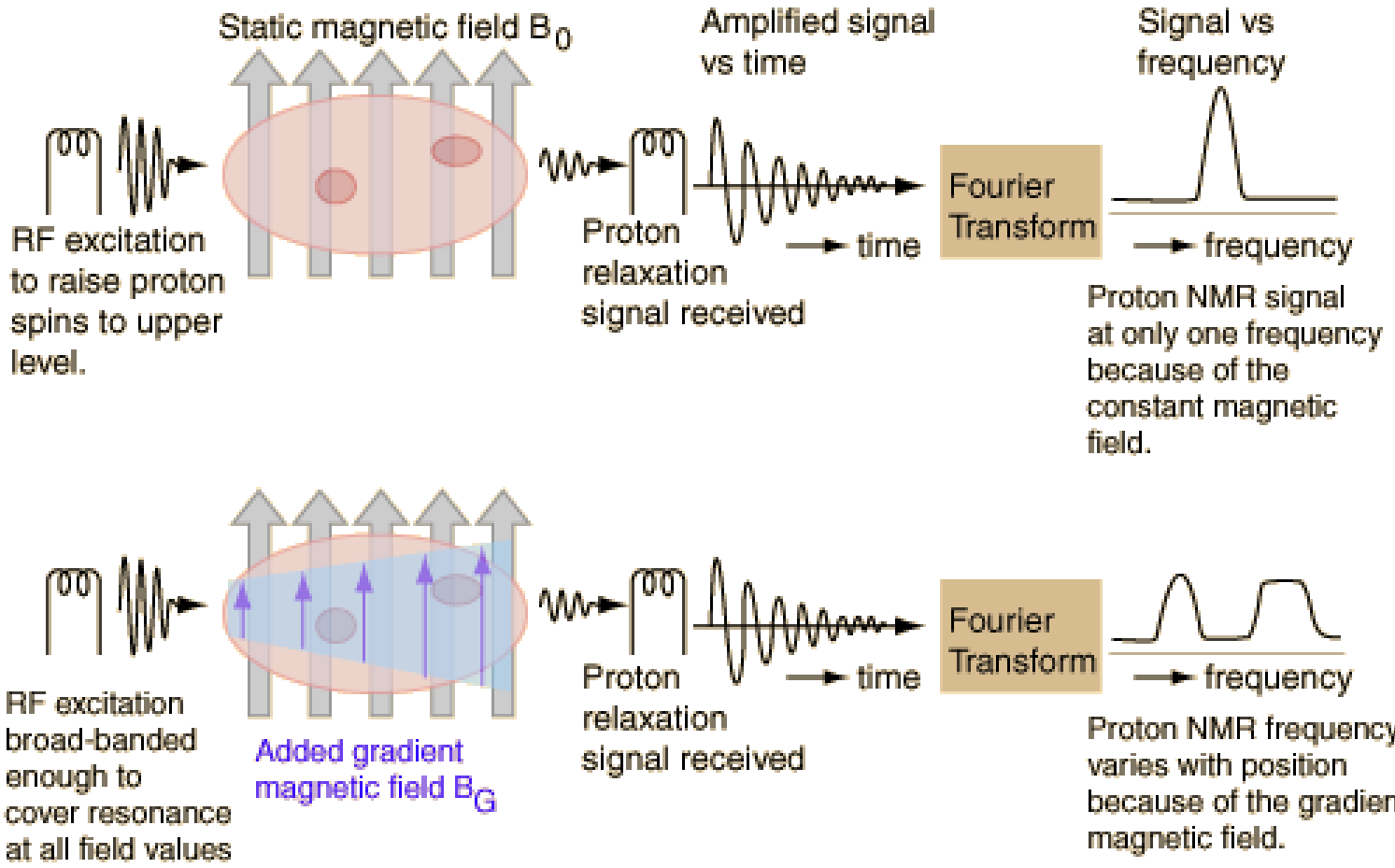
^{1}H, ^{12}C, ^{16}O ^{14}N



http://www.magnet.fsu.edu

- The word "nuclear" is eliminated.
- Hydrogen is the primary element seen.

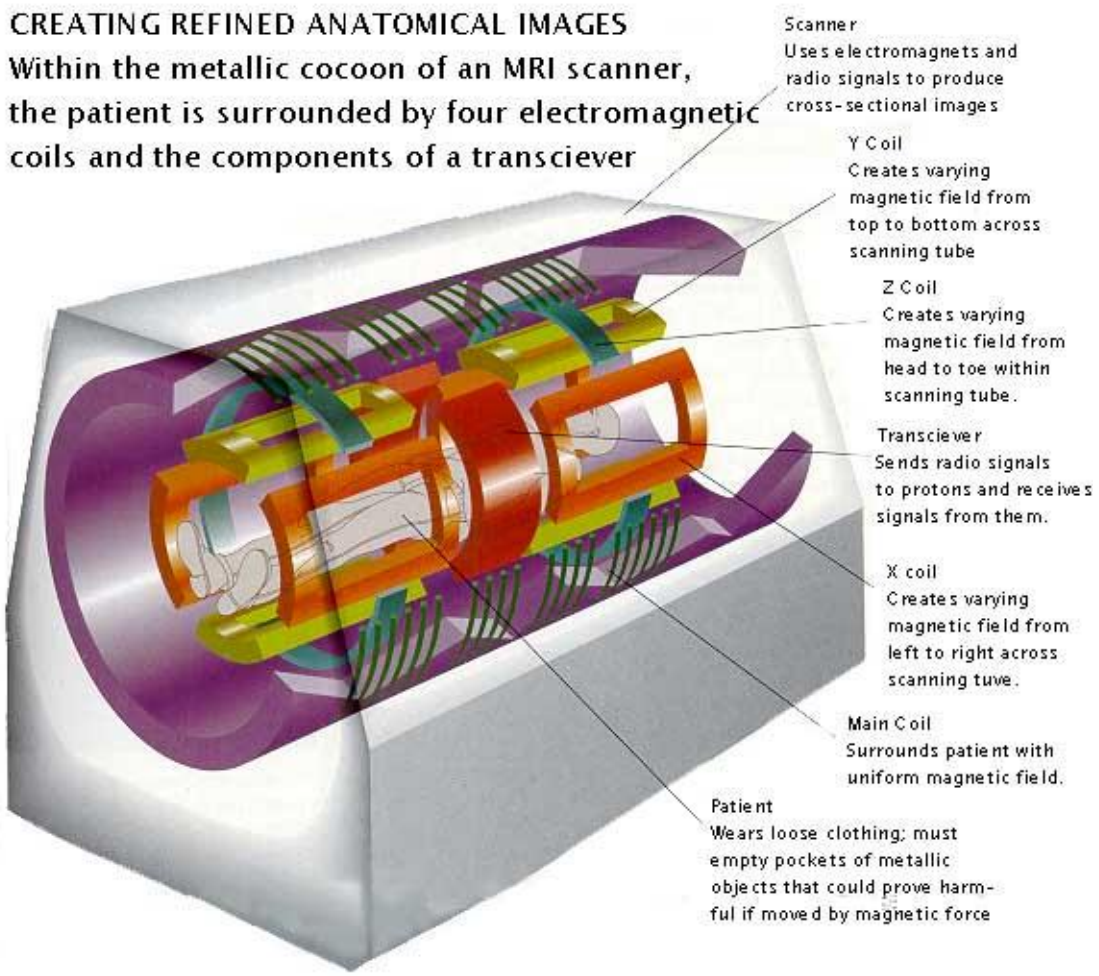
Spatial Encoding (Magnetic Field Gradient)

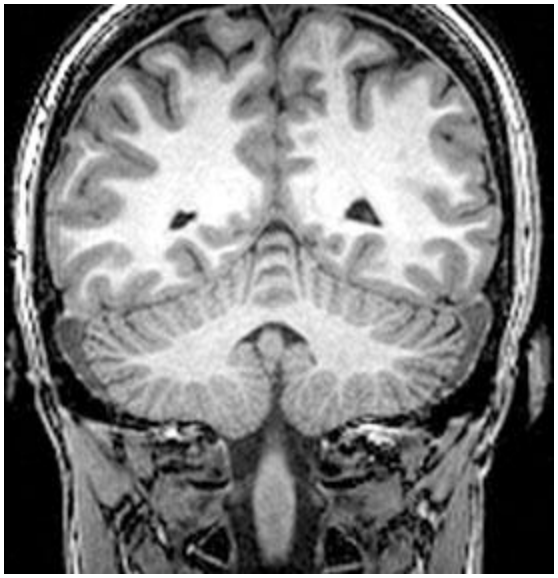


$$\omega = \frac{egB}{2m}$$

- Use non-uniform magnetic field.
- The precession frequencies at different locations are different.

Generating Magnetic Field Gradient



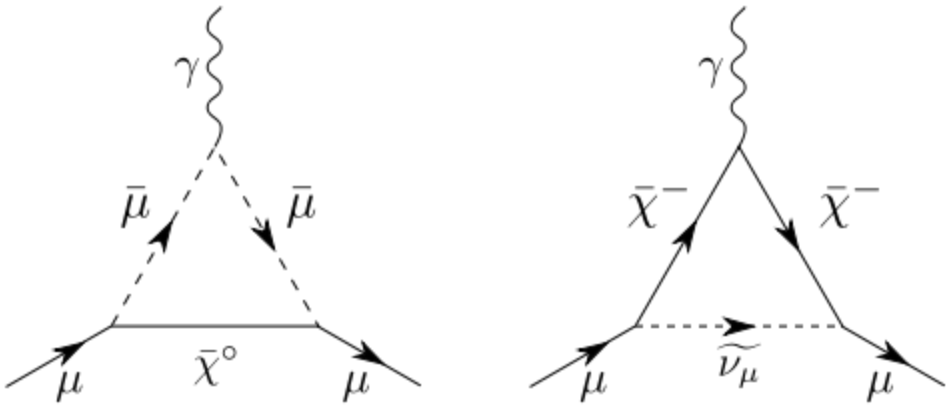


- Main Coil: Generates 1-2T magnetic field.
- X, Y, Z Coil: Cause field variations.

g-2

g-2: "GEE minus 2"

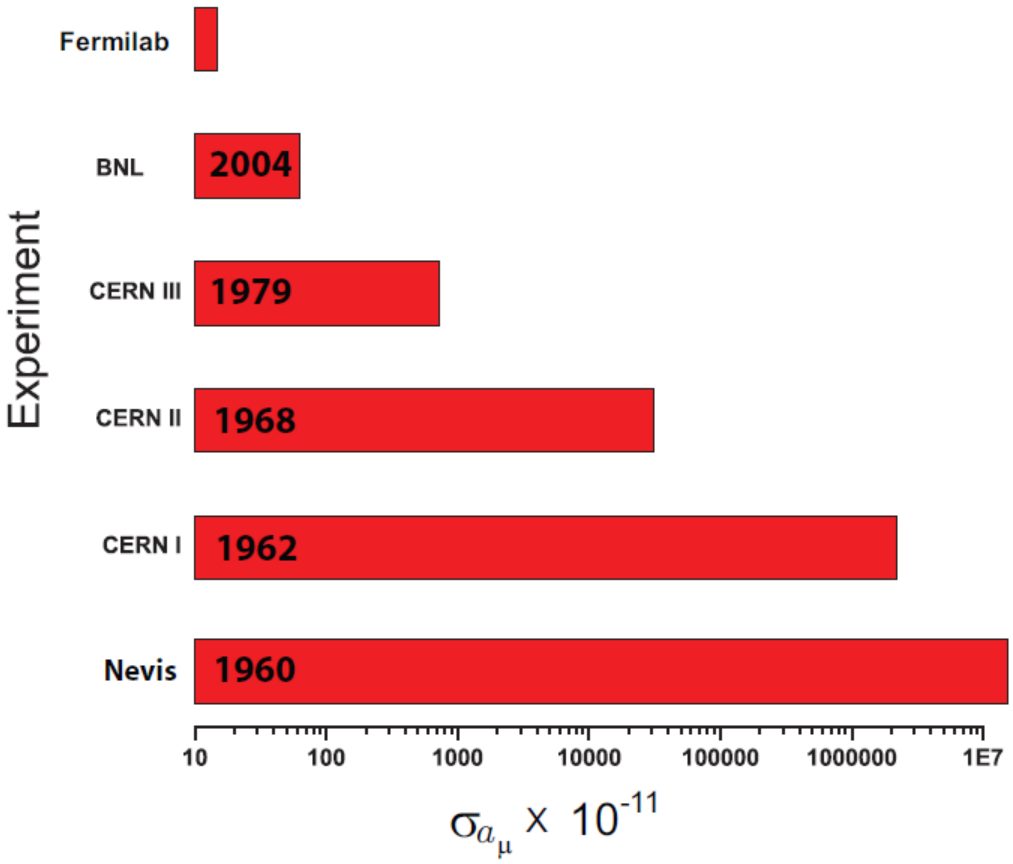
$$\vec{\mu}_\mu = g_\mu \frac{q}{2m}\vec{S}$$



Particle	g	
g_e	2.00231930436153	0.00000000000053
g_μ (Theory, SM)	2.0023318361	
g_μ (Measurement, BNL)	2.0023318414	0.0000000012

- The g-factor represents a ration between the magnetic moment and the angular momentum.
- For electron and muon, g ~ 2.
- The small difference is believed due to existence of heavy particles in vacuum.

New Fermilab g-2 Experiment



- The new Fermilab g-2 experiment will make more precise measurement.

Discovery of Argon

The mean numbers for the weights of gas contained in the globe used were as follows :—

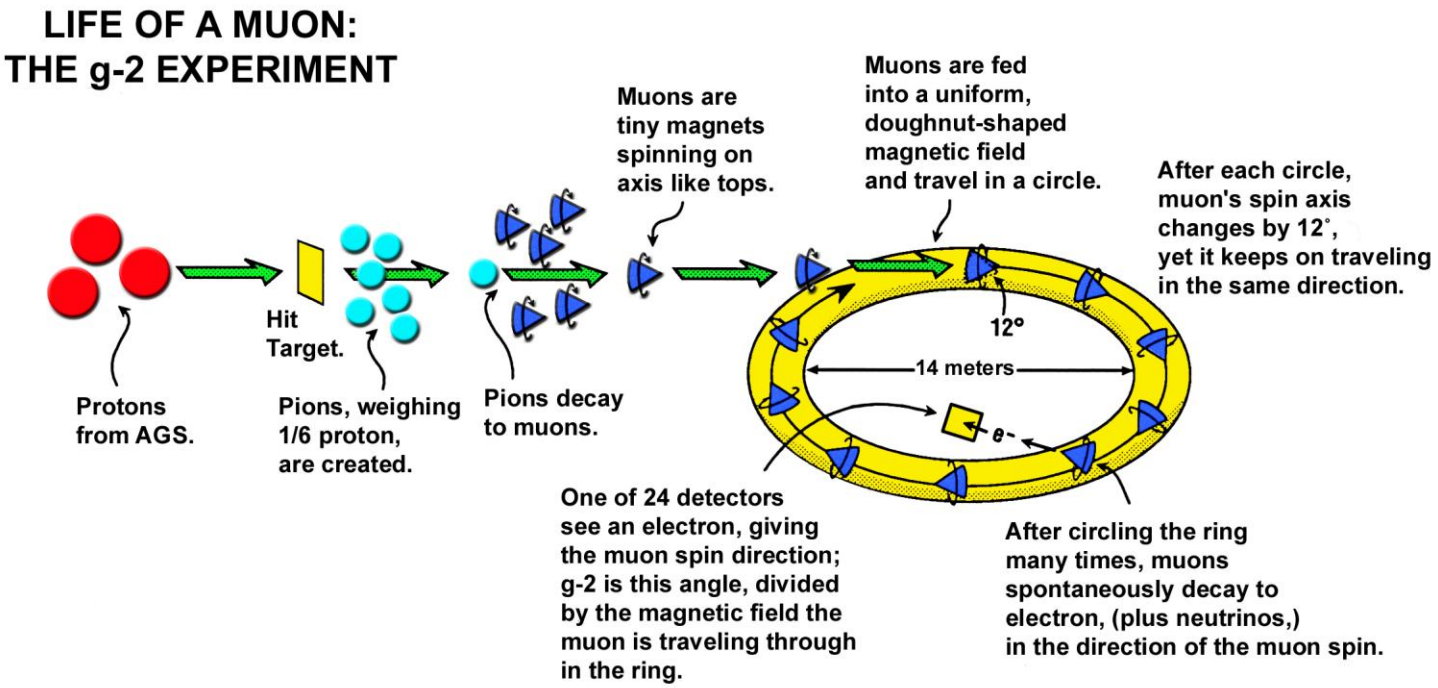
	Grams.
From nitric oxide	2·3001
From nitrous oxide.....	2·2990
From ammonium nitrite	2·2987

while for "atmospheric nitrogen" there was found—

By hot copper, 1892	2·3103
By hot iron, 1893	2·3100
By ferrous hydrate, 1894.....	2·3102

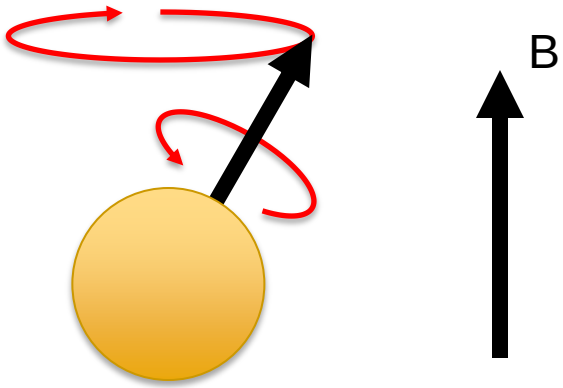
- Discovery of argon is an analogue of g-2 experiment.
- "Atmospheric nitrogen" is heavier than chemical nitrogen.
- The difference is due to existence of argon.
- Small difference in number brings large discovery.

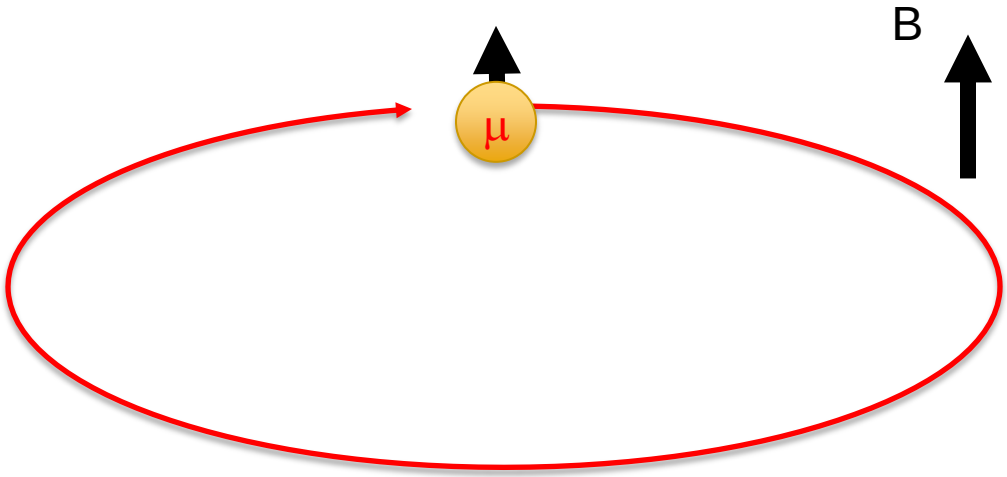
Generating Muons



- Accelerator generates proton beam to hit target producing many particles.
- Pions are selected from the secondary beam.
- The pion is further decayed into a muon (plus a neutrino) .

Flying Muons





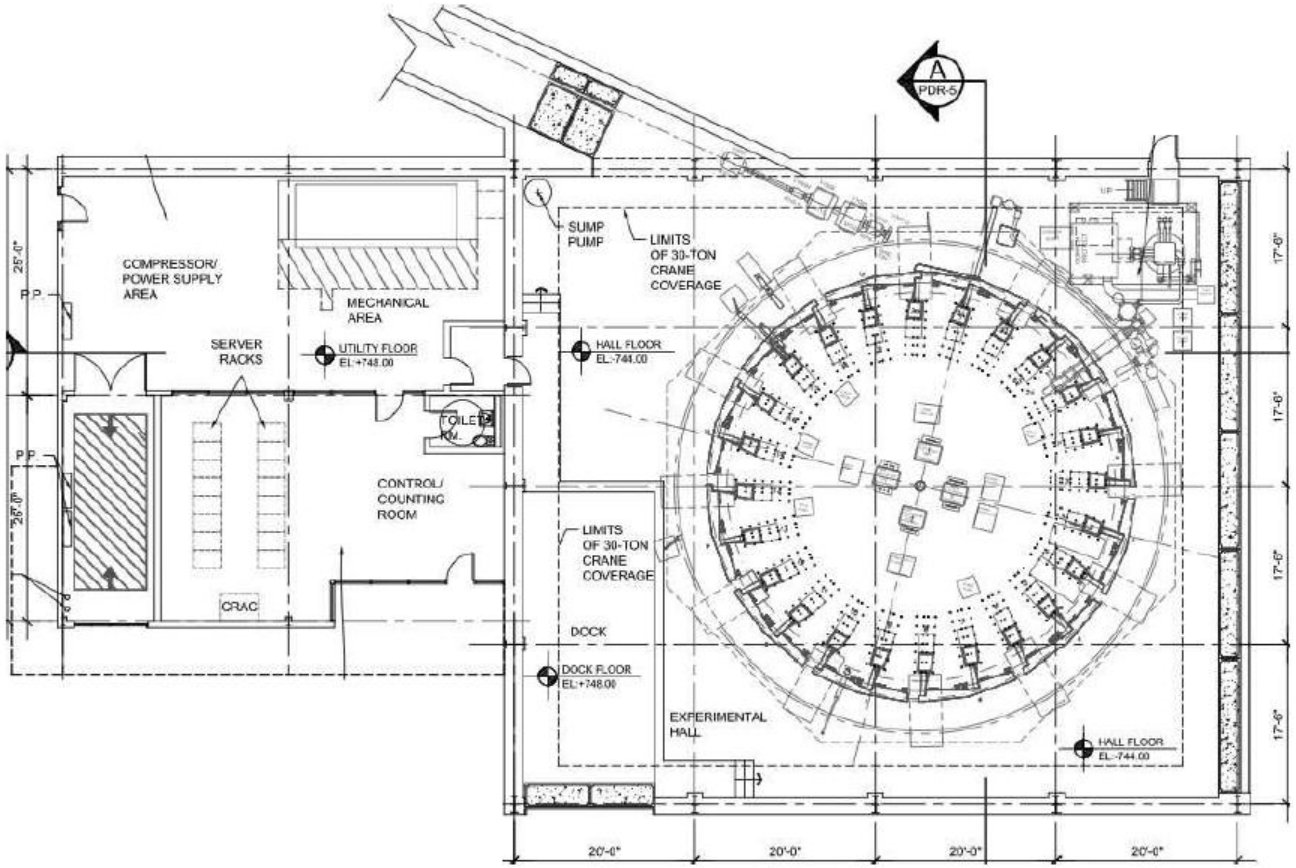
$$\omega_s = -g_\mu \frac{qB}{2m} - (1-\gamma)\frac{qB}{\gamma m}$$

$$\omega_c = -\frac{qB}{\gamma m}$$

$$\omega_s - \omega_c = -\left(\frac{g_\mu - 2}{2}\right)\frac{qB}{m}$$

- The muons from secondary beam fly at nearly speed of light.
- A ring of magnet is needed to store the mouns.
- The difference of precession frequency – cyclotron frequency is measured

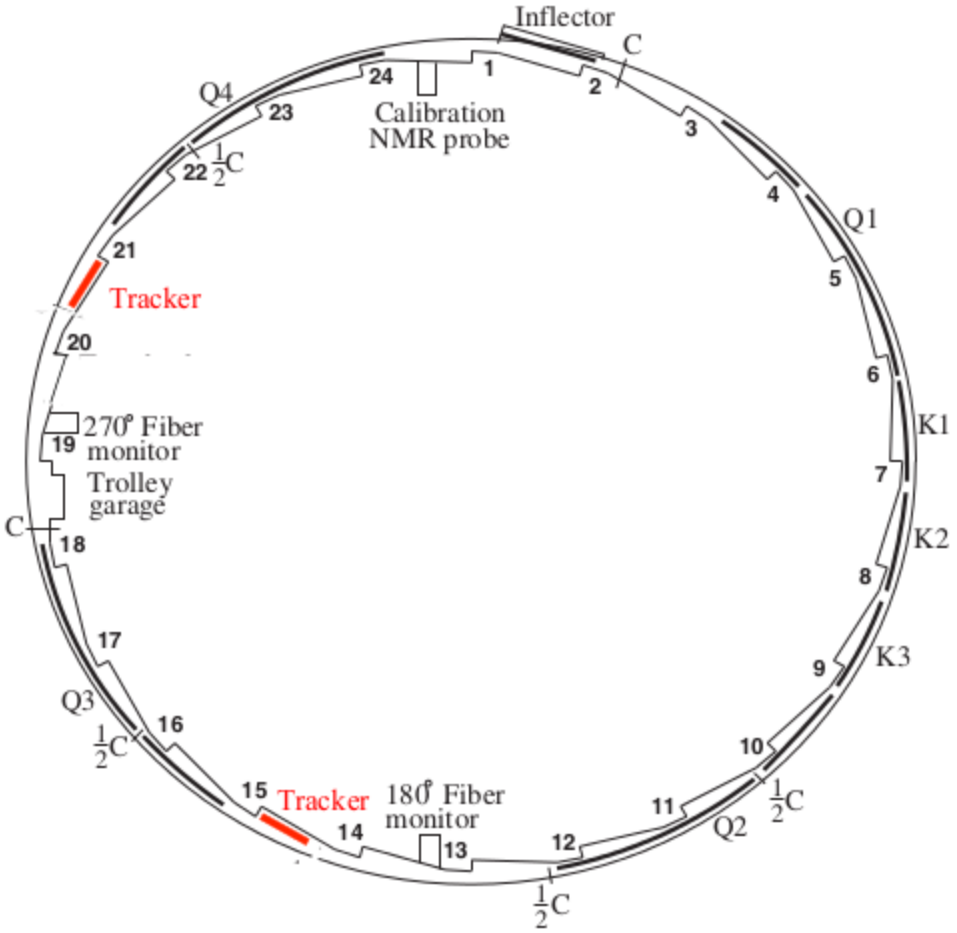
Experiment Layout



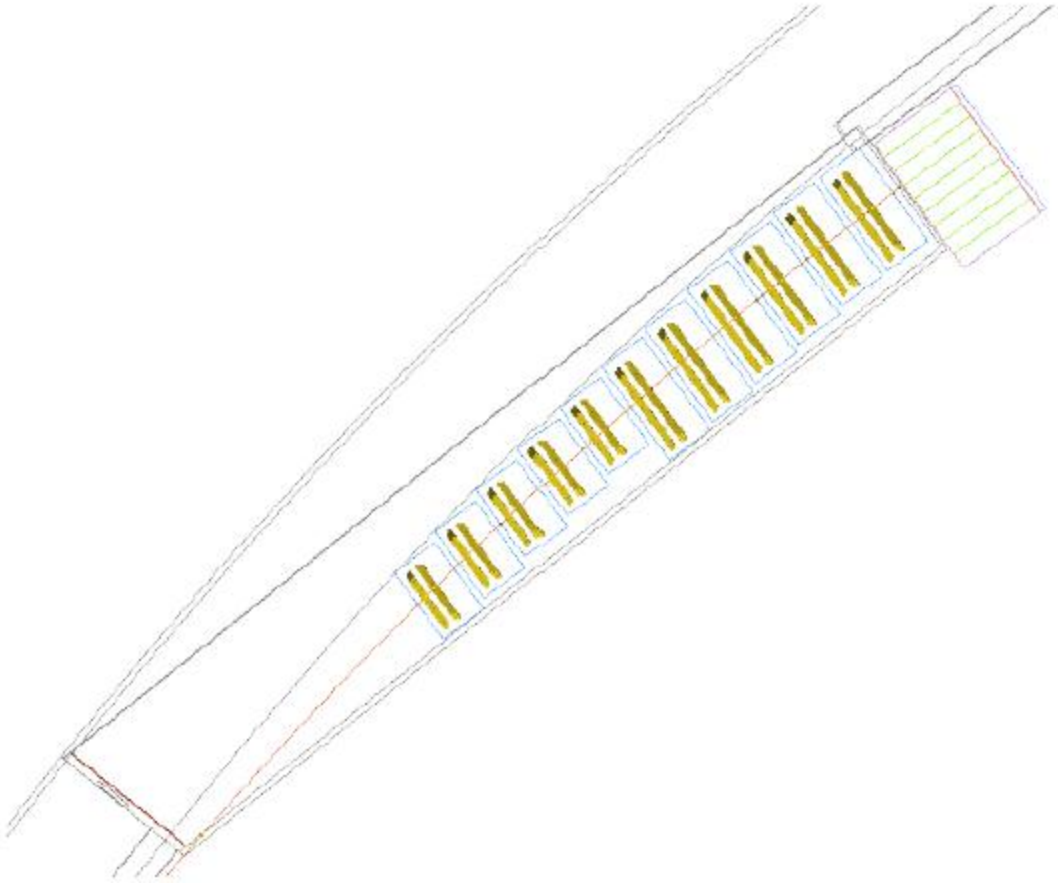
- The beam enters the storage ring.
- The beam is at a “magic” momentum of 3.094 GeV.

Detecting Muon Decays

- Muon decays into electron + neutrino.
- Various detectors are placed near the ring to detect the electron.

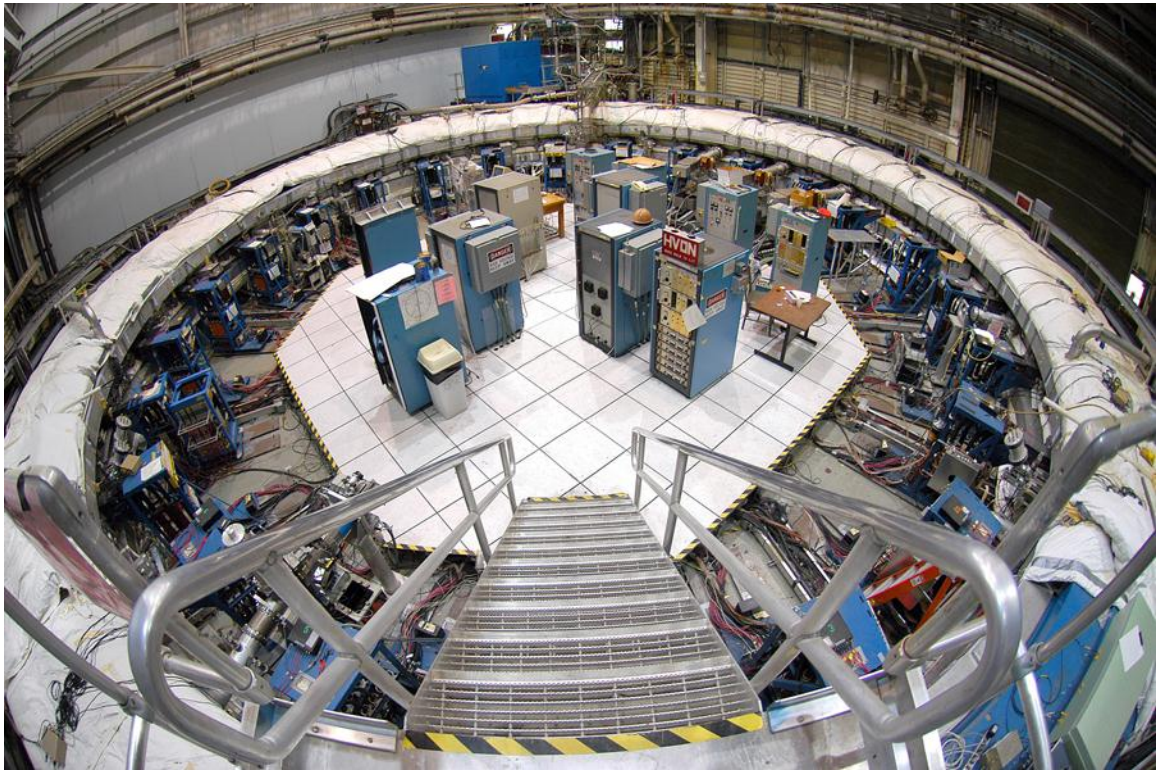


Electron in the Tracker



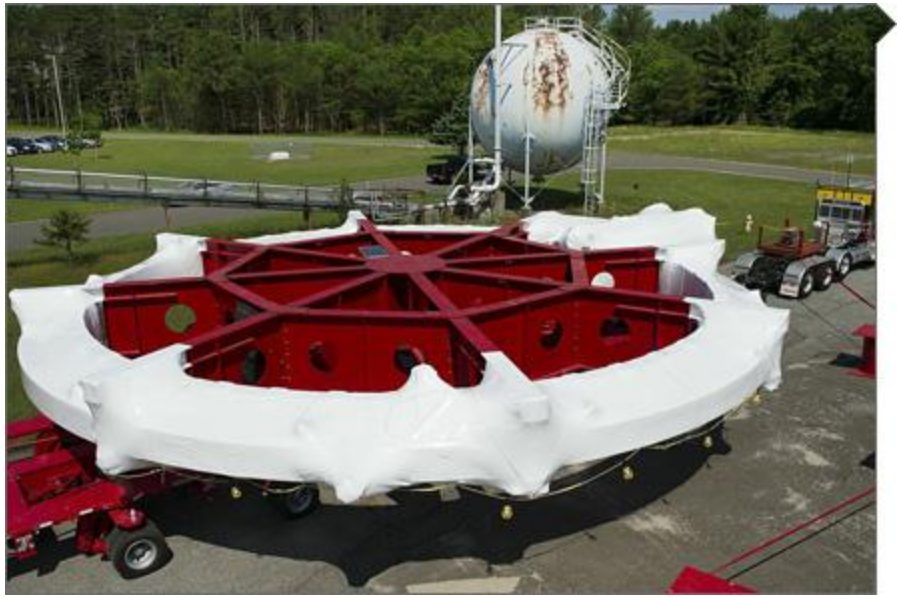
- The electron track parameters are measured.

The BNL g-2 Experiment



- The BNL g-2 was built ~20 years ago.
- The result was published in 2004.

The g-2 Magnet from BNL



- The requirement of the magnet precision is very high.
- It is cheaper to move it from BNL to FNAL than build a new one.
- The magnet was moved from BNL to FNAL in 2013.

The g-2 Magnet Arriving

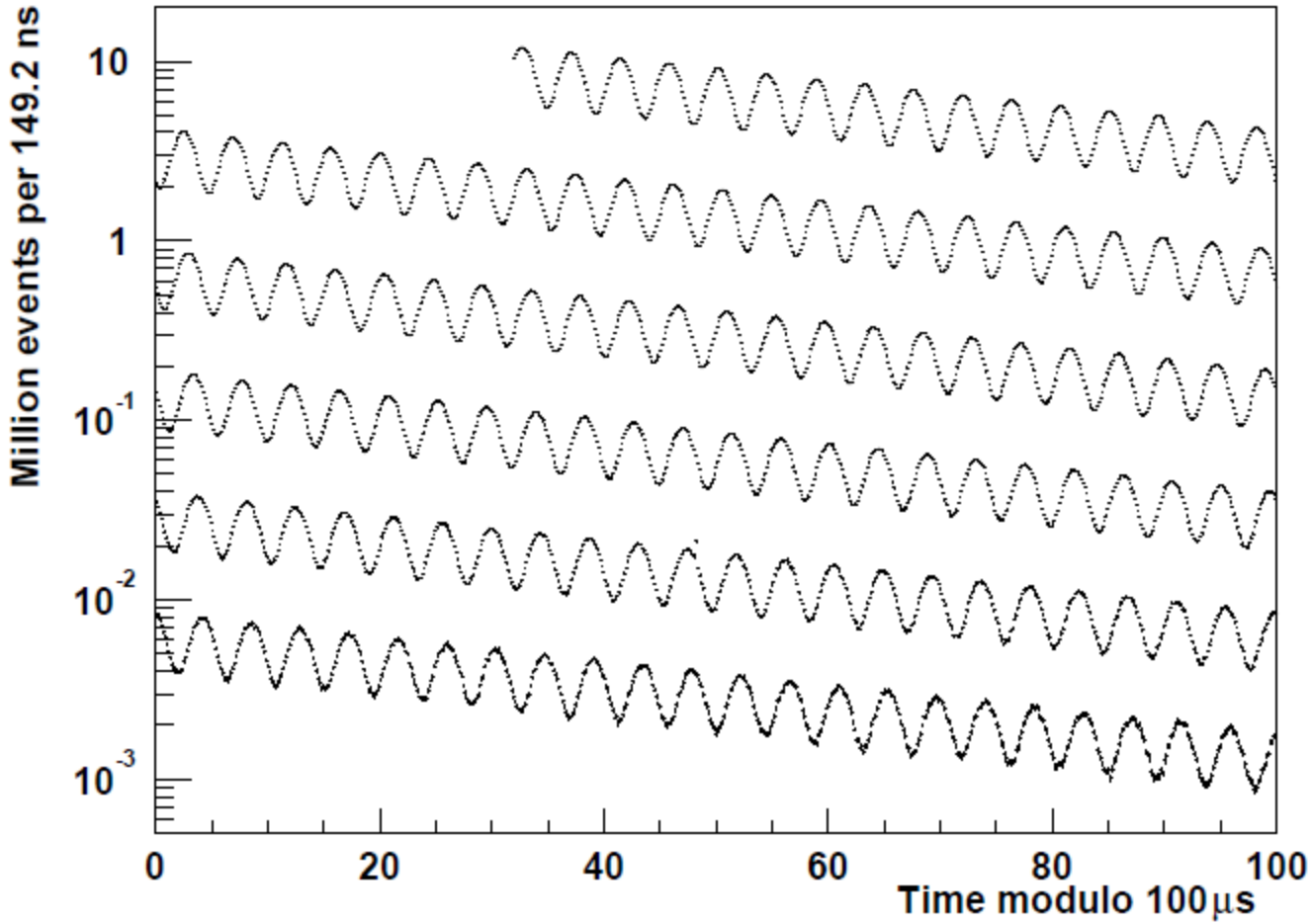
- The magnet arrived FNAL in the summer of 2013.

Final Location of the g-2 Magnet

- The magnet was moved to experiment hall in summer of 2014.

Collecting a Lot of Data

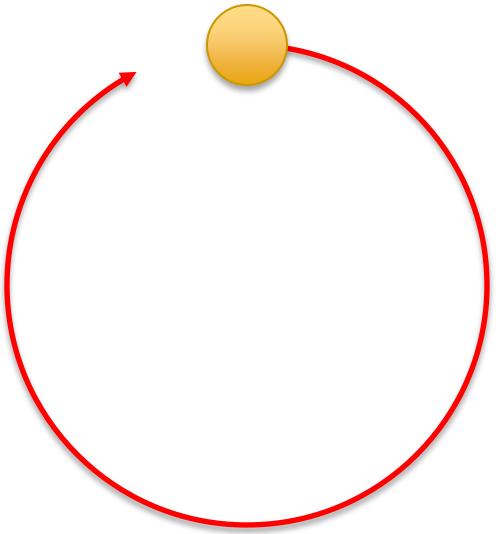
$$\omega_s - \omega_c = -\left(\frac{g_\mu - 2}{2}\right)\frac{qB}{m}$$

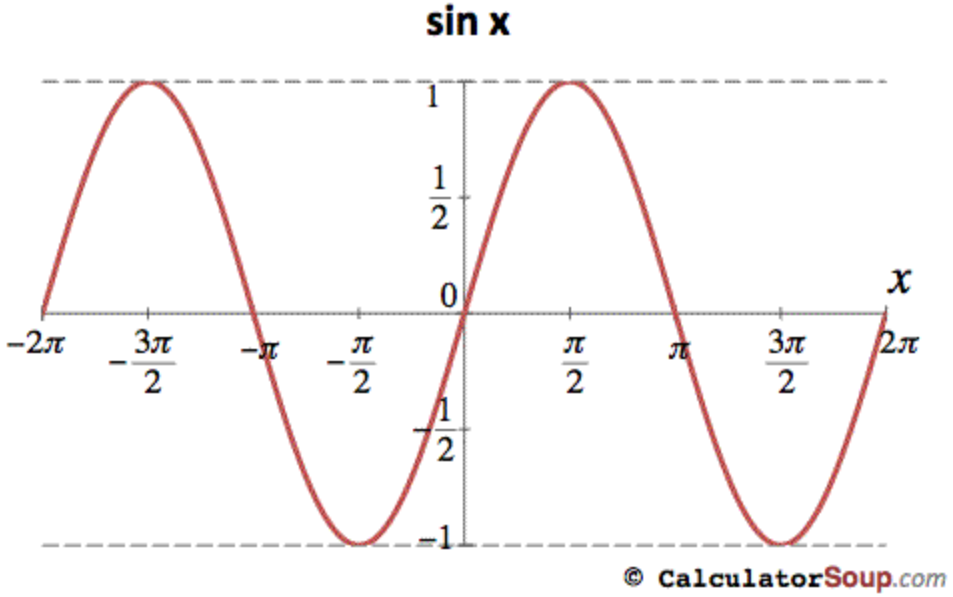


- Collect a lot of data and spend years to analyze the result.

Finale: Why Sine Function is Special

Sine Function

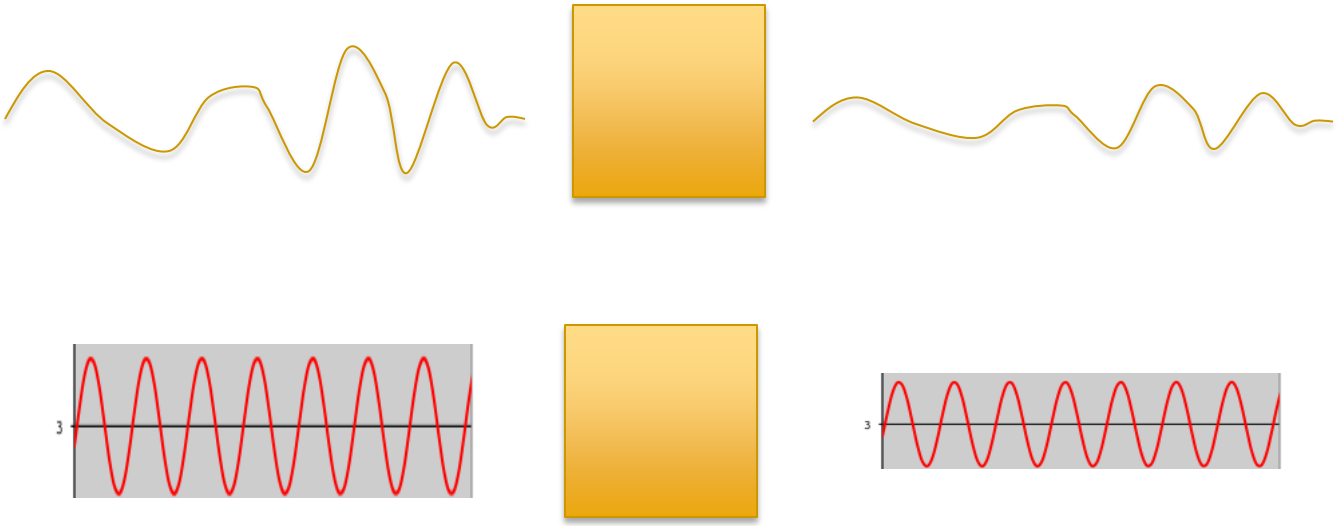




$$x(t) = \sin(\omega t)$$

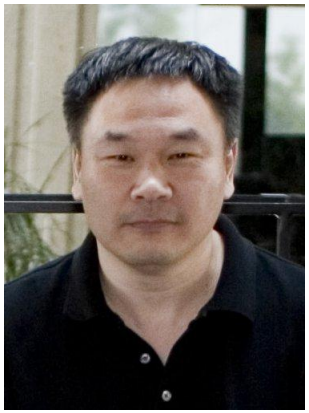
- The projection of the rotation is a sine (cosine) function.

Sine Function => Sine Function



- If an arbitrary wave is sent through a material, its shape will like be changed.
- If a sine wave is sent through a material, the output is still sine wave.

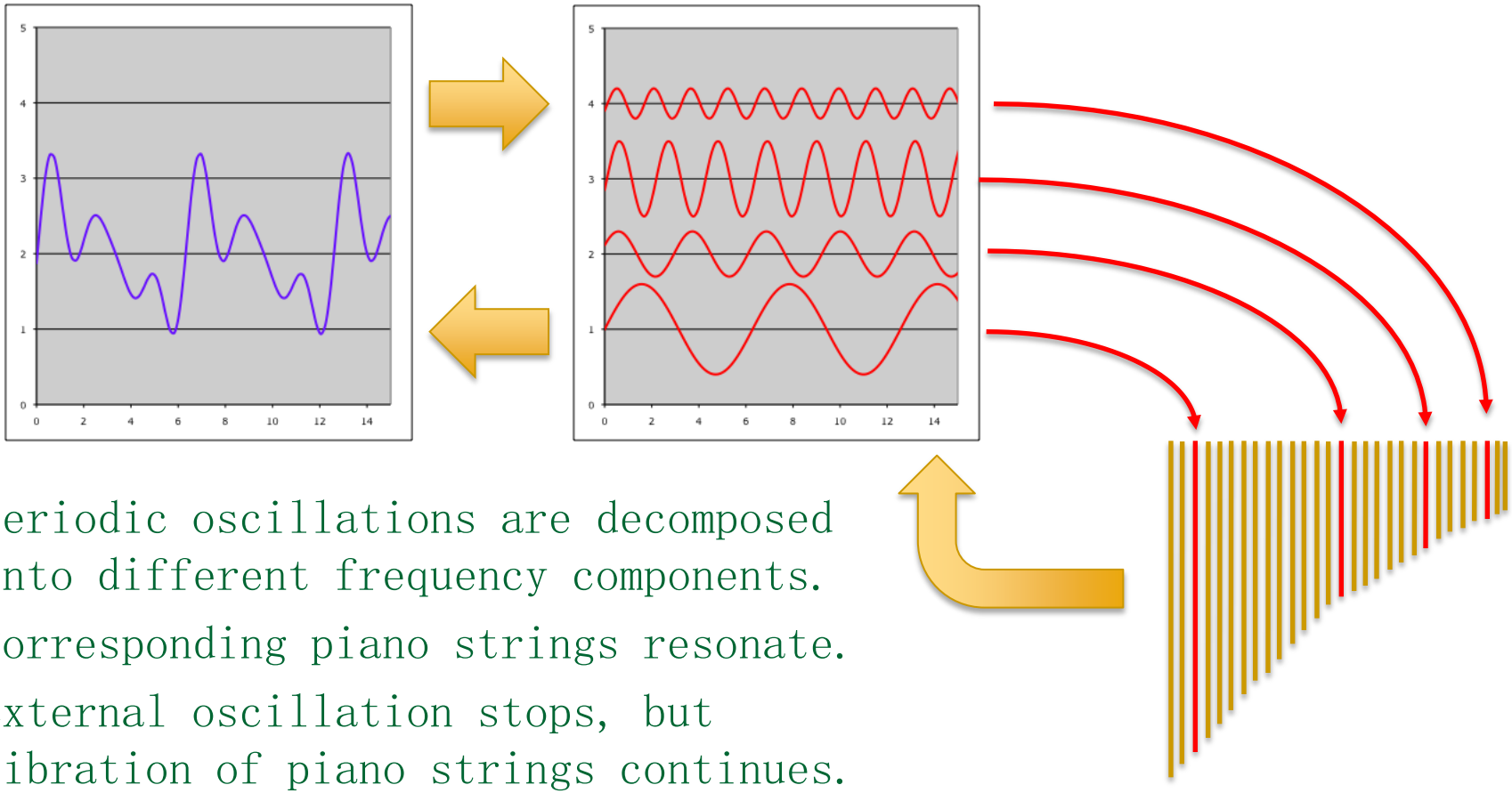
Demo: Reproducing Voice with a Piano



- All students come to stage.
- Howl a vowel into the piano.

Fourier Analysis

$$x(t) = a_0 + a_1 \sin(\omega t + \varphi_1) + a_2 \sin(2\omega t + \varphi_2) + a_3 \sin(3\omega t + \varphi_3) + \ldots$$



- Periodic oscillations are decomposed into different frequency components.
- Corresponding piano strings resonate.
- External oscillation stops, but vibration of piano strings continues.
- Human voice is thus synthesized.

Fourier Series

$$x(t) = a_0 + a_1 \sin(\omega t + \varphi_1) + a_2 \sin(2\omega t + \varphi_2) + a_3 \sin(3\omega t + \varphi_3) + \ldots$$

- Any periodic function can be expanded into a Fourier series.
- Mathematically, a periodic function can also be expanded into other series.
- Why do we choose the Fourier series?

Taylor Expansion

$$f = -kx + bx^2 + cx^3 + dx^4 + \ldots$$

- Using Taylor expansion, a function can be decomposed into linear term and non-linear terms.
- Is this decomposition artificial?

Coexistence of Fourier series and the Taylor expansion

$$f = -kx + bx^2 + cx^3 + dx^4 + \ldots$$

- The recovery force of real system may not be linear.
- But it can be arbitrarily close to linear when the amplitude is arbitrarily small.
- Linear recovery force causes simple harmonic oscillation:

$$f = -kx \quad m\frac{d^2x}{dt^2} + kx = 0 \quad x(t) = A_1\cos(\omega t) + B_1\sin(\omega t)$$

- The Fourier series is not merely a mathematic expression.
- There exist physical systems oscillating this way.

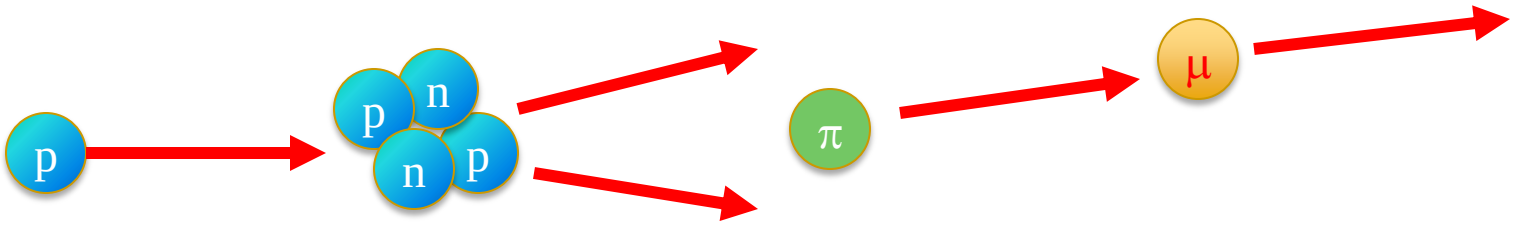
$$x(t) = a_0 + a_1\sin(\omega t + \varphi_1) + a_2\sin(2\omega t + \varphi_2) + a_3\sin(3\omega t + \varphi_3) + \ldots$$

Conclusion

- We have talked topics in various discipline: carpentry, medical imaging, elementary particle physics, etc.
- They are all connected.
- Your knowledge is not too far from researchers in these areas.

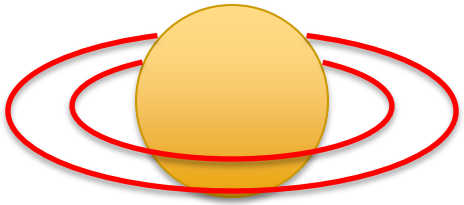

The End
Thanks

Generating Muons



- Accelerator generates proton beam to hit target producing many particles.
- Pions are selected from the secondary beam.
- The pion is further decayed into a muon (plus a neutrino) .

Repetitive Motions of many Things

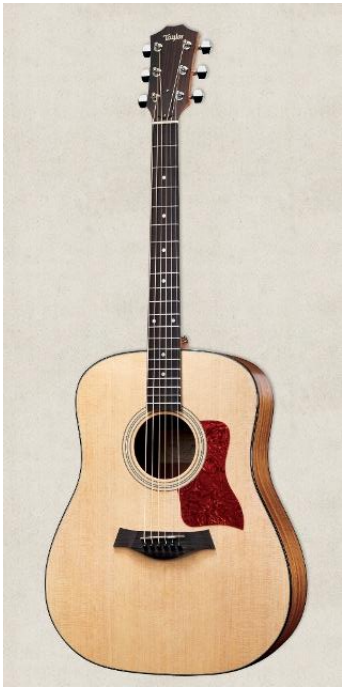


We live in a world with two type of motions:

- One pass motions.
- Repetitive motions.

Frequency vs. Length

- F = const. x (1/L)
  - Guitar
  - Violin
  - Organ
- $F = \text{const.} \times (1/L)^2$
  - Xylophone
- $F \sim \text{const.} \times (1/L)^0$
  - Rubber band string



Reflection of Waves